BOOK - II

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CONTROL SYSTEMS

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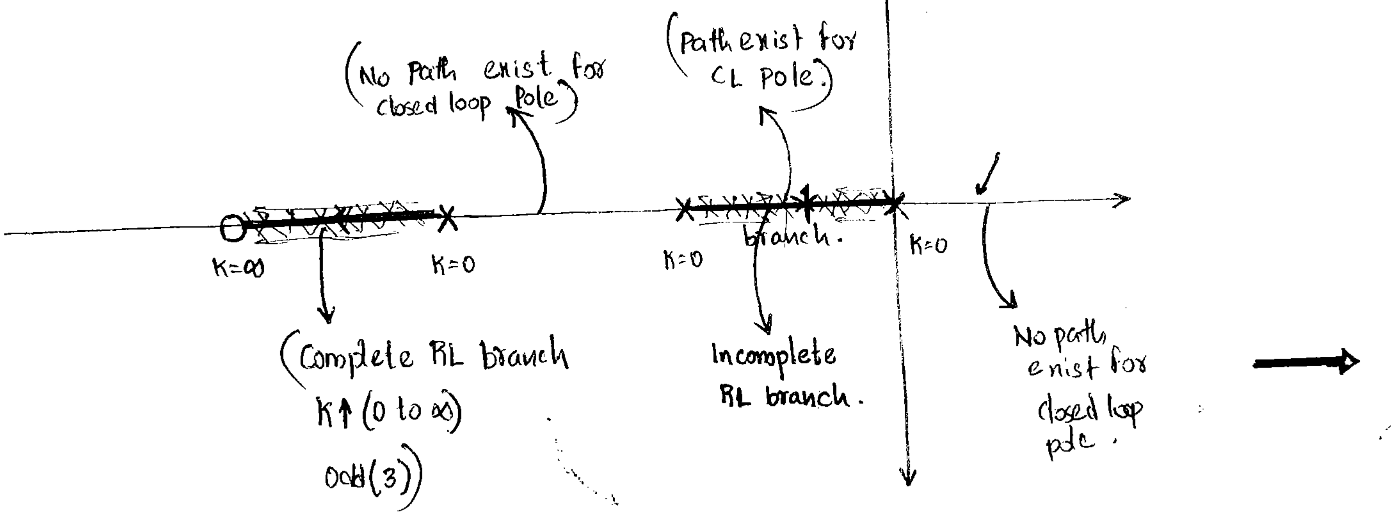
RULE 3 REAL AXIS~~$~~ LOCII

point considering.

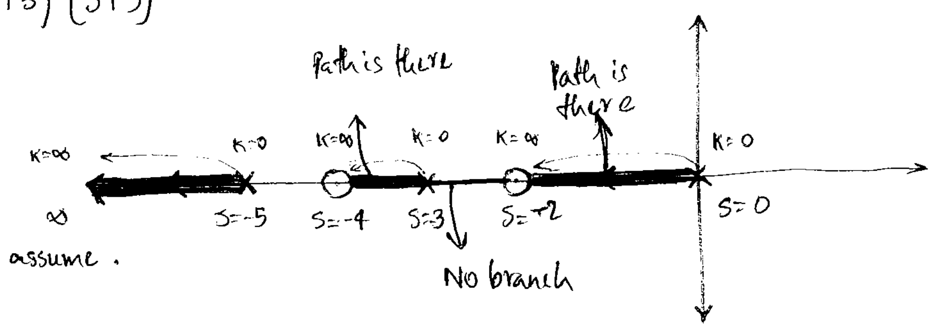


⇒ "A point exist on real axis root locus branches, the sum of the poles and zeros to the right hand side ~~side~~ of that point should be odd."

⇒ "The poles must goes only on root locus branches.

Once the pole reaches the zero (ie when K=0 to K=∞) then it becomes the complete root locus branch for that particular pole".

Q. Identify the sections of real axis which belongs to Root Locu[s] for $G(s)H(s) = \dfrac{K(s+2)(s+4)}{s(s+3)(s+5)}$



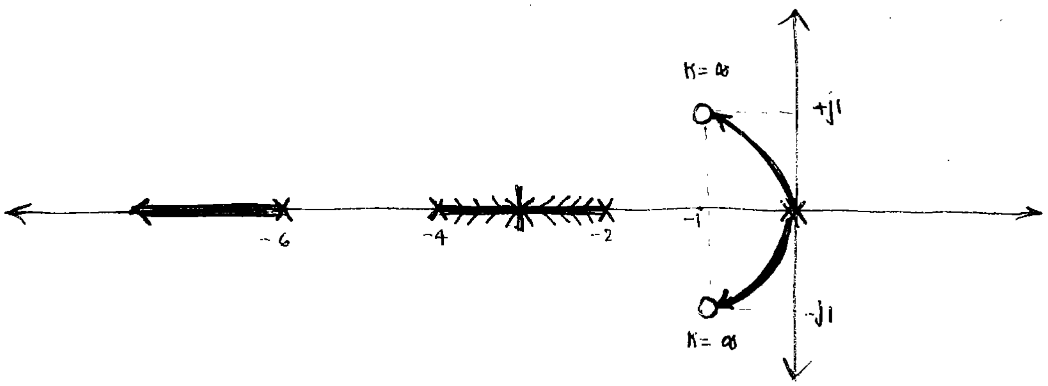
Complete Root Locus Diagram → K↑(0 to ∞)

Q. Identify the following points which are on the Root locus branch.

for $G(s)H(s) = \dfrac{K(s^2+2s+2)}{s^2(s+2)(s+4)(s+6)}$

(1) $s=0$ (2) $s=-1$ (3) $s=-2$ (4) $s=-3$ (5) $s=-4$ (6) $s=-5$

(7) $s=-6$ (8) $s=-\infty$ (9) $s=-1+j1$ (10) $s=-1-j1$



→ All the pole and zero position, there must be a Root locus branch.

$\dfrac{-2 \pm \sqrt{4-8}}{2}$

$-1 \pm j1$

So, $s=0$, $s=-2$, $s=-4$, $s=-6$, $s=-1+j1$ and $s=-1-j1$ are on Root branches for sure.

for remaining points, go for Real axis or Angle condition.

Note:

At the position of each and every pole and zero location, there must be a root locus branch because the root locus branch start at poles end at zeros.

RULE 4 ASYMPTOTES

→ The branches which are approached to the infinity is called the assymptote.

→ The number of assymptote $\boxed{N = P - Z}$

→ The angle of asymptote $\boxed{\theta = \frac{(2q+1)180^\circ}{P-Z}}$ where $q = 0, 1, 2, \ldots (P-Z-1)$

→ The asymptotes are symmetrical about the real axis.

Note: The asymptotes gives the direction of zeros when poles are greater than zeros only.

RULE 5 CENTROID (σ)

Centroid is the intersection point of asymptote on the real axis.

$$\boxed{\text{Centroid} = \sigma = \frac{\sum(\text{Real part of Poles}) - \sum(\text{Real part of Zeros})}{(P-Z)}}$$

→ The centroid may be located anywhere on the real axis. It may or may not be on root locus branch.

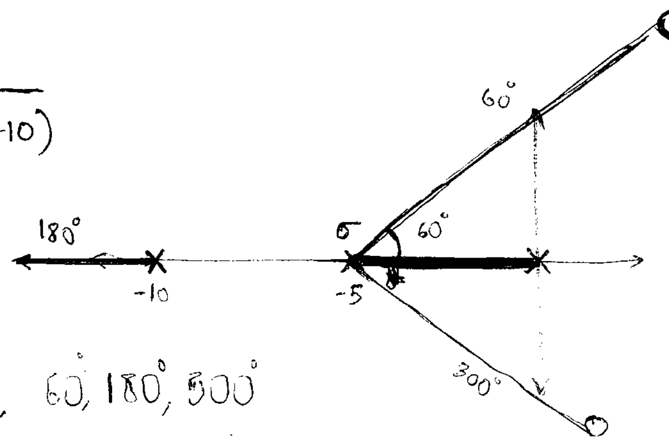
Q Find the angle of asymptote and centroid to the following system

$$G(s)H(s) = \frac{K}{s(s+5)(s+10)}$$

$P = 3$, $Z = 0$ $P - Z = 3$

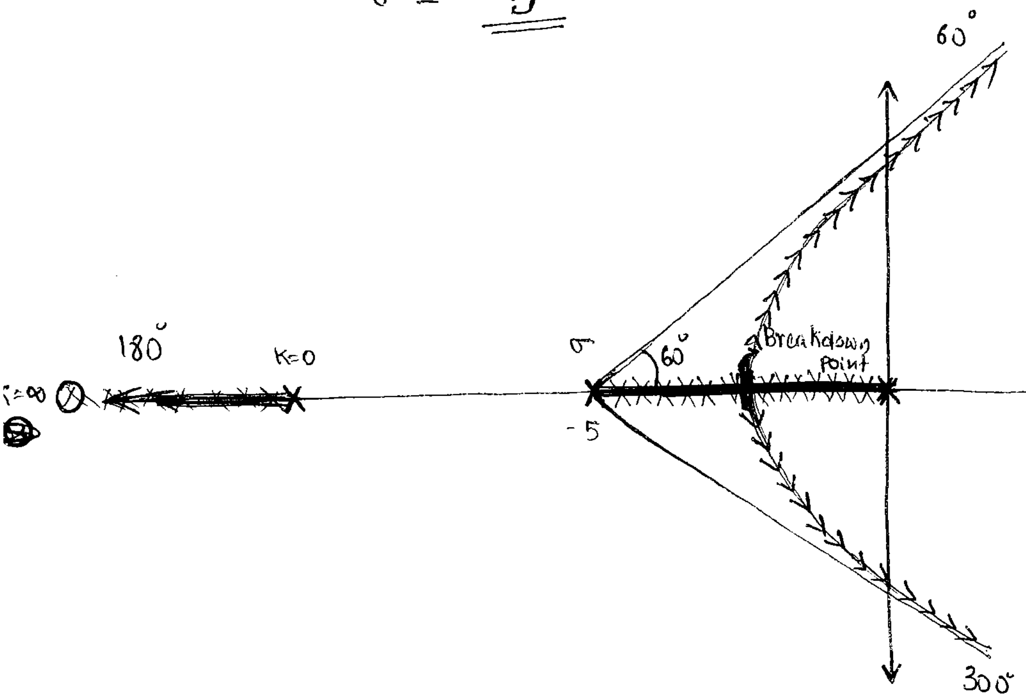
$$\theta = \frac{(2q+1)180^\circ}{3} = (2q+1)\,60^\circ$$

$60^\circ, 180^\circ, 300^\circ$



$$\text{Centroid } \sigma = \frac{-15-0}{3}$$

$$\sigma = \underline{\underline{-5}}$$



Individual pole real axis cannot leave alone. So move up to breakpoint and leaves with angle $\frac{180^\circ}{n}$

where n is the number of ~~roots~~ poles coming to break point.

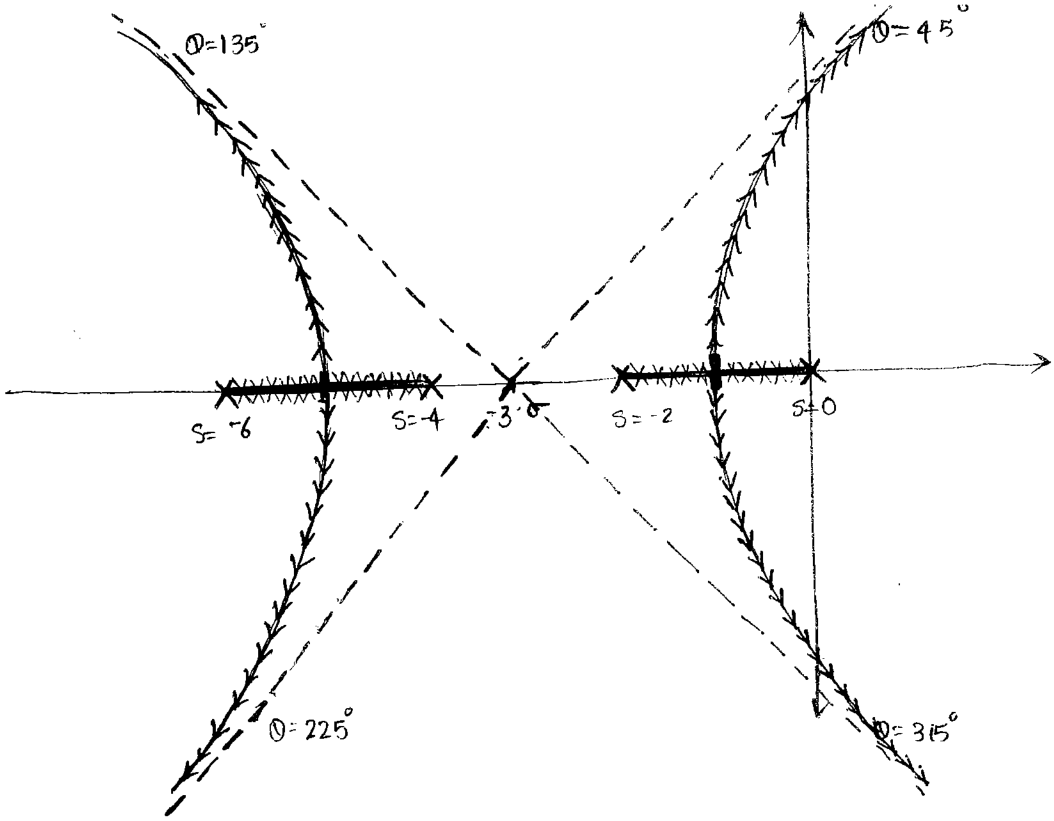
Q
$$G(s)H(s) = \frac{K}{s(s+2)(s+4)(s+6)}$$

$$\theta = \frac{(2q+1)\,180^\circ}{4}$$

$$= (2q+1)\,45^\circ$$

$$= 45^\circ, 135^\circ, 225^\circ, 315^\circ$$

$$\text{Centroid } \sigma = \frac{-12-0}{4} = \underline{\underline{-3}}$$



RULE 6 BREAKPOINT

The point at which two or more poles meet or directly locate at any location, then it is called the Breakpoint.

BREAK AWAY POINT :

The point at which, the root locus branches ~~are~~ or poles leaves the real axis is called the breakaway point.

BREAKIN POINT

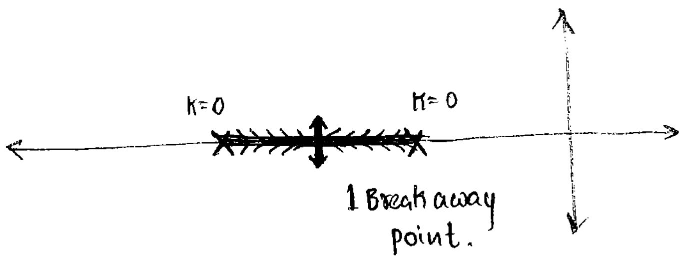
The point at which, the root locus branches enter into the real axis is called Breakin point.

The root locus branches enter or leaves the real axis with an angle of $\boxed{\frac{\pm 180^\circ}{n}}$ where n is number of poles at the breakpoint.

FINDING THE EXISTANCE OF BREAKPOINT

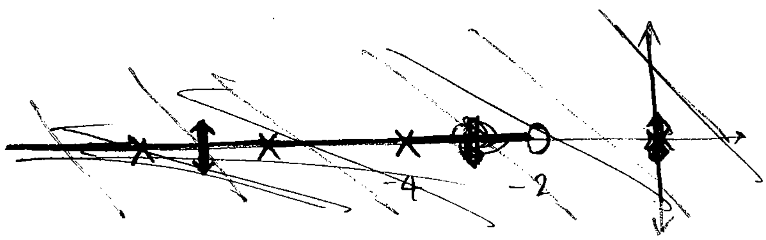
Case (i)

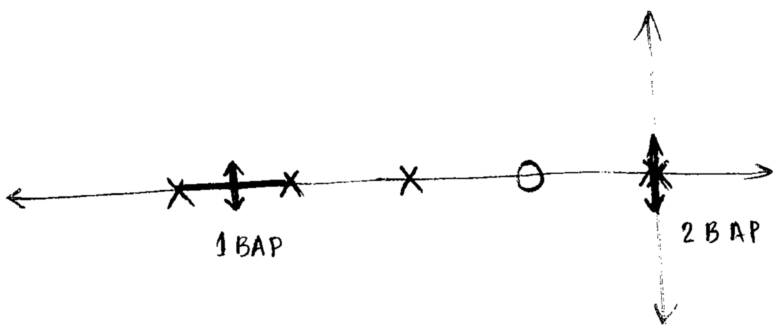
whenever there exist a ~~leftmost side~~ two adjescently placed poles in b/w there exist a root locus branch, then there should be minimum One breakaway point in b/w adjescently placed poles.



Q. Find the number of breakpoints

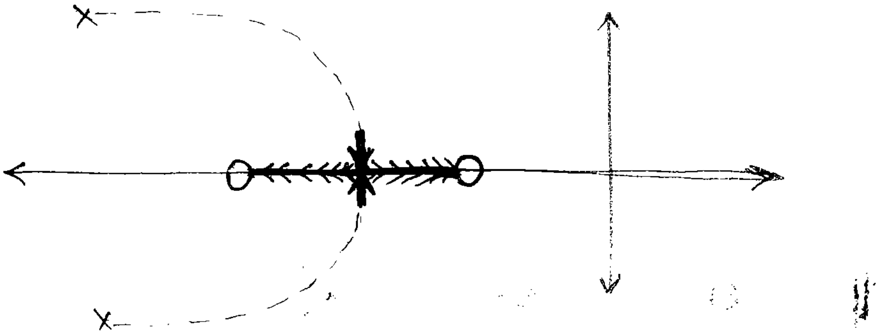
$$G(s)H(s) = \frac{K(s+2)}{s^2(s+4)(s+6)(s+8)}$$





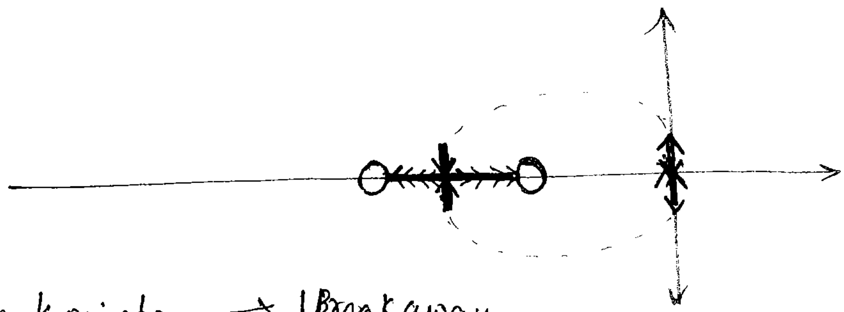
Case (ii)

whenever there exist the two adjescently placed zeros, ~~in~~ in b/w there exist a R.L branch, then there should be ~~one~~ minimum one breaking point in b/w the adjescently placed zeros.



Q. Find no of breakpoints.

$$G(s)H(s) = \frac{K(s+2)(s+4)}{s^2}$$

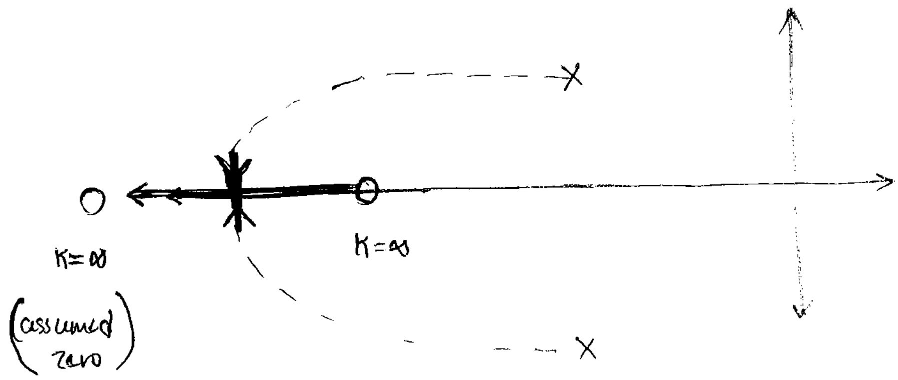


2 Break points → 1 Breakaway, 1 Break in.

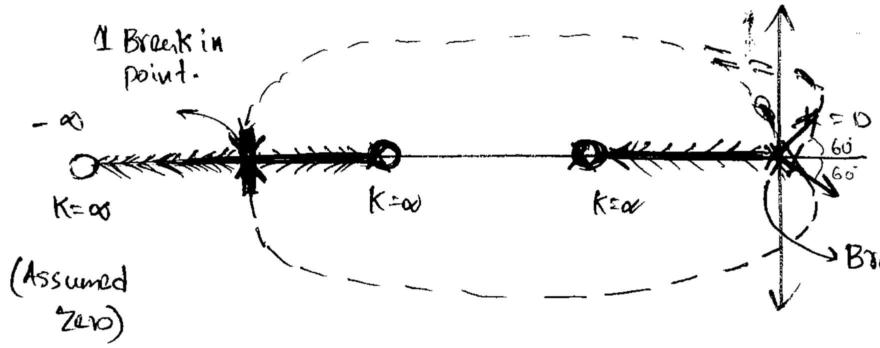
Case (iii)

whenever there exist a left most side zero, to the leftmost side of that zero, there exist a Root Locus branch, then there should be a minimum one 1 breakin point when the number of poles are greater than zeros only.

Condition: Poles > Zeros



Q $G(s)H(s) = \frac{K(s+2)(s+4)}{s^3}$

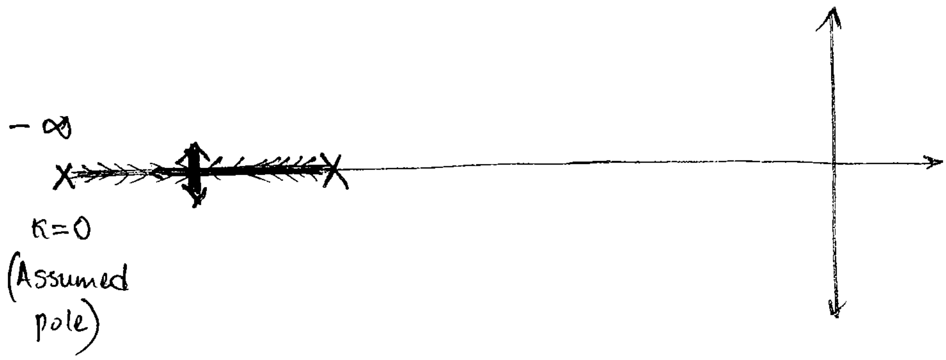


$\frac{180}{3} = 60^\circ$

Case (iv)

whenever there exist the leftmost side pole, to the leftmost side of that pole, there exist the root locus branch, then there should be the minimum one breakaway point to the ~~bo~~ left most side of that pole, when poles are less than zeros only.

Condition: Zeros > poles.



⇒ practically the above case does not exist, Because

poles > Zeros ⇒ strictly proper T.F

poles ≥ Zeros → proper T.F

poles < zeros ⇒ Improper T.F.

⇒ we use only strictly proper or proper T.F usually.

PROCEDURE TO FIND THE LOCATION OF BREAKPOINT

S1 : Form the characteristic Equ.

S2 : Rearrange the above equ in the form $K = f(s)$

S3 : Differentiate K w.r.t s and equate to zero.

ie, $\frac{dK}{ds} = 0$

CONCLU 1 → The roots of $\frac{dK}{ds} = 0$ gives the valid and invalid breakpoints.

CONCLU 2 → The valid breakpoint is the one, it must be on the root locus branch, OR for valid breakpoint, the K value should be positive.

Q. Find the location of breakpoints:

(i) $G(s)H(s) = \frac{K}{s(s+2)}$

(ii) $G(s)H(s) = \frac{K}{s(s+2)(s+4)}$

(iii) $G(s)H(s) = \frac{K(s+4)}{s(s+2)}$

(i) $G(s)H(s) = \frac{K}{s(s+2)}$

CE, ⇒ $s^2 + 2s + K = 0$

$K = -s^2 - 2s$

$\frac{dK}{ds} = -2s - 2 = 0$

$-2s = 2$

$s = -1$

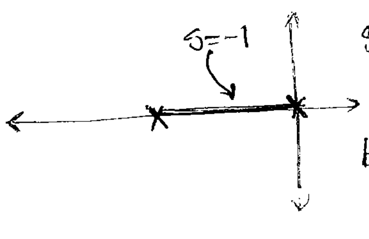
Short cut.

For only pole case take directly openloop pole equ instead of C.E

$s^2 + 2s = 0$

$2s + 2 = 0$

$s = -1$



$s = -1$ on root locus.

Hence Valid Breakpoint.

(ii) CE ⇒

$$\cancel{s^2}\, s(s+2)(s+4)+K=0$$

$$s^3+6s^2+8s+K=0$$

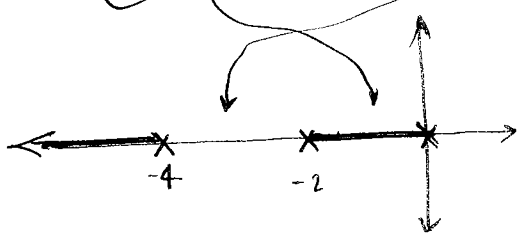
d $\cancel{3s^2+12s+8=0}$

$$K=-s^3-6s^2-8s$$

$$=-3s^2-12s-8$$

$\cancel{K=}$

$s=-0.84\,,\ -3.15$ ✗



(iii) CE ⇒

$$s(s+2)+K(s+4)=0$$

$$s^2+(2+K)s+8=0$$

$\cancel{(2+K)s=-s^2-8}$

$\cancel{Ks\ s=\frac{-s^2-8}{2+K}}$

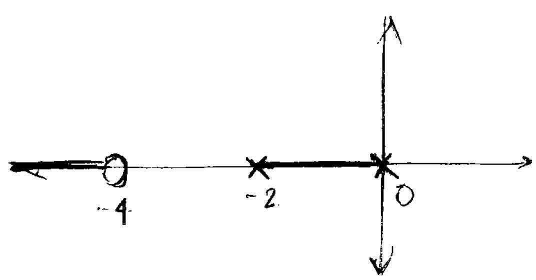
$$K=\frac{-s^2-2s}{s+4}$$

$$\frac{dK}{ds}=\frac{(s+4)(-2s-2)+(s^2+2s)}{(s+4)^2}=0$$

$$-2s^2-8s-\cancel{2s}-8+s^2+\cancel{2s}=0$$

$$-s^2-8s-8=0$$

$\cancel{-1.17}\quad \cancel{-6.83}$



RULE 7 INTERSECTION POINT WITH IMAGINARY AXIS

The intersection point with imaginary axis obtained by RH criteria,

S1 : Form the characteristic equation.

S2 : write the Routh table (Array)

S3 : Find $K_{marginal}$.

S4 : Form the auxillary equation. The roots of auxillary equation gives the valid and invalid intersection point with imaginary axis.

→ For valid intersection point, $K_{marginal}$ is +ve,

Q Find the intersection point with imaginary axis. For

$$G(s)H(s) = \frac{K}{s(s+2)(s+4)}$$

CE ⇒

$$s^3 + 6s^2 + 8s + K = 0$$

Internal > External.

$$K_{Marginal} = 48$$

s^3	1	8
s^2	6	K
s^1	$\frac{48-K}{6}$	0
s^0	K	

$$A.E \Rightarrow 6s^2 + 48 = 0$$

$$s^2 + 8 = 0$$

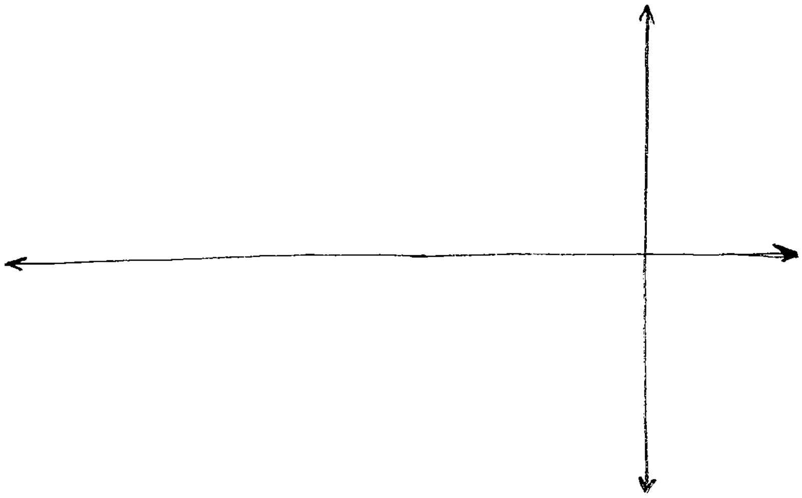
$$s = \pm j\sqrt{8}$$

It is valid because for this K_{mar} value is positive.

RULE 8 ANGLE OF DEPARTURE AND ARRIVAL

→ The angle of departure is ~~at~~ calculated at ~~the~~ a complex conjugate poles.

→ The angle of arrival is calculated at a complex conjugate Zeros.



Angle of departure: It gives the angle at which the pole depart or leaves from the initial position.

Angle of departure $\boxed{\phi_d = 180 + \angle GH \Big|_{\text{at a +ve Imag Complex pole.}}}$

$$\boxed{\phi_d = 180^\circ - \phi}$$

where $\boxed{\phi = \sum \phi_p - \sum \phi_z}$

Angle of Arrival: It gives the angle at which the pole arrives or terminates or end near the complex Zero.

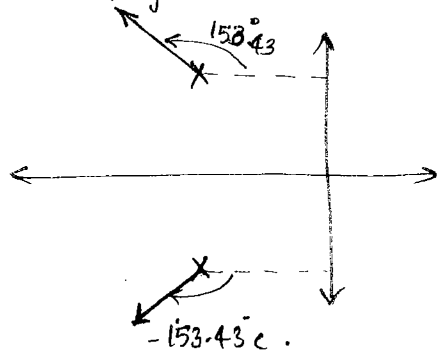
$$\boxed{\phi_a = 180 - \angle GH \Big|_{\text{at a +ve Imag Complex Zero}}}$$

$$\boxed{\phi_a = 180 + \phi}$$

where $\boxed{\phi = \left(\sum \phi_p - \sum \phi_z\right)}$

Q Find the angle of departure at a complex conjugate poles for

$$G(s)H(s) = \frac{K(s+2)(s+4)}{(s^2+2s+2)}$$



~~$\angle GH$~~ $-1 \pm j1$

$$\angle G(s)H(s) = \frac{K(-1 \pm j1 + 2)(-1 \pm j1 + 4)}{\left((-1+j1)^2 + 2(-1+j1) + 2\right)}$$

$$= \frac{K(1+j1)(3+j1)}{1 - \cancel{2j} - 1 \oplus -2 + \cancel{2j} + 2}$$

$$= 0^\circ + 45^\circ + 18.43^\circ - 0 - 90^\circ$$

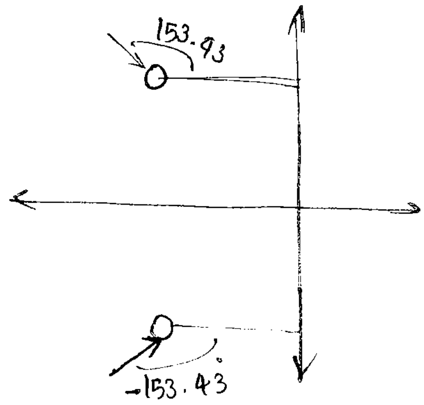
$$= \underline{\underline{-26.57^\circ}}$$

$$\phi_d = 180 + (-27.57^\circ)$$

$$\underline{\underline{\phi_d = 153.43^\circ}}$$

Q, Calculate the angle of arrival at a complex conjugate zero.

$$G(s)H(s) = \frac{K(s^2+2s+2)}{(s+2)(s+4)}$$



$$= \frac{K(s+1+j)(s+1-j)}{(s+2)(s+4)}$$

$$= \frac{K\left((-1+j+1+j)(-1+j+1-j)\right)}{(1+j)(3+j)}$$

$$= \frac{\angle K \ \cancel{\angle 0} \cdot \angle 0 \ \angle 2}{\angle 1+j \ \ \angle 1+3j}$$

$$= 0^\circ + \cancel{8}90 + 0 - 45^\circ - 18.43$$

$$= \underline{26.57^\circ}$$

$$\phi_a = 180 - 26.57^\circ$$

$$= \underline{153.43^\circ}$$

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ie, whenever all the poles and zeros are interchanged

⇒ then angle of departure = angle of arrival.

⇒ Breakin point = Break away point.

⇒ The shape of Root Locus diagram also same except the direction.

H.W Calculate the BA breakawaypoints, Angle of departure (ϕ_d) and Angle of arrival (ϕ_a)

(1) $GH = \dfrac{K}{s(s+2)}$

(2) $GH = \dfrac{K}{s(s^2+2s+2)}$

(3) $GH = \dfrac{Ks}{s^2+4}$

(4) $GH = \dfrac{K}{s^4-1}$

(5) $GH = \dfrac{K(s+2)(s+4)}{(s^2+2s+2)}$

(6) $GH = \dfrac{K(s^2+2s+2)}{(s+2)(s+4)}$

(7) $GH = \dfrac{K(s^2-2s+2)}{(s^2+2s+2)}$

(8) $GH = \dfrac{K}{s}$, $GH = \dfrac{K}{s^2}$, $GH = \dfrac{K}{s^3}$, $GH = \dfrac{K}{s^4}$

(9) $GH = \dfrac{K}{s(s+1)^2(s+2)}$

(10) $GH = \dfrac{K(s+1)^2}{s(s+2)}$

(11) $GH = \dfrac{K}{s(s^2+2s+2)(s+4)}$

(12) $GH = \dfrac{K}{s(s^2+2s+2)(s+1)}$

(13) $GH = \dfrac{K}{s(s^2+2s+2)(s+2)}$

(14) $GH = \dfrac{K}{s(s^2+2s+1.25)(s+2)}$

(15) $GH = \dfrac{K}{s(s^2+2s+10)(s+2)}$

(16) $GH = \dfrac{K(s+1)}{s^2(s+K_1)}$

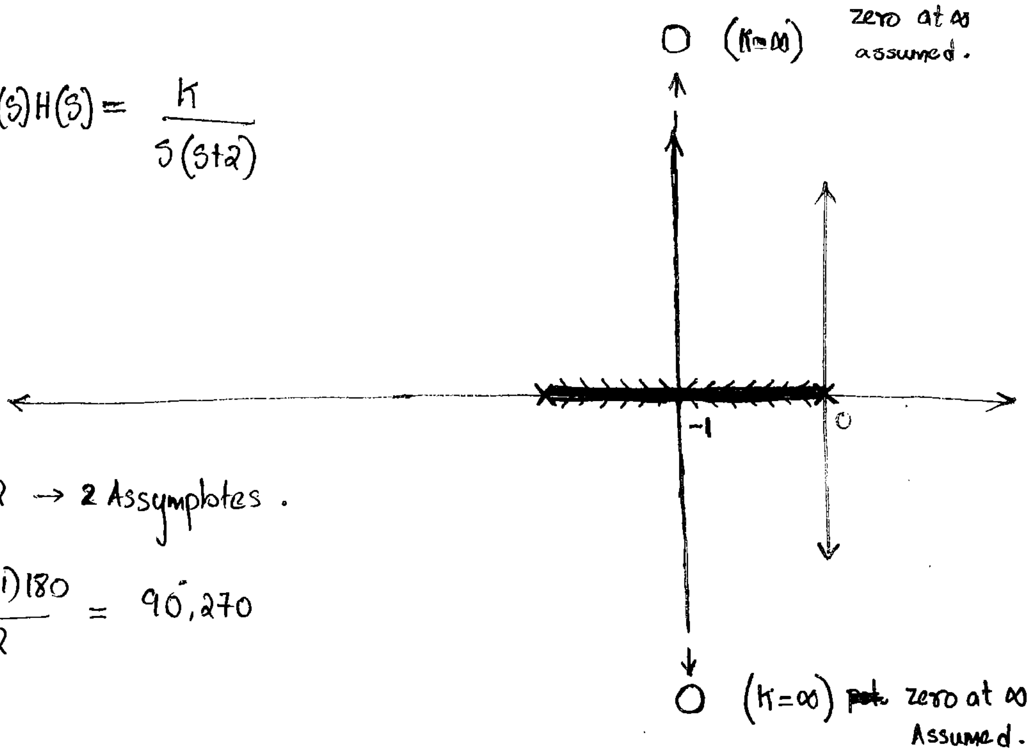
(i) $K_1 = 20$ (3 BP's)

(ii) $K_1 = 9$ (2 BP's)

(iii) $K_1 = 2$ (1 BPs)

(iv) $K_1 = 0.1$ (1 BP)

1. $G(s)H(s) = \dfrac{K}{s(s+2)}$



$P - Z = 2 \rightarrow$ 2 Assymptotes.

$\theta = \dfrac{(2q+1)180}{2} = 90°, 270$

$\sigma = -1$

B.P position.

CE ⇒ $s^2 + 2s + K = 0$

$K = -s^2 - 2s$

$\dfrac{dK}{ds} = 0 \qquad -2s - 2 = 0$

$\underline{\underline{s = -1}}$

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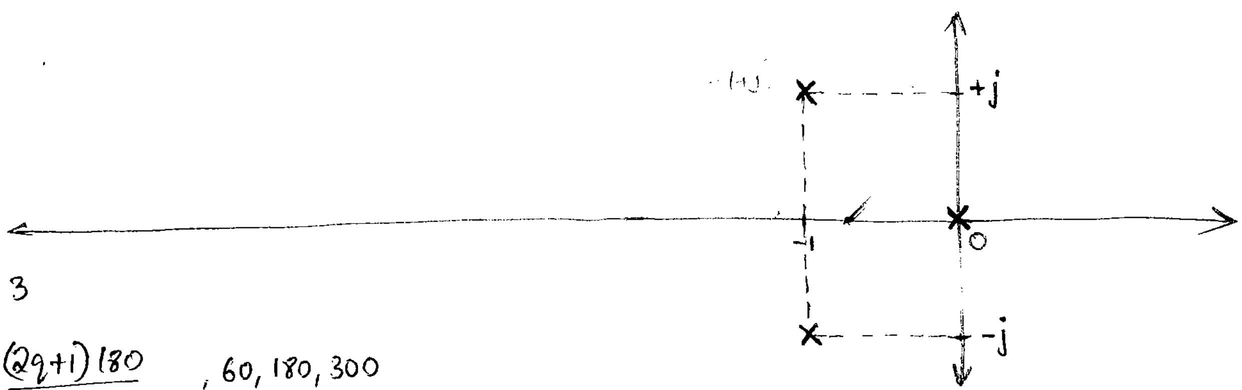
2. $G(s)H(s) = \dfrac{K}{s(s^2+2s+2)}$



$P - Z = 3$

$\theta_n = \dfrac{(2q+1)180}{3}$, 60, 180, 300

$\sigma_n = -2/3$

~~P position~~ No Breakpoint here.

$\cancel{s^3 + 2s^2 + 2s = 0}$

$\cancel{3s^2 + 4s + 2 = 0}$

Angle of Departure

$\cancel{\phi_d = \angle}$ $\qquad \angle G(s)H(s) = \dfrac{\angle K}{\angle s \, \angle s^2 + 2s + 2}$

$\angle K$

$$= \frac{0}{\angle -1+j \ \angle \cancel{-1}+j+\cancel{1}+j \ \angle \cancel{-1}\cancel{+j}+\cancel{1}-j}$$

$$= 0 - \tan^{-1} -1 - \tan^{-1}\frac{2j}{0} - \tan^{-1} 0$$

$$= \frac{-3\pi}{4} - \frac{\pi}{2}$$

$$= 225$$

No Breakpoint

Angle of DeParture $\phi_d = 45^\circ, 315^\circ$

$$\phi_d = 180 + 225$$

$$= 405 \Rightarrow 360 + 45$$

Hence $\phi_d = 45^\circ$

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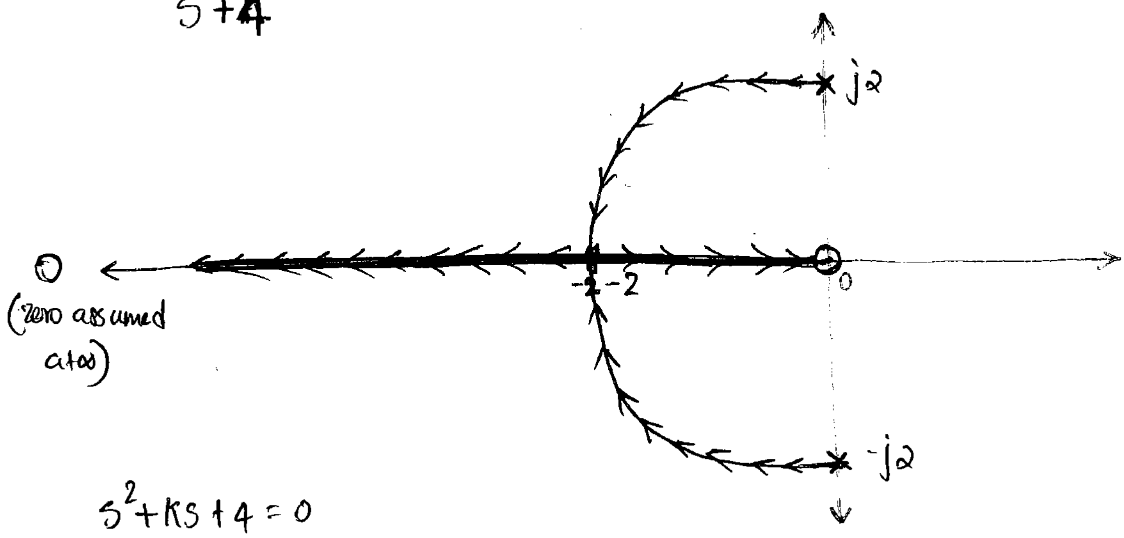
3. $G(s)H(s) = \frac{Ks}{s^2+4}$



CE ⇒ 0

$s^2 = -4$

$s = \pm j\sqrt{4}$

$s = \pm j2$

$$s^2 + Ks + 4 = 0$$

$$K = -\frac{(s^2+4)}{s} \qquad \frac{dK}{ds} = 0 \Rightarrow s^2 = 4 \quad s = \pm 2$$

1 Valid B.P at -2

Break point

Angle of Departure

$$\angle G(s)H(s) = \frac{\angle K \ \angle s}{\angle s^2+4} = \frac{\angle K \ \angle s}{\angle(s+j2)\ \angle s-j2)}$$

$$= \frac{\angle K \ \angle 2j}{\angle(j2+j2) + \angle j2-j2}$$

$$= 90^\circ - \tan^{-1}\frac{2j}{0} - \tan^{-1} 0 = 0$$

$\phi_d = 180 + (0)$

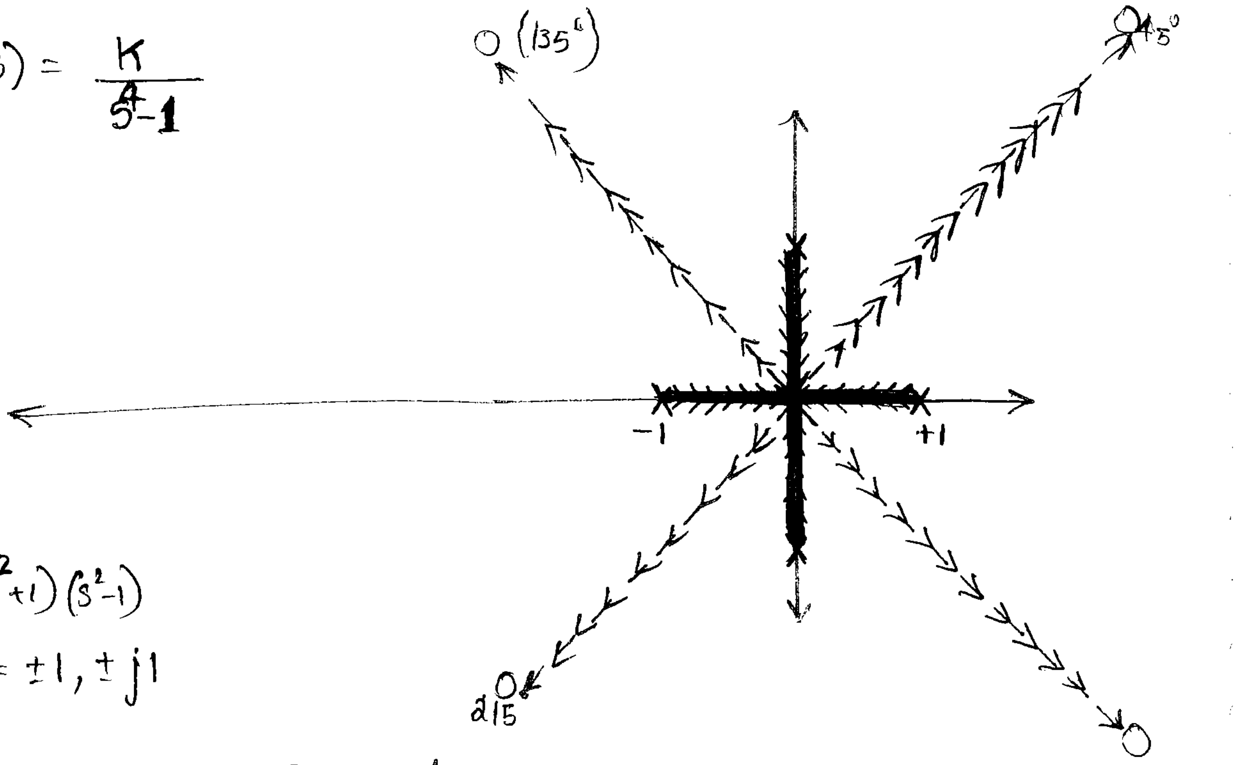
$= 180^\circ$

No Breakpoint

Angle of Departure ±180

4. $G(s)H(s) = \dfrac{K}{s^4-1}$

$P-Z=2$



<u>Break point</u>

$s^4-1=0 = (s^2+1)(s^2-1)$

~~$2s=0$~~ $\quad s = \pm 1, \pm j1$

$s=0$

Valid B.P (on RL path).

$\phi_p = 180+(270)$

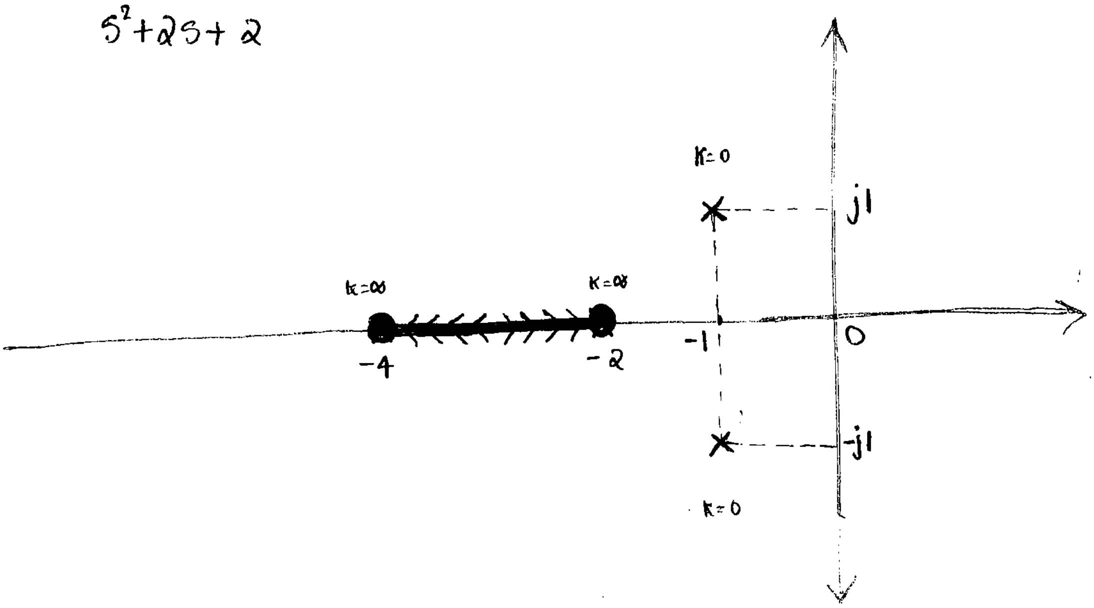
$= -90^\circ$

One 1 B.P at origin

~~No Angle of departure and Angle of arrival~~ (∵ No conjugate ~~poles & zeros~~)

Angle of departure $= \mp 90^\circ$

5. $G(s)H(s) = \dfrac{K(s+2)(s+4)}{s^2+2s+2}$



<u>Break point</u>.

$CE \Rightarrow 1+GH=0$

$s^2+2s+2+K(s^2+6s+8)=0$

$K = \dfrac{-(s^2+2s+2)}{s^2+6s+8}$

$$\frac{dK}{ds} = 0 \qquad (s^2+6s+8)(-2s-2) + (s^2+2s+2)(2s+6) = 0$$

$$\cancel{-2s^3} - 12s^2 - 16s - 2s^2 - 12s - 16 + \cancel{2s^3} + 4s^2 + 4s + 6s^2 + 12s + 12 = 0$$

$$-4s^2 - 28s + 16s - 16 + 12 = 0$$

$$-4s^2 - 12s - 4 = 0 \qquad \times\ s = -0.38$$

$$s^2 + 3s + 1 = 0 \qquad \checkmark\ s = -2.61 \quad \text{valid B.P (Break in pt)}$$

Angle of Departure

$$\angle G(s)H(s) = \frac{\angle K \angle s+2 \angle s+4}{\angle(s+1+j)\ \angle(s+1-j)}$$

$$= \frac{\angle K\ \angle -1+j+2\ \ \angle -1+j+4}{\angle -1+j+1+j\ \ \angle -1+j+1-j} = \frac{\angle K\ \angle 1+j\ \angle 3+j}{\angle 2j\ \ \angle 0}$$

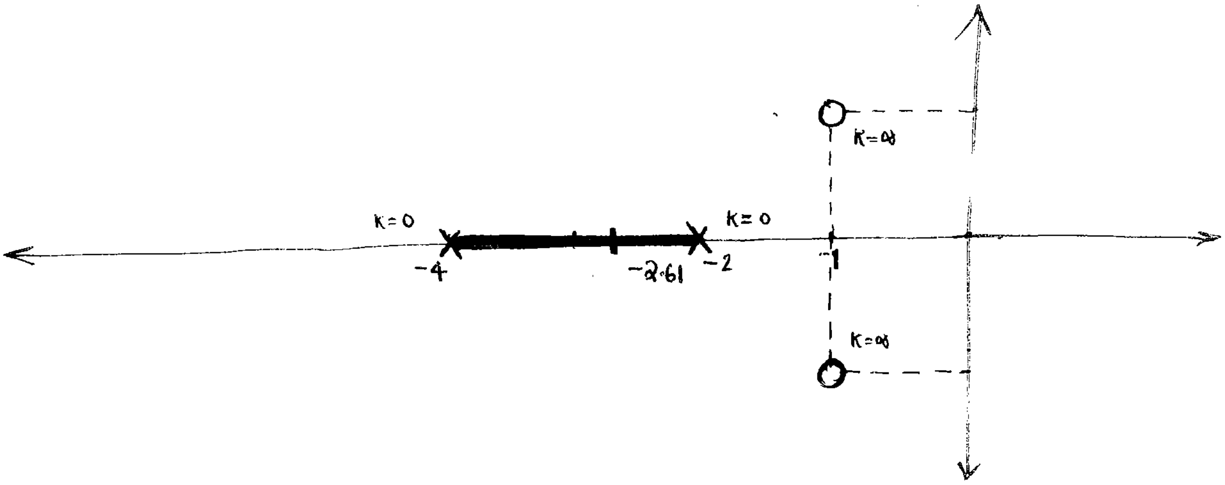
$$= 0 + 45^\circ + 18.435 - 90^\circ - 0^\circ$$

$$= -26.565^\circ \qquad \text{1 Breakpoint at } -2.61$$

$$\phi_d = 180 + (-26.565^\circ) \qquad \text{Angle of depature} = 153.435,$$

$$= \underline{153.435}$$

6. $G(s)H(s) = \dfrac{K(s^2+2s+2)}{(s+2)(s+4)}$



Breakpoint.

$CE \Rightarrow 1+GH = 0$

$$s^2+6s+8+K(s^2+2s+2) = 0$$

$$K = -\frac{(s^2+6s+8)}{s^2+2s+2}$$

$$\frac{dK}{ds} = 0 \Rightarrow -(s^2+2s+2)(2s+6) + (s^2+6s+8)(2s+2) = 0$$

$$s^2+3s+1 = 0 \qquad \times\ s = -0.38$$

$$s = -2.61 \quad \text{valid Breakaway point.}$$

Angle of arrival

$$\angle G(s)H(s) = \frac{\angle K\ \angle(s+1+j)\ \angle(s+1-j)}{\angle(s+2)\ \angle(s+4)}$$

$$= 0^\circ + 90^\circ + 0 - 45^\circ - 18.435^\circ$$

$$= 26.565^\circ.$$

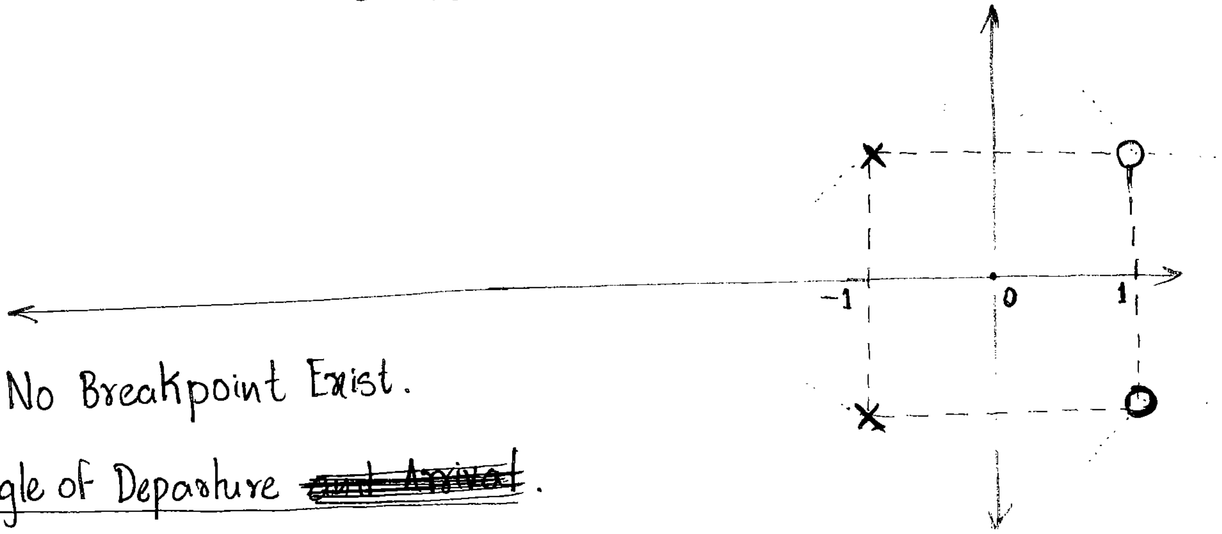
$$\phi_d = 180 - 26.565^\circ$$

$$= \underline{153.435^\circ}$$

1 Breakaway point at -2.61

Angle of Arrival $= 153.435^\circ$

7. $G(s)H(s) = \dfrac{K(s^2-2s+2)}{s^2+2s+2}$



No Breakpoint Exist.

Angle of Deparhure ~~and Arrival~~.

$$\angle G(s)H(s)\Big|_{s=1+j} = \cancel{\frac{\angle K \ \angle(s+1+j)\angle(s+1-j)}{\angle s}}$$

$$= \frac{\angle K \ \angle(s-1+j)\angle(s-1-j)}{\angle(s+1+j) \ \angle(s+1-j)}$$

$$= \frac{\angle K \ \angle -1+j-1+j \ \ \angle \cancel{-1+j-1-j}}{\angle \cancel{-1}+j\cancel{+1}+j \ \ \angle \cancel{-1+j+1-j}}$$

$$= 0 + \angle -2+2j + 0 - 90 - 0$$

$$= 135 - 90$$

$$= 45^\circ$$

$$\phi_d = 180 + 45^\circ = \underline{\underline{225^\circ}}$$

Angle of Arrival

$$\angle G(s)H(s)\Big|_{s=1+j} = \frac{\angle K \ \angle \cancel{1}+j\cancel{-1}+j \ \angle \cancel{1+j-1-j}}{\angle 1+j+1+j \ \ \angle 1\cancel{+j}+1\cancel{-j}} = 0+90^\circ+0-45-0$$

$$= \underline{\underline{45^\circ}}$$

$$\phi_a = 180 - 45 = \underline{\underline{135^\circ}}$$

No Breakpoint.

Angle of Arrival $= 135^\circ$ Angle of deparhure $= 225^\circ$

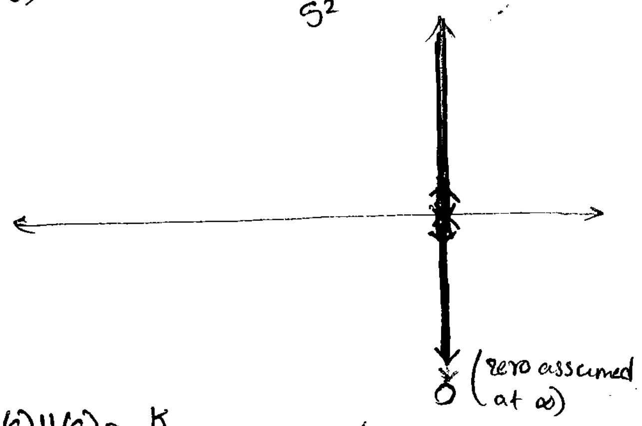
8. (i) $G(s)H(s) = \frac{K}{s}$

No Breakpoint.

No conjugate poles or zeros.

Hence No Angle of Arrival and Departure.

(ii) $G(s)H(s) = \frac{K}{s^2}$

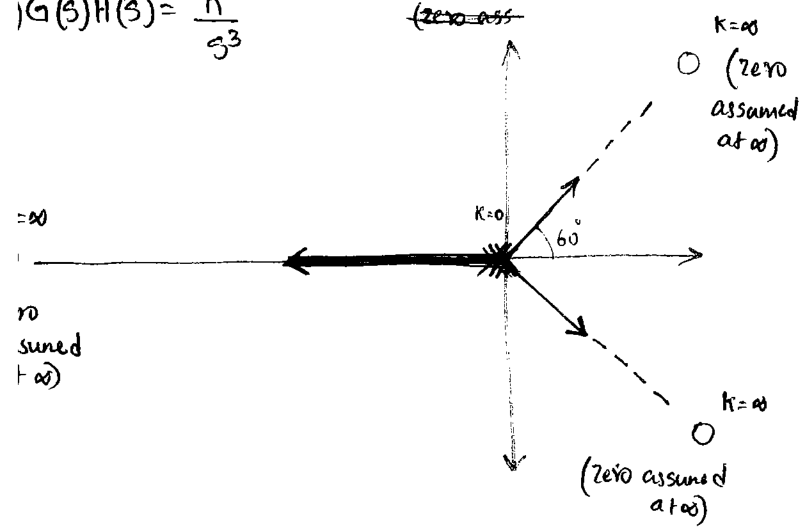


1 Breakpoint at origin.

No conjugate poles or zeros

Hence No Angle of arrival & Departure.

)$G(s)H(s) = \frac{K}{s^3}$

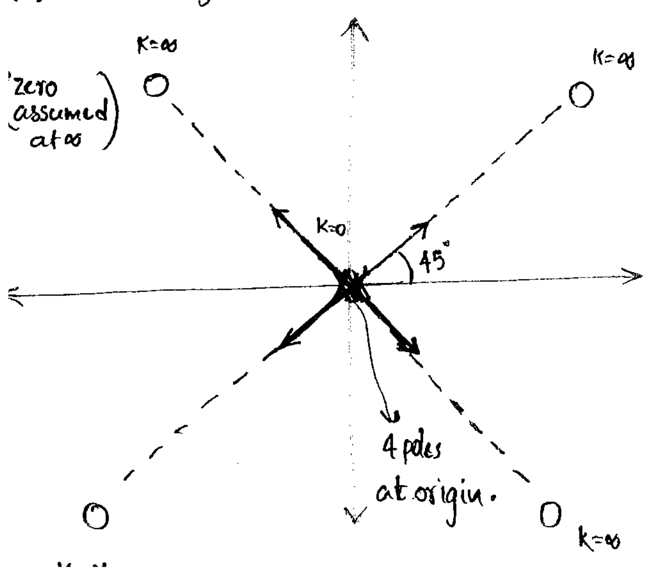


1 Breakpoint at origin.

No conjugate poles & zeros.

Hence no angle of departure or arrival.

(iv) $G(s)H(s) = \frac{K}{s^4}$

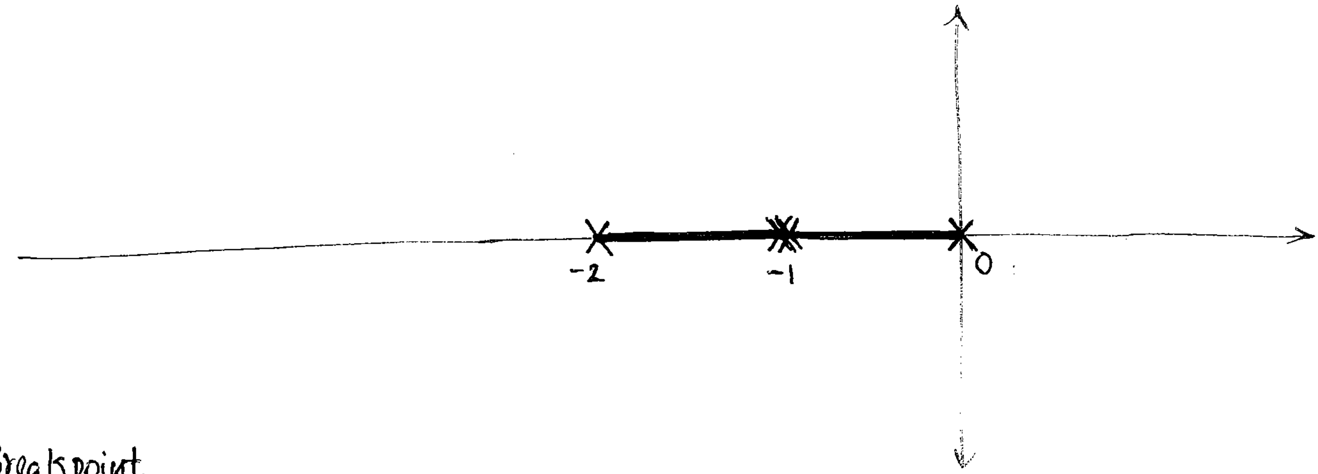


1 Breakpoint at origin

No conjugate poles and zeros.

Hence no angle of departure or arrival.

9. $G(s)H(s) = \dfrac{11}{s(s+1)^2(s+2)}$



Breakpoint

$s(s^2+2s+1)(s+2) = 0$

$(s^3+2s^2+s)(s+2) = 0$

$s^4+2s^3+s^2+2s^3+4s^2+2s = 0$

$s^4+4s^3+5s^2+2s = 0$

Diff:

$4s^3+12s^2+10s+2 = 0$

No conjugate poles. Hence no Angle of departure & arrival.

$s = -1.707$

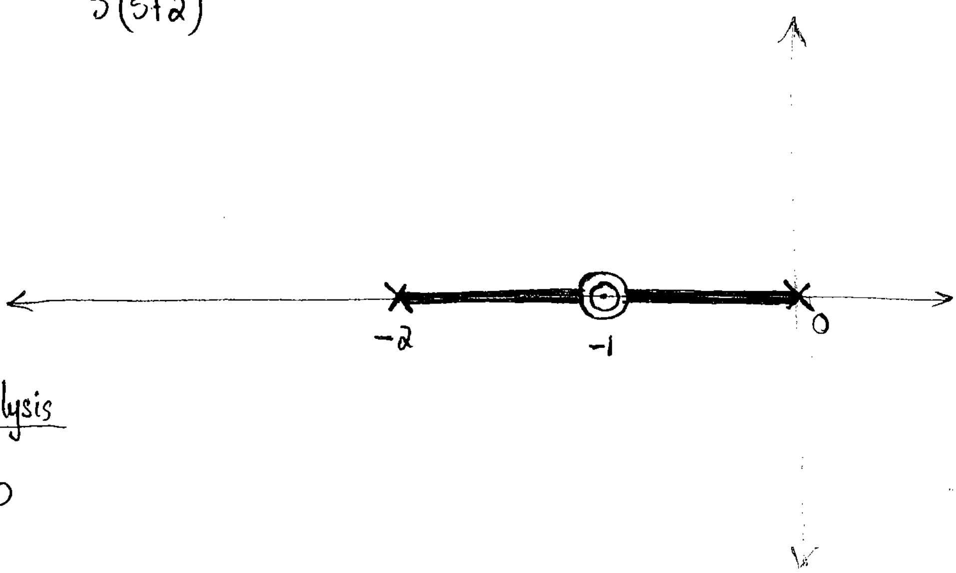
$s = -0.293$

$s = -1$

3 Valid Breakpoints at -1.707, -0.293, -1

---

10. $G(s)H(s) = \dfrac{K(s+1)^2}{s(s+2)}$



Breakpoints Analysis

$1+G(s)H(s) = 0$

$$s(s+2) + K(s+1)^2 = 0$$

$$s^2+2s + K(s^2+2s+1) = 0$$

$$K = \frac{-(s^2+2s)}{s^2+2s+1}$$

$$\frac{dK}{ds} = 0 \Rightarrow -(s^2+2s+1)(2s+2) + (s^2+2s)(2s+2) = 0$$

$$(2s+2)\left[\cancel{s^2}\cancel{+2s}\cancel{-s^2}\cancel{-2s}-1\right] = 0$$

$$2s = -2$$
$$s = \underline{\underline{-1}}$$

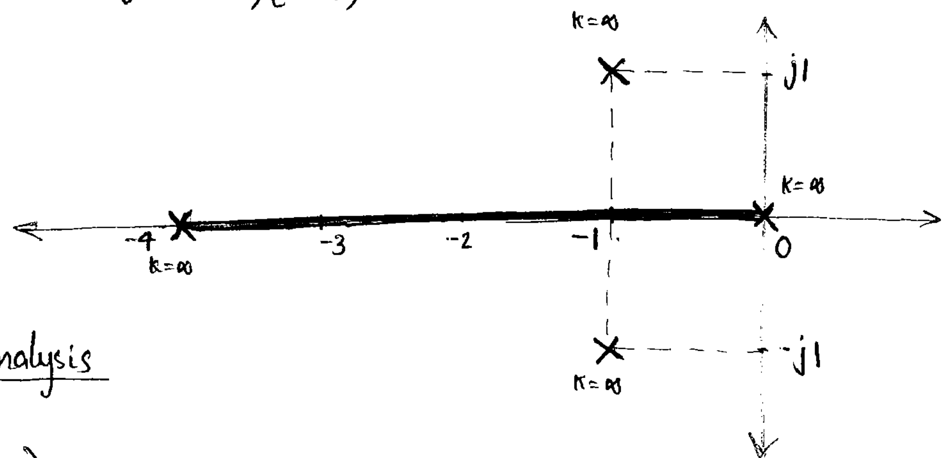
1 Valid Breakpoint at -1

No conjugate poles & zeros.

Hence No angle of departure & Arrival.

---

11. $$G(s)H(s) = \frac{K}{s(s^2+2s+2)(s+4)}$$



Breakpoint analysis

$$(s^2+4s)(s^2+2s+2) = 0$$

$$s^4+2s^3+2s^2+4s^3+8s^2+8s = 0$$

$$s^4+6s^3+10s^2+8s = 0$$

Diff..

$$4s^3+18s+20s+8=0$$

$$s = -3.092$$

$$s = -0.704 + .389j$$

$$s = -0.704 - .389j$$

Angle of departure.

$$\angle G(s)H(s) = \left.\frac{\angle K}{\angle s \angle(s+1+j) \angle(s+1-j) \angle s+4}\right|_{s=-1+j}$$

$$= \frac{\angle K}{\angle -1+j \, \angle \cancel{-1}+j\cancel{+1}+j \, \angle \cancel{-1+j+1}-j \, \angle -1+j+4}$$

$$= 0 - \tan^{-1}(-1) - 90^\circ - 0^\circ - \tan^{-1} 1/3$$

$$= -135 - 90 - 18.435$$

$$= -243.435^\circ$$

$$\phi_d = 180 + (-243.435)$$

$$= -63.435$$

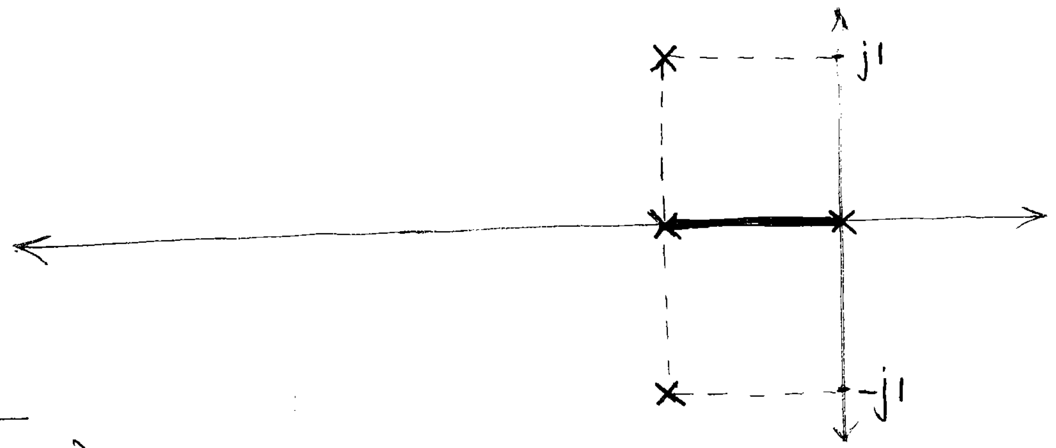
$$= \underline{296.565}$$

1 Valid Breakpoint at $\underline{-3.092}$

Angle of Departure at $\underline{-63.435^\circ}$ or $\underline{296.565^\circ}$

---

12 $\quad G(s)H(s) = \dfrac{K}{s(s^2+2s+2)(s+1)}$



Breakpoint

$$(s^2+s)(s^2+2s+2) = 0$$

$$s^4 + 2s^3 + 2s^2 + s^3 + 2s^2 + 2s = 0$$

$$s^4 + 3s^3 + 4s^2 + 2s = 0$$

Diff.

$$4s^3 + 9s^2 + 8s + 2 = 0$$

$s = -0.394$

$s = -0.928 + 0.638j$

$s = -0.928 - 0.638j$

Angle of Departure.

$$\angle G(s)H(s)\Big|_{-1+j} = \frac{\angle K}{\angle -1+j + \angle \cancel{-1}+j+\cancel{1}+j \cdot \angle \cancel{-1}+j+\cancel{1}-j \;\; \angle -\cancel{1}+j+\cancel{1}}$$

$$= 0 - 135 - 90 - 90$$

$$= -315$$

$$\phi_d = 180 + (-315)$$

$$= \underline{-135^\circ}$$

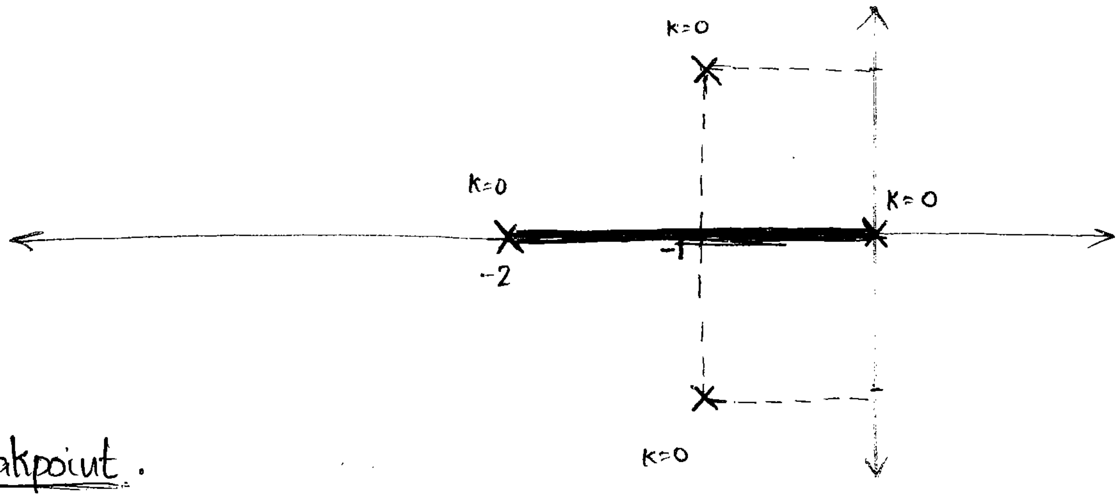
$$= \cancel{225}^\circ$$

1 Valid Breakpoint at -0.394

Angle of Departure at -135° or $\cancel{225}^\circ$

---

13. $$G(s)H(s) = \frac{K}{s(s^2+2s+2)(s+2)}$$



Breakpoint.

$$(s^2+2s)(s^2+2s+2) = 0$$

$$s^4 + 2s^3 + 2s^2 + 2s^3 + 4s^2 + 4s = 0$$

$$s^4 + 4s^3 + 6s^2 + 4s = 0$$

$$4s^3 + 12s^2 + 12s + 4 = 0$$

$s = -1$

Angle of Departure

$$\angle G(s)H(s)\Big|_{-1+j} = \frac{\angle K}{0-135-90-45}$$

$\angle -1+j+2$

$\angle 1+j$

$$= -270$$

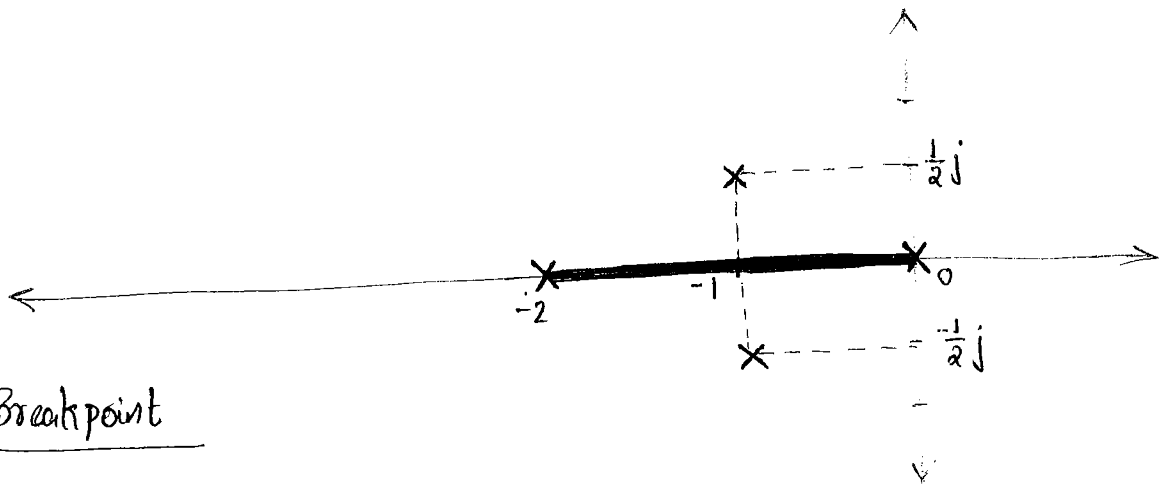
$$\phi_d = 180+(-270)$$
$$= -90^\circ$$

1 Valid Breakpoint at -1

Angle of Departure at -90°

---

14. $$G(s)H(s) = \frac{K}{s(s^2+2s+1.25)(s+2)}$$



Breakpoint

$$(s^2+2s)\left(s^2+2s+\frac{5}{4}\right) = 0$$

$$(s^2+2s)(4s^2+8s+5) = 0$$

$$(4s^4+8s^3+5s^2+8s^3+16s^2+10s = 0$$

$$4s^4+16s^3+21s^2+10s = 0$$

$$16s^3+48s^2+42s+10 = 0$$

$s = -1.6124$

$s = -0.388$

$s = -1$

Angle of Departure

$$\angle G(s)H(s)\Big|_{-1+j/2} = \frac{\angle K}{\angle s\ \angle s+1+j/2\ \angle s+1-j/2\ \angle s+2}\Bigg|_{-1+j/2}$$

$$= \frac{\angle K}{\angle -1+j/2\ \ \angle -1+j/2+1+j/2\ \ \angle -1+j/2+1-j/2\ \ \angle -1+j/2+2}$$

$$= \frac{\angle K}{\angle -1+j/_2 \;\; \angle j \;\; \angle 0 \;\; \angle 1+j/_2}$$

$$= 0 - \overset{153.435}{\cancel{135}} - 90 - \overset{26.565}{\cancel{45}}$$

3 Valid Breakpoint at $-1, -0.388, -1.6124$

$$\cancel{= +270} = {}^{-}270$$

Angle of Departure $= \mp 90^\circ$

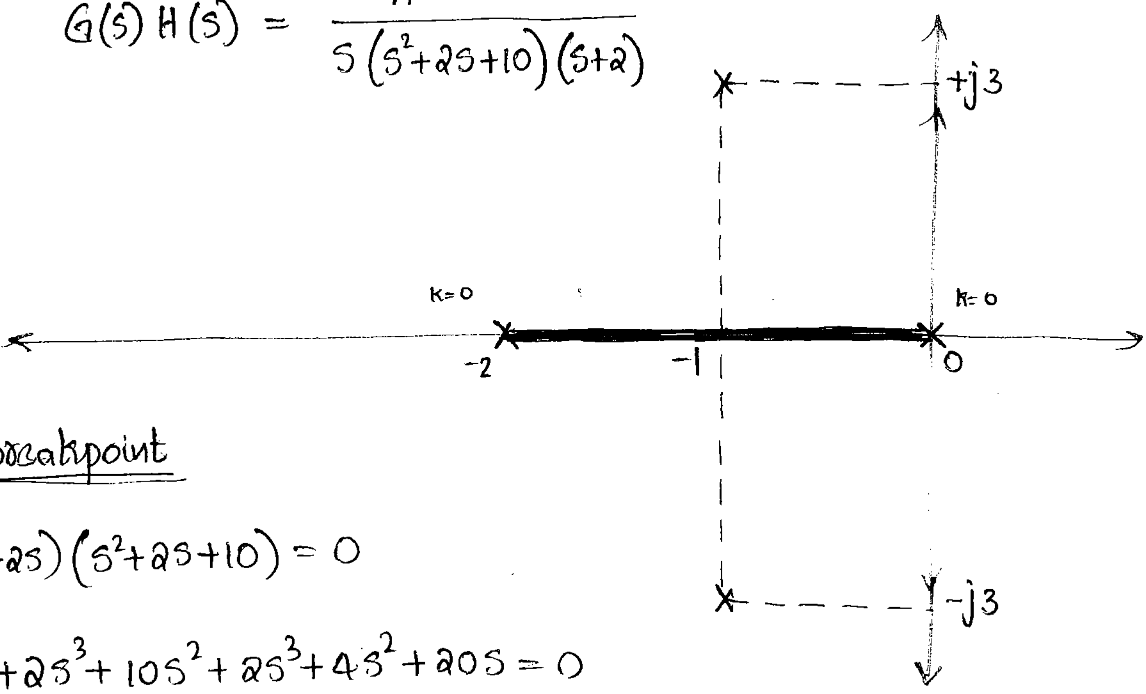
$\cancel{270^\circ}$

$$\phi_d = 180 - 270$$

$$= -90^\circ$$

---

15. $$G(s)\,H(s) = \frac{K}{s\,(s^2+2s+10)\,(s+2)}$$



<u>Breakpoint</u>

$$(s^2+2s)\,(s^2+2s+10) = 0$$

$$s^4+2s^3+10s^2+2s^3+4s^2+20s = 0$$

$$s^4+4s^3+14s^2+20s = 0$$

$$4s^3+12s^2+28s+20 = 0$$

$s = -1$

$s = -1+2i$, $s = -1-2i$ } explained later.

<u>Angle of Departure.</u>

$$\angle G(s)H(s)\Big|_{-1+3j} = \frac{\angle K}{\angle s \;\; \angle(s+1+3j) \;\; \angle(s+1-3j) \;\; \angle s+2}\Bigg|_{-1+3j}$$

$$= \frac{\angle K}{\quad}$$

$$= \frac{\angle K}{\angle -1+3j \;\; \angle 6j \;\; \angle 0 \;\; \angle 1+3j}$$

$$= 0 - 108.435 - 90 - 71.565$$

$$= \underline{\underline{-270}} \qquad\qquad \phi_d = 180 - 270 = -90^\circ \text{ or } 225^\circ$$

1 valid Breakpoint at -1

Angle of Departure = $\mp 90^\circ$ or 225°

---

16. (i) $G(s)H(s) = \dfrac{K(s+1)}{s^2(s+K_1)}$ $\qquad K_1 = 20$

$$G(s)H(s) = \frac{K(s+1)}{s^2(s+20)}$$

CE $\Rightarrow$

$1 + G(s)H(s) = 0$

$$s^2(s+20) + K(s+1) = 0$$

$$K = \frac{-s^2(s+20)}{s+1} = \frac{-(s^3+20s^2)}{s+1}$$

$$\frac{dK}{ds} = 0 \Rightarrow \frac{-(s+1)(3s^2+40s) + (s^3+20s^2)}{(s+1)^2} = 0$$

$$-(3s^3 + 40s^2 + 3s^2 + 40s) + (s^3 + 20s^2) = 0$$

$$-3s^3 - 40s^2 - 3s^2 - 40s + s^3 + 20s^2 = 0$$

$$-2s^3 - 23s^2 - 40s = 0$$

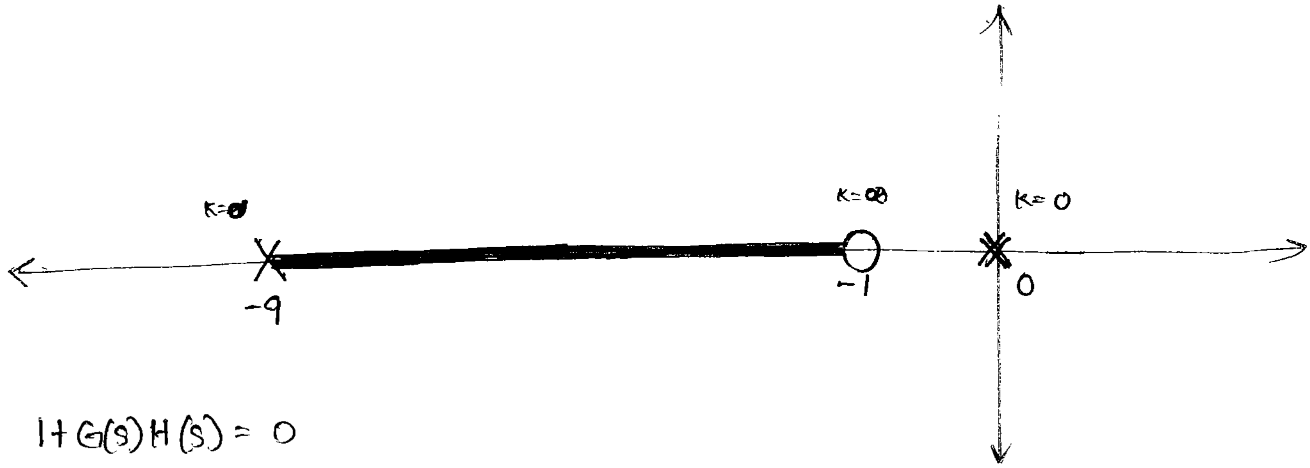
$$s(2s^2 + 23s + 40) = 0$$

$s = 0$

$s = -2.136$

$s = -9.3642$

(ii) $G(s)H(s) = \dfrac{K(s+1)}{s^2(s+9)}$



$1 + G(s)H(s) = 0$

$s^2(s+9) + K(s+1) = 0$

$K = \dfrac{-(s^3+9s^2)}{s+1}$

$\dfrac{dK}{ds} = 0 \Rightarrow -(s+1)(3s^2+18s) + (s^3+9s^2) = 0$

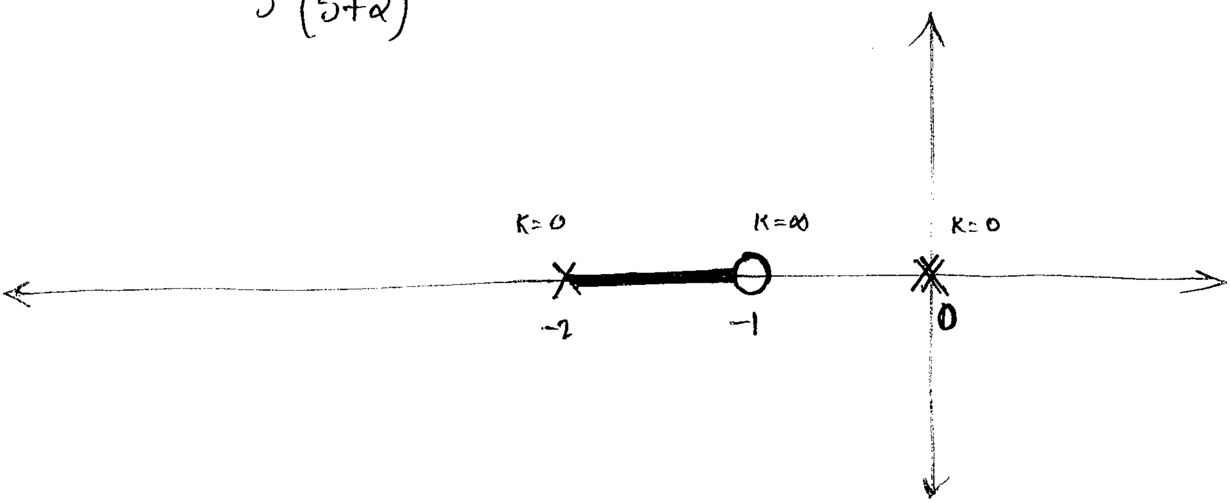
$-[3s^3+18s^2+3s^2+18s] + s^3+9s^2 = 0$

$-3s^3 - 21s^2 - 18s + s^3 + 9s^2 = 0$

$-2s^3 - 12s^2 - 18s = 0$ $\qquad \cancel{s=0}$

$s(2s^2+12s+18) = 0$ $\qquad \cancel{s=-3}$

(iii) $G(s)H(s) = \dfrac{K(s+1)}{s^2(s+2)}$



$1+G(S)H(S)=0$

$S^2(S+2)+K(S+1)=0$

$$K=\frac{-(S^3+2S^2)}{S+1}$$

$$\frac{dK}{dS}=0 \Rightarrow \quad -(S+1)(3S^2+4S)+(S^3+2S^2)=0$$

$$-3S^3-4S^2-3S^2-4S+S^3+2S^2=0$$

$$-2S^3-5S^2-4S=0$$

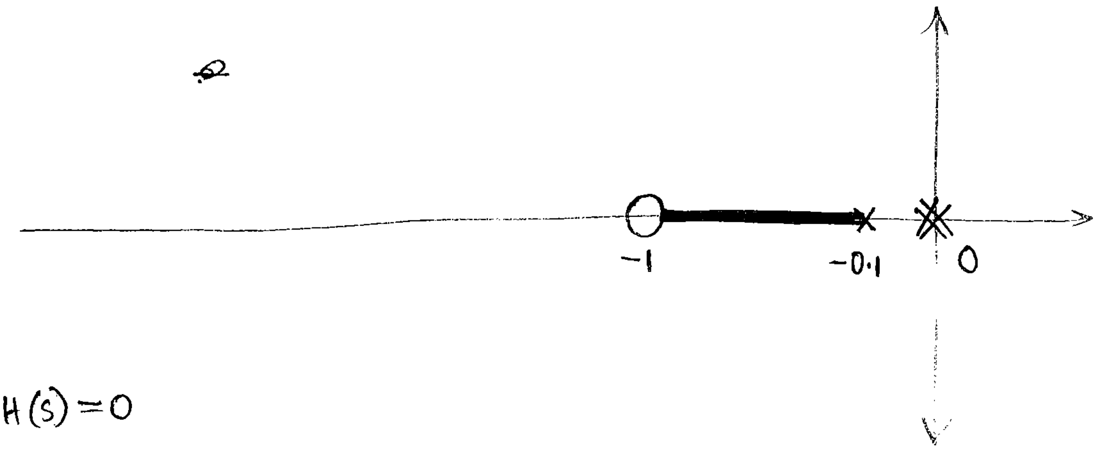
$$S[2S^2+5S+4]=0$$

$\checkmark$ $\cancel{S=0}$

$\times\ S=-\frac{5}{4}+0.661j$

$\times\ S=-\frac{5}{4}-0.661j$

(iv) $$G(S)H(S)=\frac{K(S+1)}{S^2(S+K_1)} \qquad K_1=0.1$$



CE $\Rightarrow$

$1+G(S)H(S)=0$

$S^2(S+0.1)+K(S+1)=0$

$$K=\frac{-(S^3+0.1S^2)}{S+1}$$

$$\frac{dK}{dS}=0 \Rightarrow -(S+1)(3S^2+0.2S)+(S^3+0.1S^2)=0$$

$$-[3S^3+0.2S^2+3S^2+0.2S]+S^3+0.1S^2=0$$

$$-3S^3-3.2S^2-0.2S+S^3+0.1S^2=0$$

$$-2s^3 - 3.1s^2 - 0.2s = 0$$

$$s\left[2s^2 + 3.1s + 0.2\right] = 0$$

~~$s = 0$~~

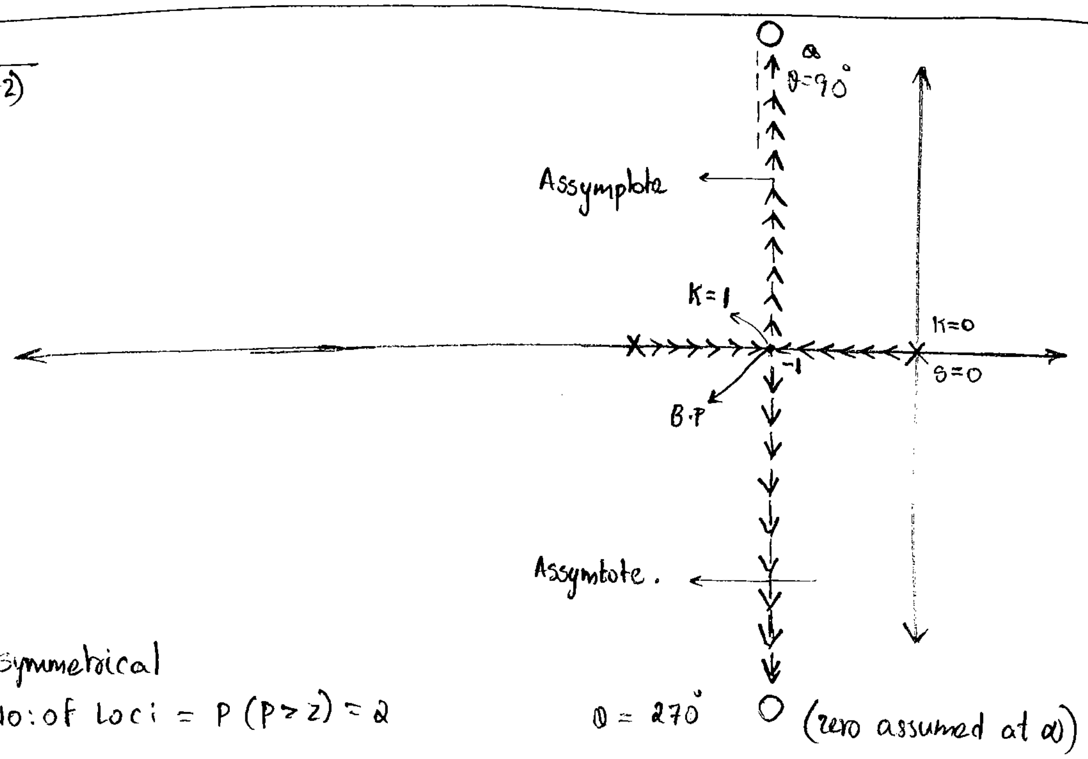
$\times$ $s = -0.0674$

~~$s = -1.4825$~~

2 Breakpoints.

---

1) $GH = \frac{K}{s(s+2)}$



S1 : symmetrical

S2 : No: of Loci = P (P > Z) = 2

S3 : .

S4 : Assymptote.

$$P - Z = 2$$

$$\theta = \frac{(2q+1)180}{2} = 90, 270°$$

S5: $a = \frac{-2-(0)}{2} = -1$

S6: $s^2 + 2s + K = 0$

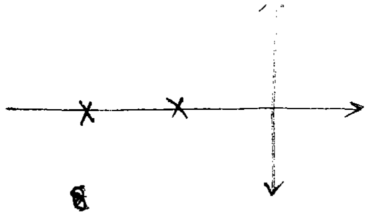
$$K = -s^2 - 2s$$

$$\frac{dK}{ds} = -2s - 2 = 0 \quad s = -1$$

S7: No intersection point with Imaginary.

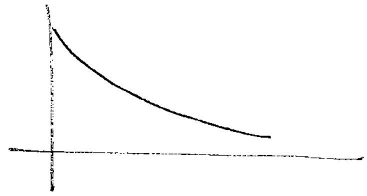
S8: No complex poles and zero.

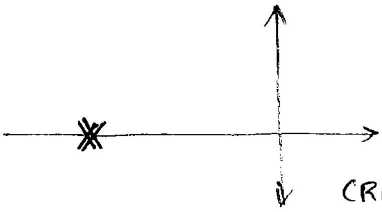
STABILITY : For $0 < K < \infty$, the poles lies in the left side. Hence stable.



$\xi_p > 1$

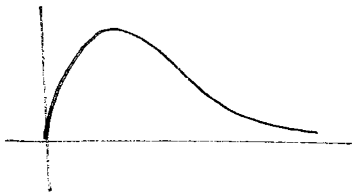
OVERDAMPED

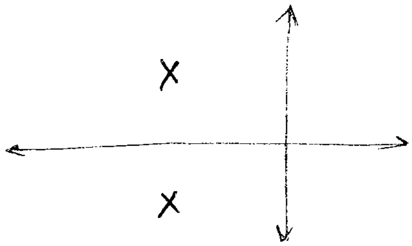




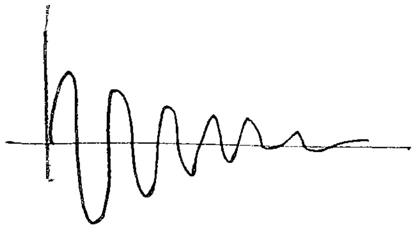
$\xi_p = 1$

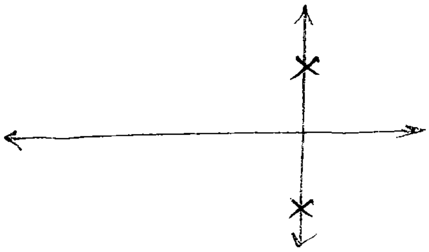
CRITICAL DAMPED



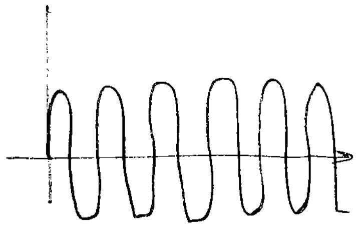


$0 < \xi_p < 1$





$\xi_p = 0$



→ Upto B·P, it is overdamped, at B·P, critical damped & after B·P, it is under damped.

→ The above system is having overdamped, critical damped, and under damped natures, but not undamped.

→ To get the K values for different nature of the system, we require to find K value at the break point.

~~Upto Breakpoint in~~

→ Magnitude condition at B·P

$$\left| \frac{K}{s(s+2)} \right| = 1$$

$$\left| \frac{K}{-1(-1+2)} \right| = 1 \qquad \text{ie, B·P, } K = 1$$

$$\frac{K}{1 \times 1} = 1 \qquad \underline{\underline{K = 1}}$$

(OR)

$$\boxed{K = \frac{\text{Product of lengths from the pt to poles}}{\text{product of lengths from the pt to zeros}}}$$

$K = 1 \times 1 = 1$

Hence conclusion.

$0 < K < 1$: overdamped

$K = 1$: Critical damped

$1 < K < \infty$: Under damped.

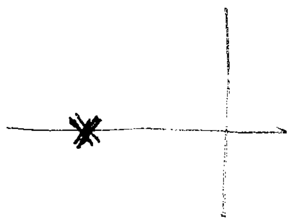
checking

Consider $K = 1$

$\therefore G(s)H(s) = \dfrac{1}{s(s+2)}$

$CLTF = \dfrac{1}{s^2+2s+1}$

$s = -1, -1$



$\omega_n = 1$

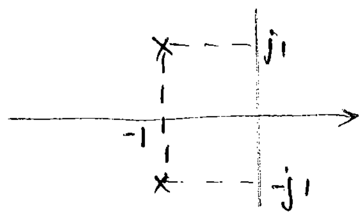
$\xi = 1$

Critical damped.

Consider $K = 2$ (>1)

$\therefore G(s)H(s) = \dfrac{2}{s(s+2)}$

$= \dfrac{2}{s^2+2s+2}$



Underdamped nature.

<u>Note</u> : 1. whenever the root locus diagram having more than or equal to two real axis root locus branches, then the system should have overdamped ~~da~~ nature.

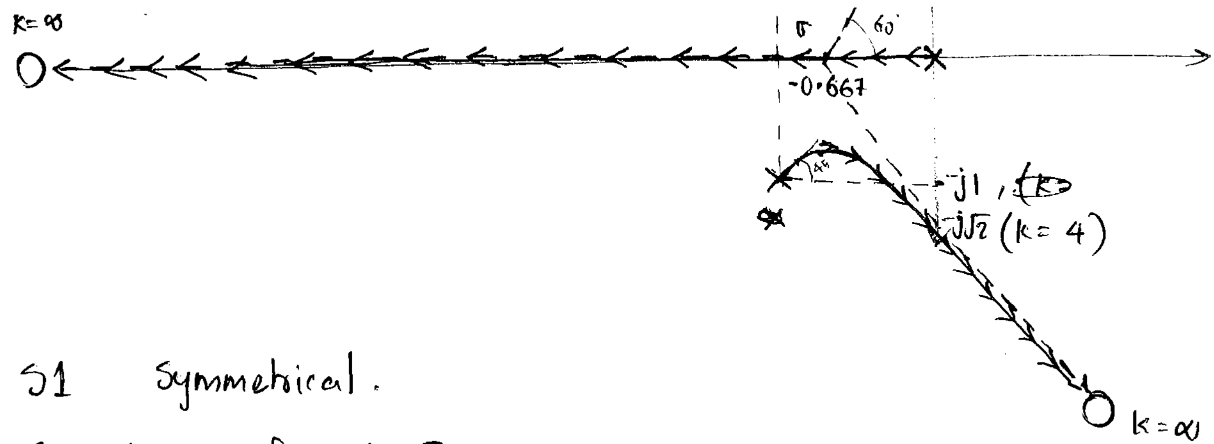
2. whenever the root locus diagram having Breakpoint, then the system should have critical damped nature.

under

any

n

have



S1 Symmetrical.

S2 ~~Loc~~ No:of Loci = **3**

S3 :

S4 : Assymptote.

$P - Z = 3$

$$\theta_n = \frac{(2q+1)180}{3} = 60^\circ, 180^\circ, 300^\circ$$

S5 : $\sigma_n = \frac{-2}{3}$

S6 : BP

$s^3 + 2s^2 + 2s = 0$

$3s^2 + 4s + 2 = 0$

$s = -0.66 \pm j0.42$

57: Intersection with imaginary by RH criteria

$$s^3+2s^2+2s+K=0$$

$K_{margin} = 4$

s^3	1	2
s^2	2	K
s^1	$\frac{4-K}{2}$	
s^0	K	

A.E $\Rightarrow$ $2s^2+4=0$

$s^2 = \pm j\sqrt{2}$

58: $\phi_d = \pm 45^\circ$

Stability : $0<K<4$: $\rightarrow$ stable.

$0<K<4$: Under damped.

$K=4$: Undamped. (with $\omega_n = \sqrt{2}$ rad/s).

Note: To get the correct RL diagram on graph, the X-axis scale must be equal to Y AXIS scale.

At $\xi = 0.5$, $\theta = \cos^{-1}\frac{1}{2} = 60^\circ$.

At 60°, $\longrightarrow$ from graph $s = -0.4 + j0.75$.

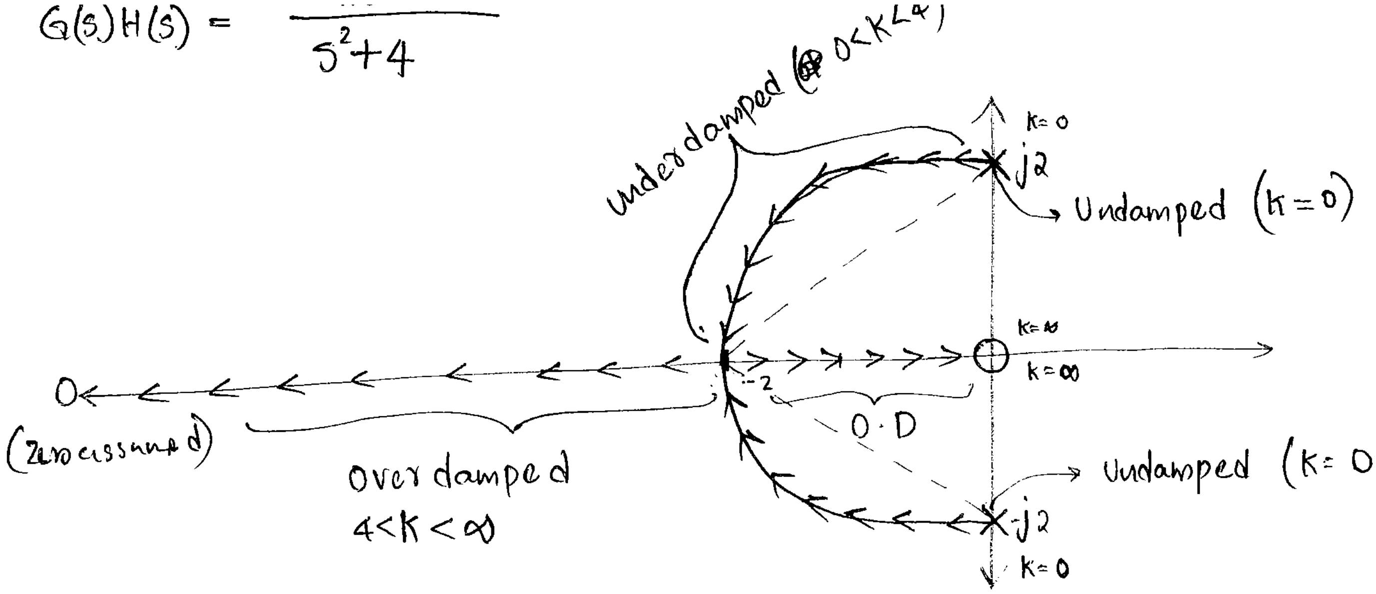
Now Apply Magnitude condition.

$$\left|\frac{K}{s(s+1+j1)(s+1-j1)}\right| = 1$$

$$\left|\frac{K}{(-0.4+j0.75)(-0.4+j0.75+1+j1)(-0.4+j0.75+1-j1)}\right| = 1$$

$$\left|\frac{K}{|1.7 \angle\ 1.85\angle\ 0.6}\right| \qquad \frac{K}{0.921} \qquad K = 1.022$$

$G(s)H(s) = \dfrac{\ldots}{s^2+4}$



S1: Symmetrical

S2: No: of Loci = 2

S3:

S4: Assymptote.

$N = P - Z = 2 - 1 = \underline{\underline{1}}$

$\theta = \dfrac{(2q+1)\,180}{1} = 180^\circ$

Note: whenever the number of asymptote more than ~~equal to~~ one. Then only required the centroid. No centroid for one assymptote only case.

S6: B.P

$s^2+4 = 0$

~~$2s$~~

$s^2+4+Ks = 0$

$K = \dfrac{-(s^2+4)}{s}$

$= -s(2s) + (s^2+4)$

$= -2s^2+s^2+4 = 0$

$s^2 = 4 \qquad s = \pm 2$

1 Valid B.P at $s = -2$

$$\angle G(s)H(s) = \left.\frac{\angle K \ \angle s}{\angle(s+j2)\ \angle(s-j2)}\right|_{j2}$$

$$= \frac{0 + \angle j2}{\angle j2+j2 \ \ \angle 0}$$

$$= 0+90-90-0$$

$$= \underline{\underline{0}}$$

$$\phi_d = 180+0$$

$$= 180^\circ$$

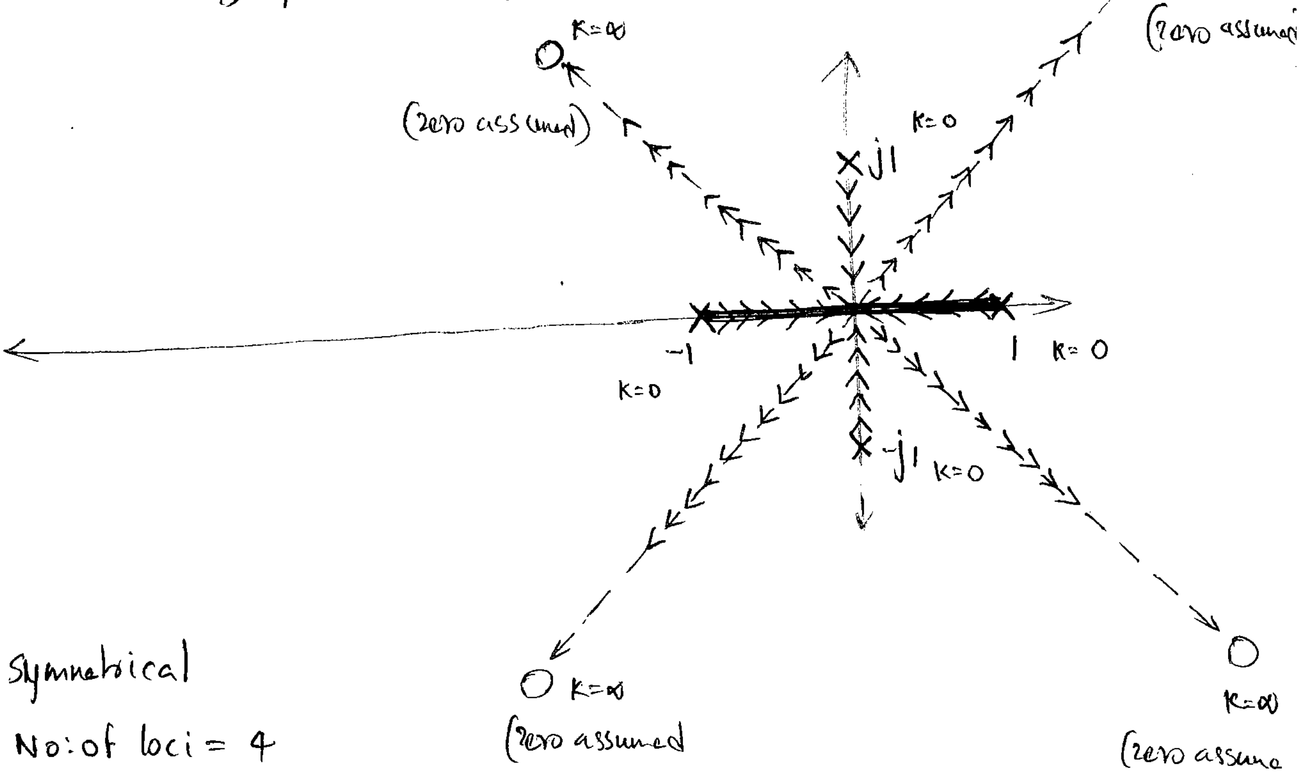
Angle of departure $= \pm 180^\circ$

K at B.P

$$\left|\frac{K \ s}{s^2+4}\right| = 1$$

$$\frac{K \times -2}{4+4} = 1 \qquad \underline{\underline{K = 4}}$$

4 Q, $G(s)H(s) = \dfrac{K}{s^4 - 1} = \dfrac{K}{(s^2-1)(s^2+1)}$



S1 : Symmetrical

S2 : No:of loci = 4

S3 :

S4 : Assymtote.

$N = P - Z = 4$

$\theta = \dfrac{2q+1}{4}$, $(45^\circ, 135^\circ, 225^\circ, 315^\circ)$

S5 : $\sigma = 0$

: BP $s^4 - 1 = 0$

$4s^3 = 0$

$s = 0$

Angle of departure

$$\angle GH\Big|_{j1} = \frac{\angle K}{\angle s+1 \ \angle s-1 \ \angle s+j1 \ \angle s-j1}$$

$$= \frac{\angle K}{\angle 1+j1 \ \angle -1+j1 \ \angle 2j \ \angle 0} = 0 - 45 - 135 - 90 + 0^\circ$$

$$= -270$$

$\phi_d = 180 + (-270)$
$= -90^\circ$

$\phi_d = \mp 90^\circ$

Note: ~~B.P = σ~~

$\boxed{\text{If BREAK POINT = CENTROID and all the poles symmetric about the B.P. Then all the poles meet at the B.P}}$

- In the above problem, 4 poles meet at the B.P

→ K value at the B.P = $1 \times 1 \times 1 \times 1 = 1$

→ The CLTF at B.P is

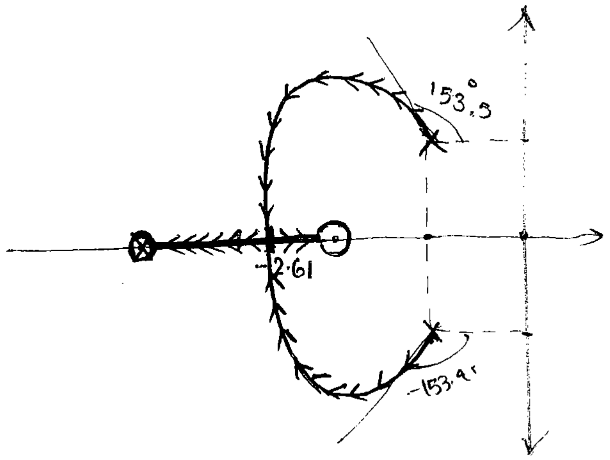
$$CLTF\Big|_{B.P} = \frac{1}{s^4 - 1 + 1} = \frac{1}{s^4}$$

→ leaving angle $= \frac{\pm 180}{n}$

$= \pm 45^\circ$

Stability : For all values of K, system is unstable.

Q $G(s)H(s) = \dfrac{K(s+2)(s+4)}{(s^2+2s+2)}$



S1 : Symmetrical

S2 : No: of loci = 2

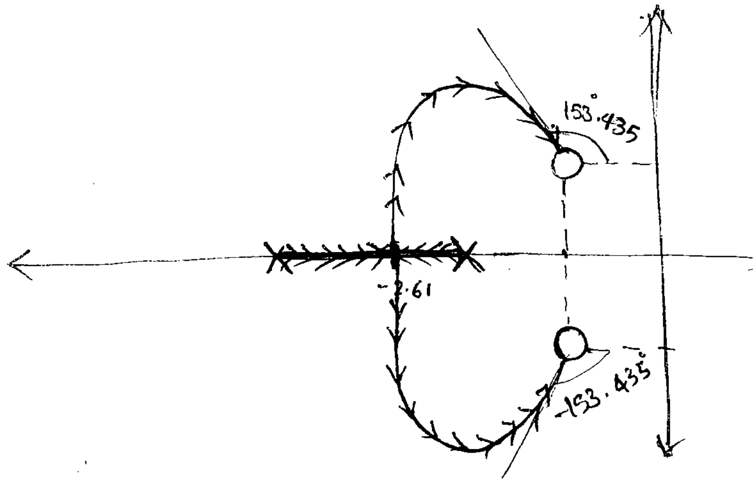
S3 :

S4 : No assymptote · $P - Z = 0$

BP = -2.61 } check

S6 : $\phi_d = \cancel{180}\ \pm 153.435$ } back

$G(s)H(s) = \dfrac{K(s^2+2s+2)}{(s+2)(s+4)}$



S1 : Symmetrical

S2 : No: of loci = 2

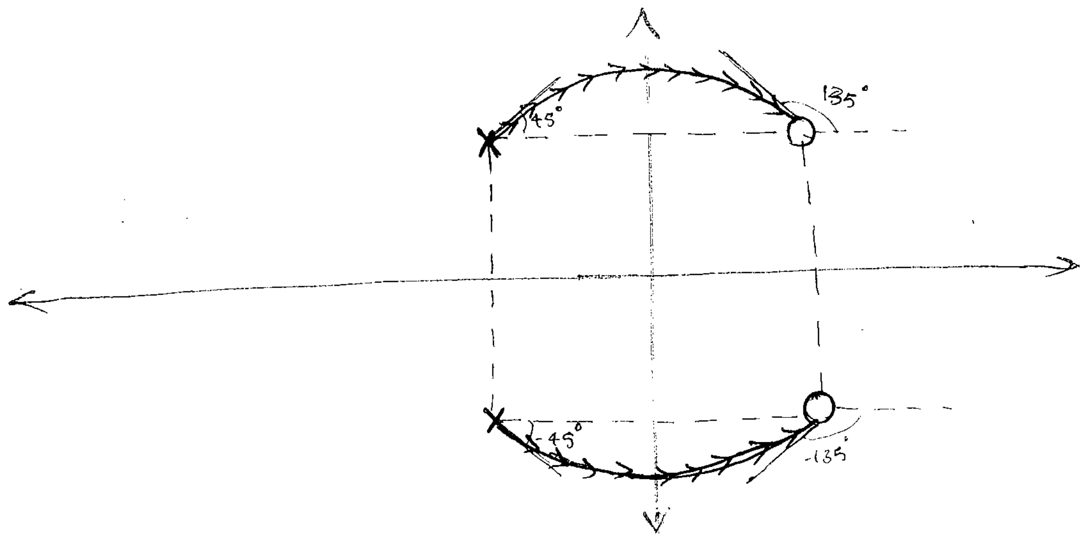
S3 :

S4 : No assymptote $P - Z = 0$

BP = -2.61

$\phi_d = \pm 153.435$

Q7 $G(s)H(s) = \dfrac{K(s^2-2s+2)}{s^2+2s+2}$ to the given RL diagram.



S1 : symmetrical

S2 : No of Loci = 2

S3 :

S4 : No assymplote.

No Valid breakpoint.

S7 : $\phi_d = \pm 45^\circ$

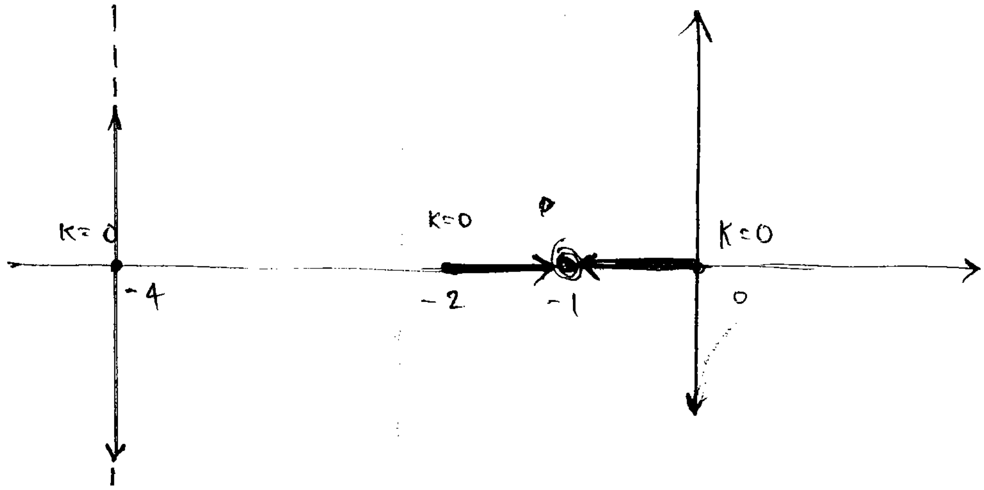
$\phi_a = \pm 135^\circ$

Intersection with imaginary axis. $\underline{\pm j\sqrt{2}}$ with $K=1$

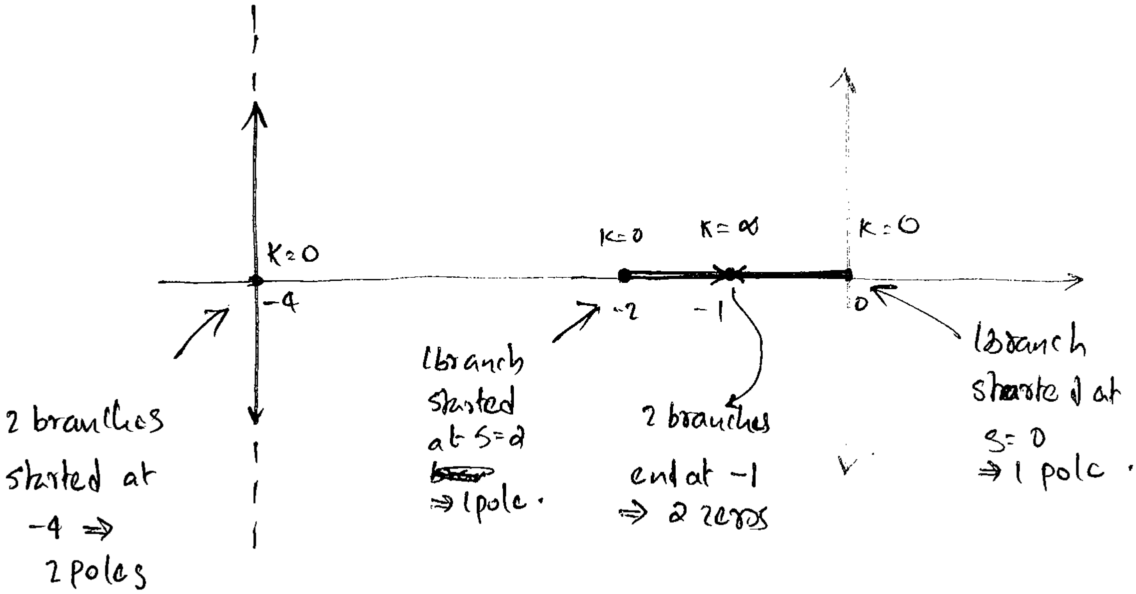
CE ⟹

$s^2+2s+2 + K(s^2-2s+2) = 0$

Q,

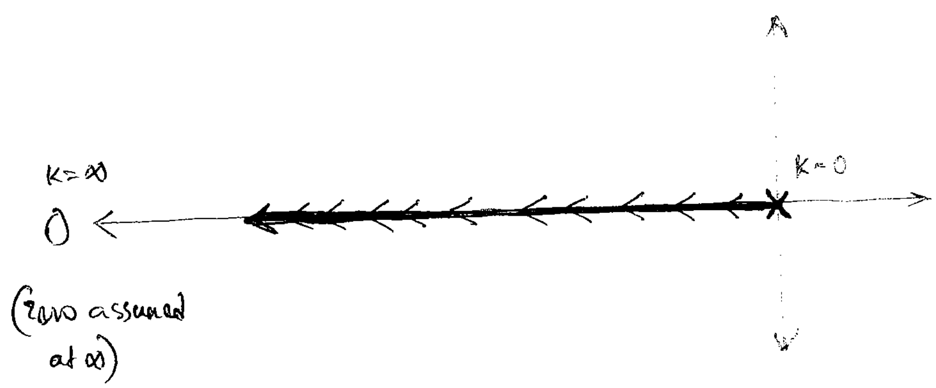


Transfer function to above RL diagram?



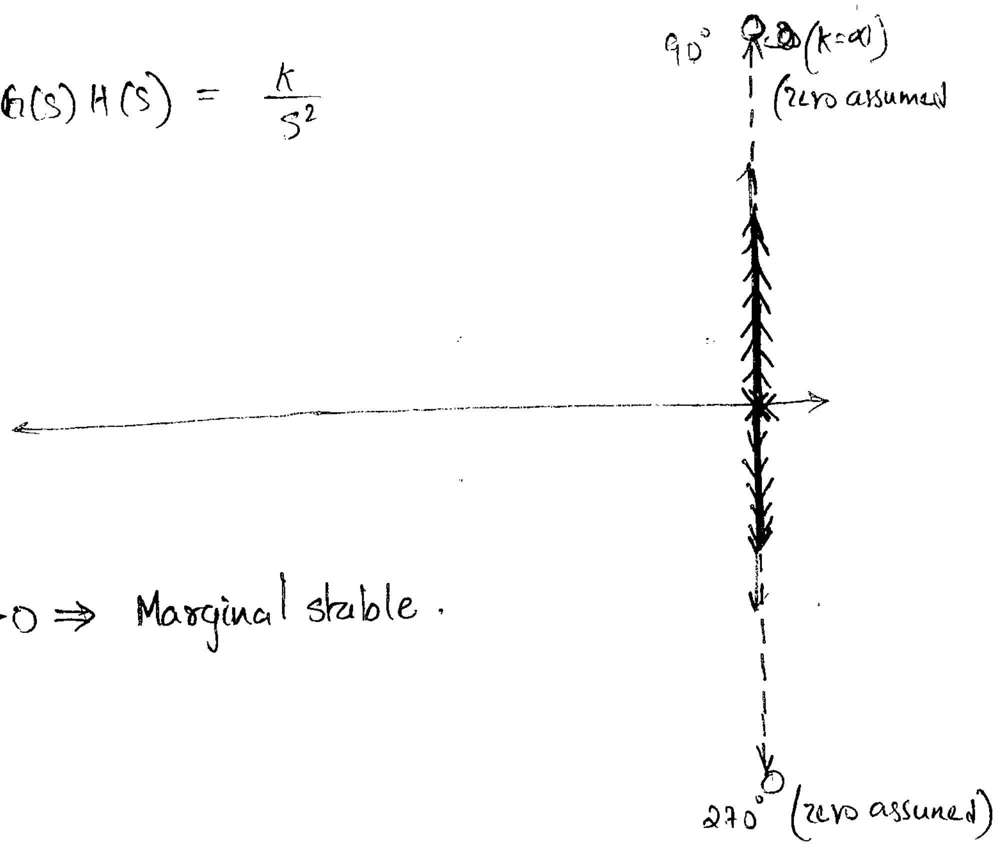
NOTE: whenever the transfer function consists only poles at origin Then the root locus diagram is nothing but angle of assymptote line.

8, (i) $G(s)\,H(s) = K/s$



$K>0 \rightarrow$ stable system.

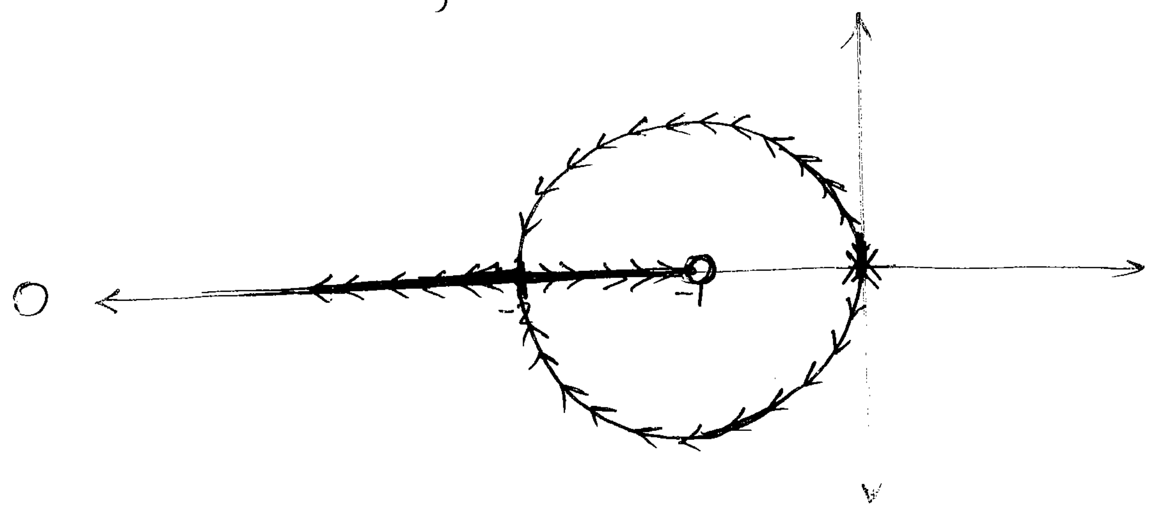
(ii) $G(s)H(s) = \frac{K}{s^2}$



$K>0 \Rightarrow$ Marginal stable.

NOTE : whenever the system is marginal stable for large range of K value, then we require to make it stable by adding a finite zero in the left hand side.

for eg: $GH(s) = \frac{K(s+1)}{s^2}$



$s^2 + K(s+1) = 0$

$K = \frac{-s^2}{s+1}$

$K = -(s+1)2s + s^2$

$= -2s^2 - 2s + s^2 = 0$

$s^2(s+2)$

NOTE : If n poles at origin, required (n-1) finite zeros in the LHS of splane, to avoid the effect on stability.

A ~~The addi~~

ADDITION OF ZEROS

→ The zeros should be added only in the left hand side.

→ The Addition of zero shift the root locus branches towards the left of s plane.

→ The Breakpoint shift towards the leftside.

→ Relative stability Improved.

→ The addition of zeros improves the bandwidth. (no: of zero increases goes to highpass filter).

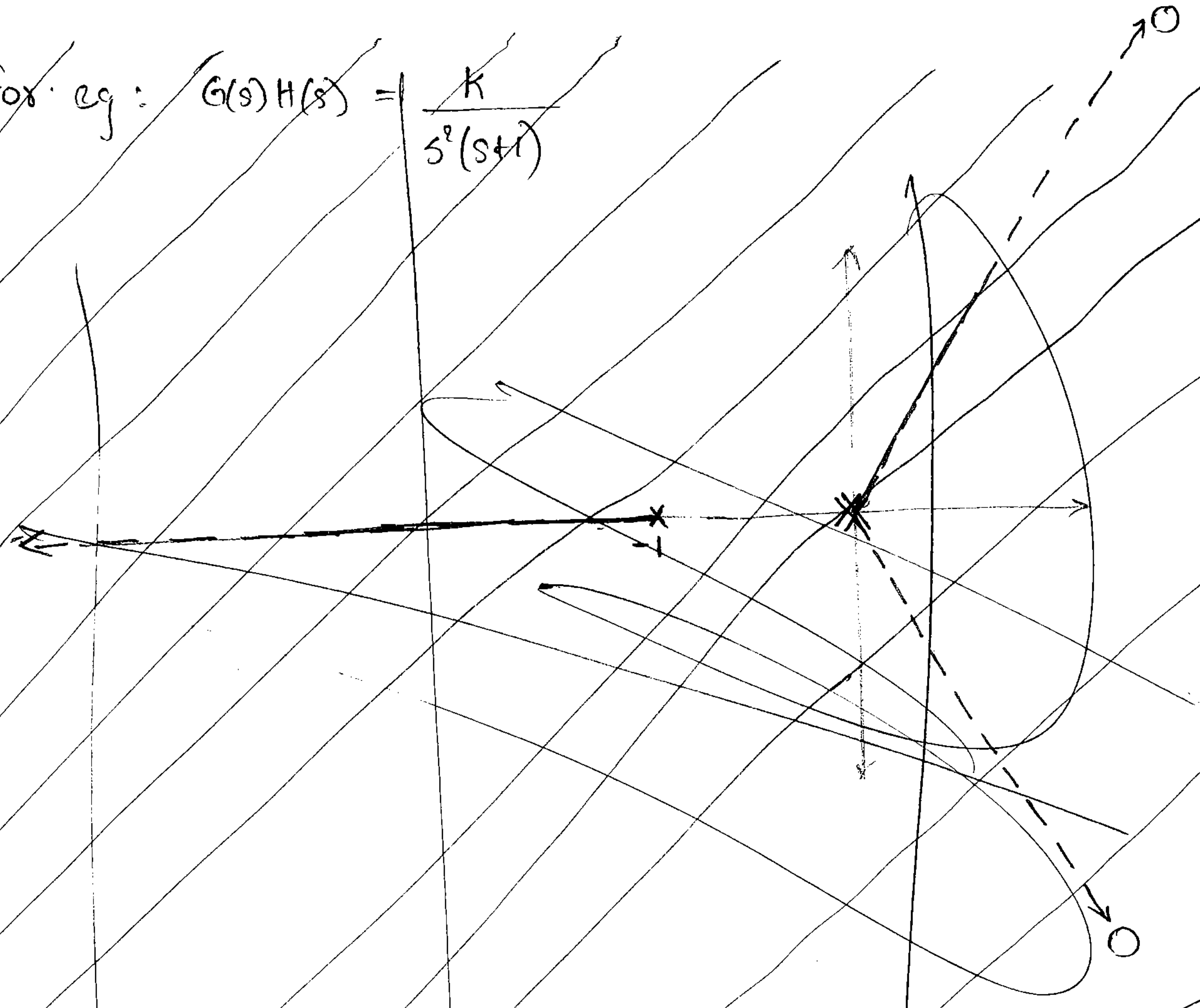
→ Bandwidth $\alpha \frac{1}{\text{Rise Time}}$.

→ when zero is added, Rise time decreases.

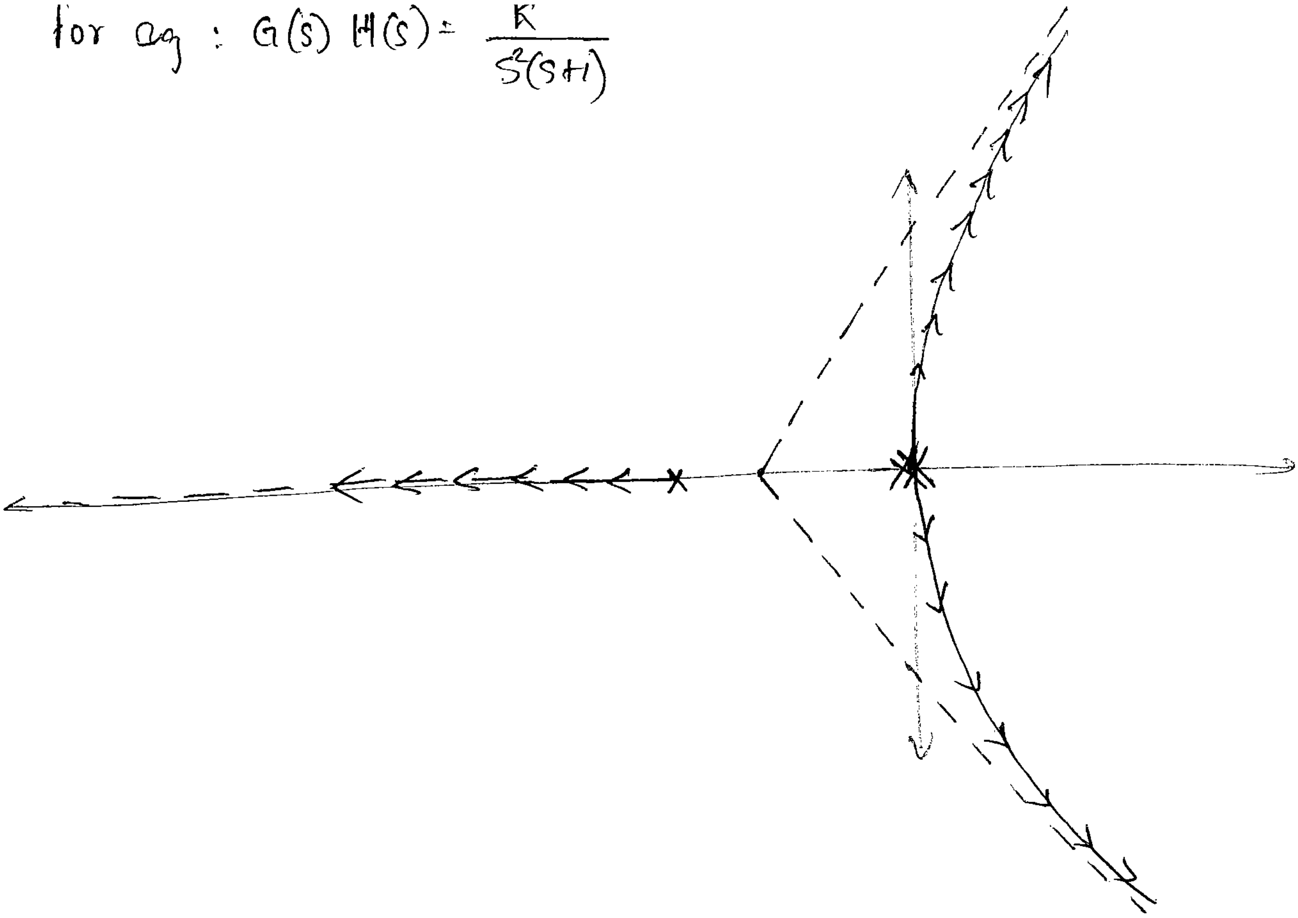
→ poles moves towards left - so the damping ratio increases. percentage overshoot ~~incr~~ decreases. Hence more stable

→ Time constant decreases. in turn Settling time decreases.

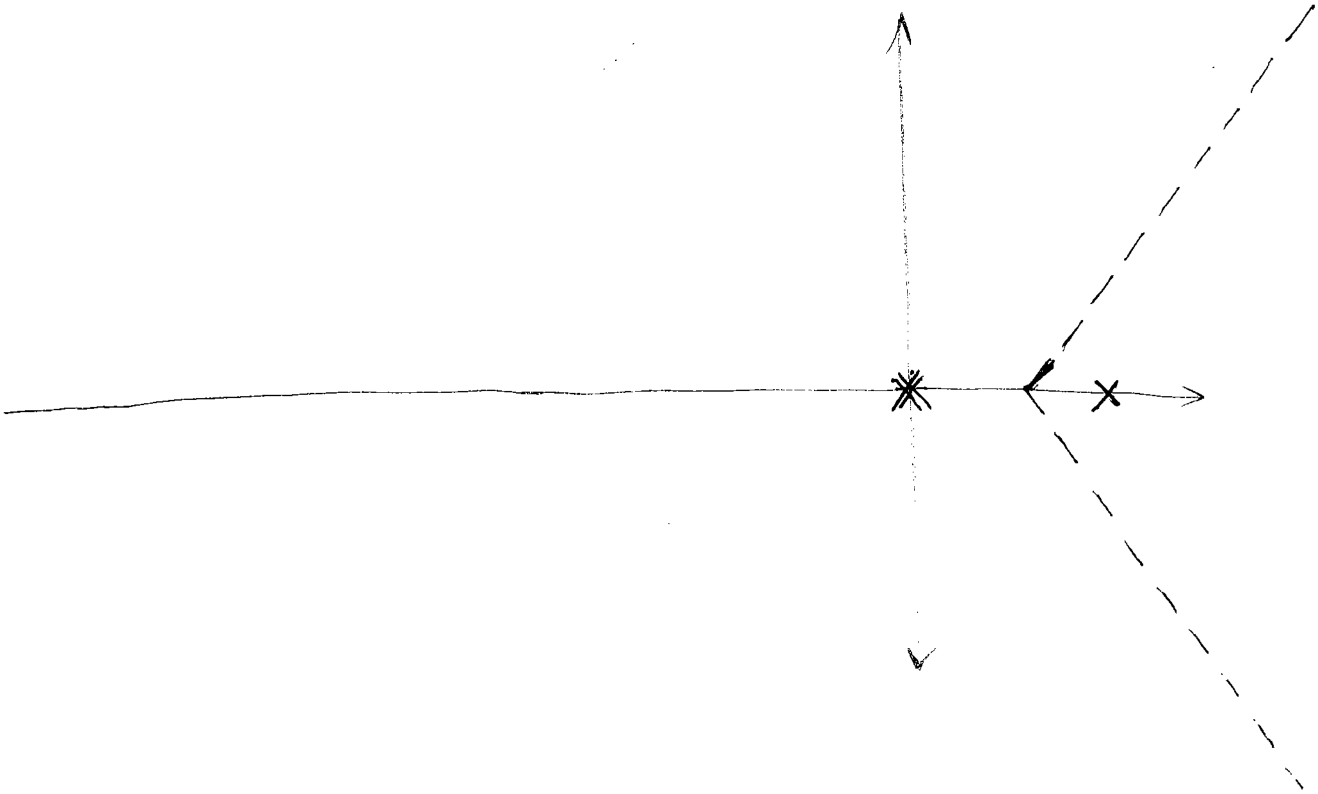
for eg: $G(s)H(s) = \dfrac{K}{s^2(s+1)}$



for eg : $G(s)H(s) = \frac{K}{s^2(s+1)}$



$G(s)H(s) = \frac{K}{s^2(s-1)}$



ADDITION OF POLES

→ The pole should

→ The root locus branches shifted towards the right side.

→ The relative stability decreases.

→ The breakpoint shifted towards right.

→ The range of K value for stability decreases.

→ Addition of poles decrease the B.W

→ $BW \propto \frac{1}{\text{Rise time}}$

→ ∴ Rise time increases.

→ poles move toward right. ξ_p decreases. %M_p increases.

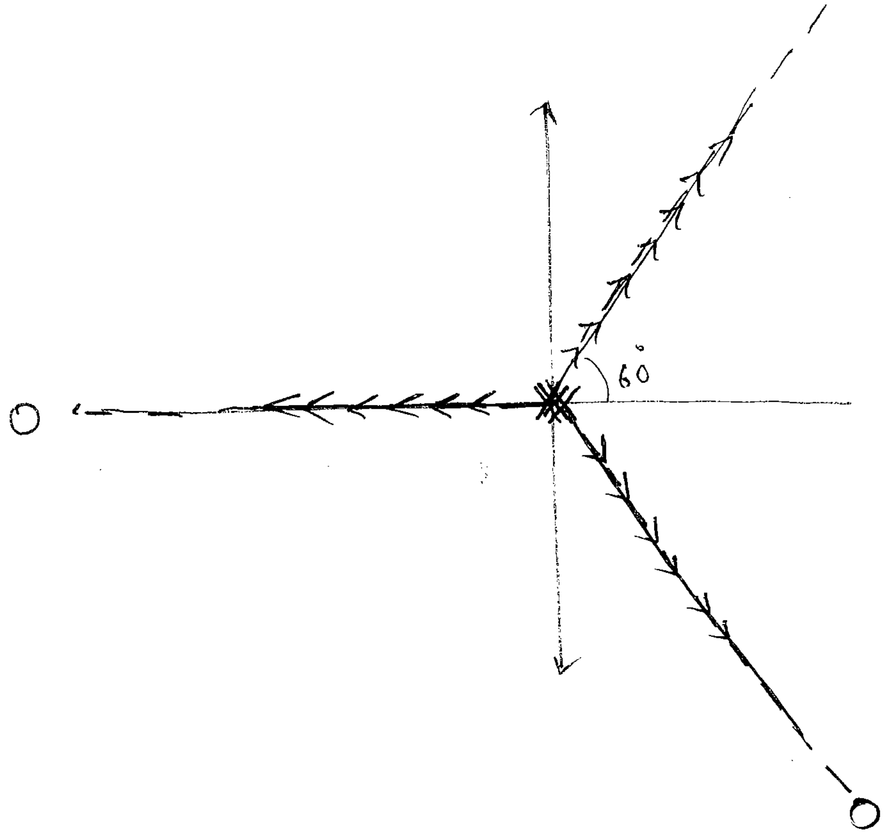
→ Hence become less stable.

→ Time constant increases, settling time increases.

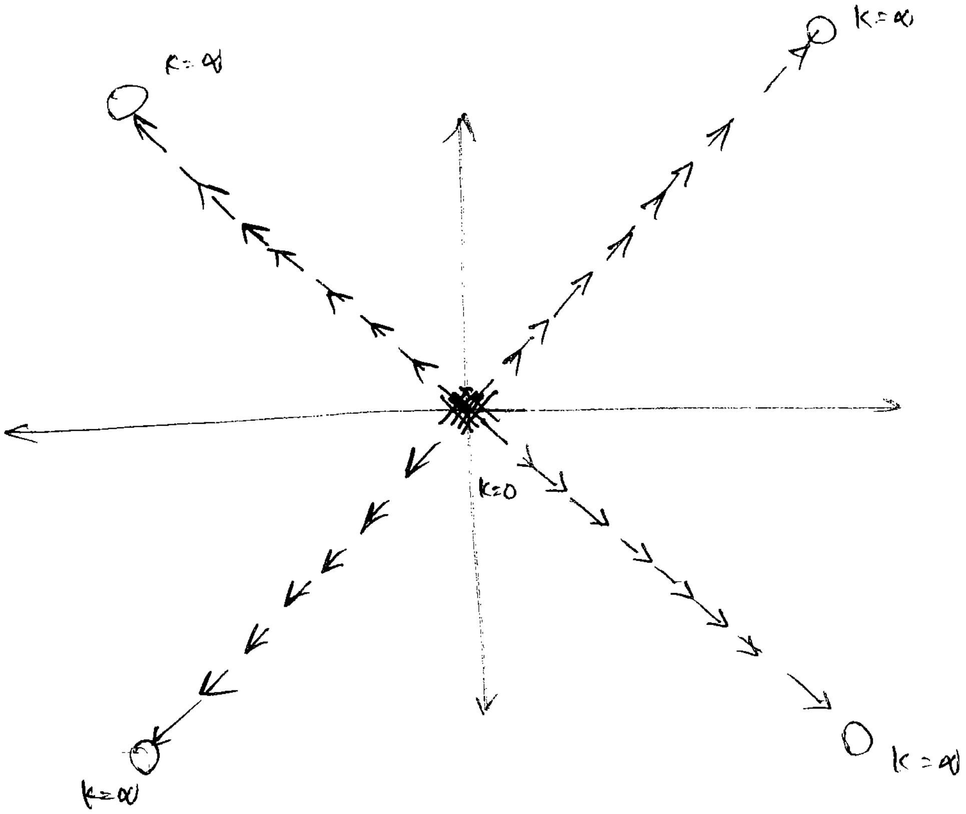
→ More number of poles → Low pass filter.

8 (iii)

Unstable.

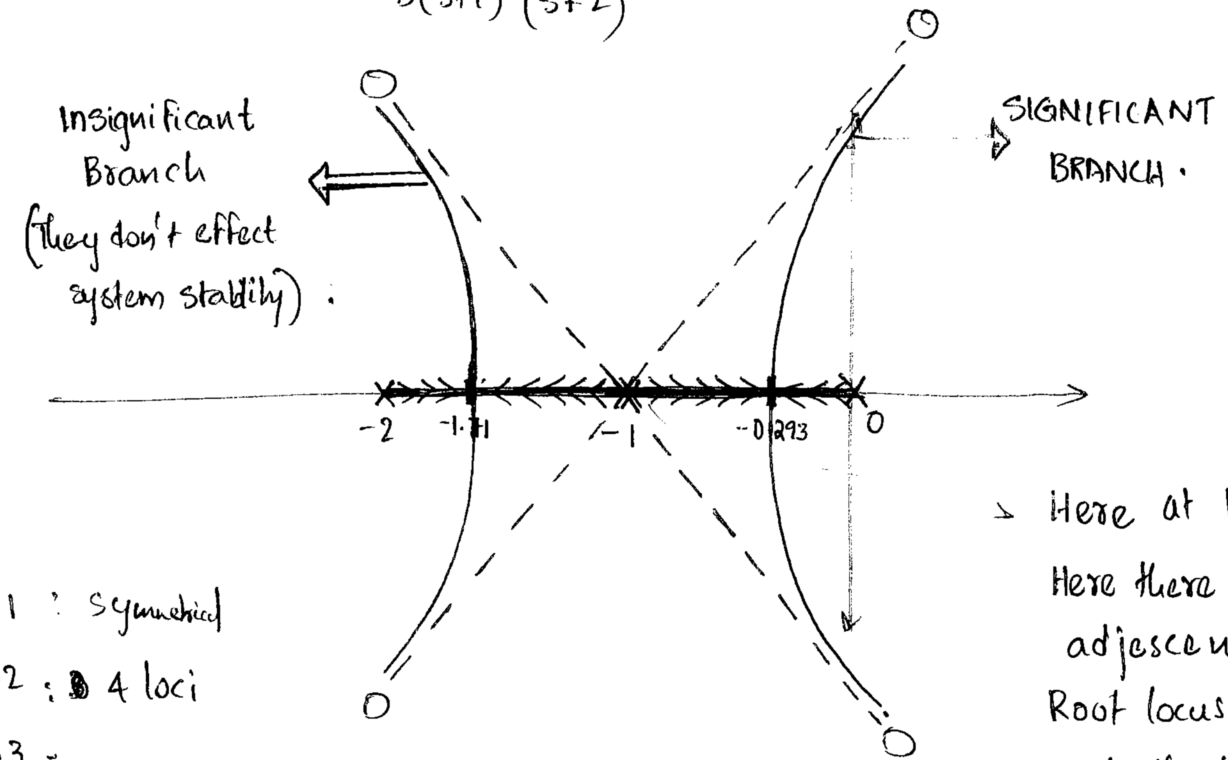


8(iv)



Unstable.

9, $G(s)H(s) = \dfrac{k}{s(s+1)^2(s+2)}$



S1 : Symmetrical

S2 : 4 loci

S3 :

S4 : Assymptotes,

$P - Z = \underline{\underline{4}}$

$\theta = (45°, 135°, 225, 315)$

$\sigma = \dfrac{-2-1-1}{4} = \underline{\underline{-1}}$

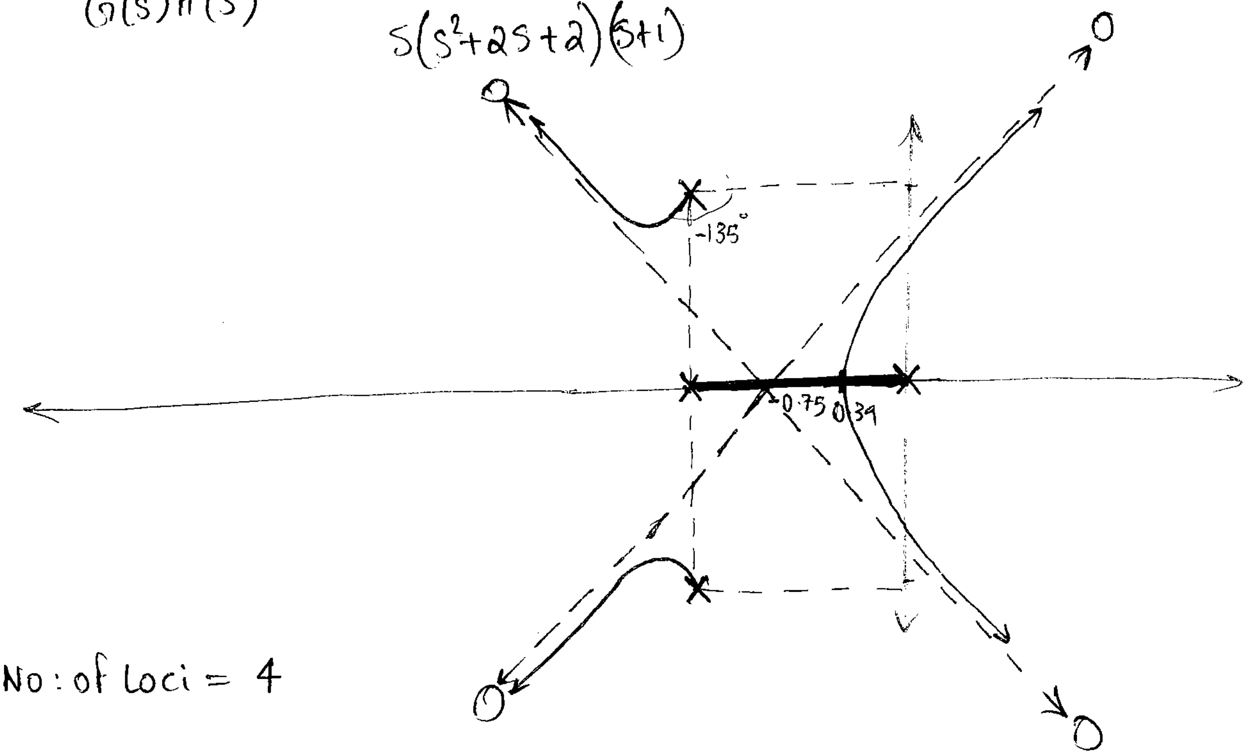
$B \cdot P = -0.293, -1, -1.71$

Here at B.P at -1, Here there are adjescent ~~breakpoi~~ Root locus branch on both sides. Hence will not ~~mo~~ leave real axis at -1. will move to either sides.

Note : whenever there having conjugate poles and also $|BP| > |\sigma|$ ie, the magnitude of the Breakpoint is greater than magnitude of centroid. Then the angle of departure at a complex pole is $< \mp 90^\circ$.

Q 12,

$$G(s)H(s) = \frac{K}{s(s^2+2s+2)(s+1)}$$



No: of Loci = 4

$\sigma = \underline{\underline{-0.75^\circ}}$

B.P = -0.394

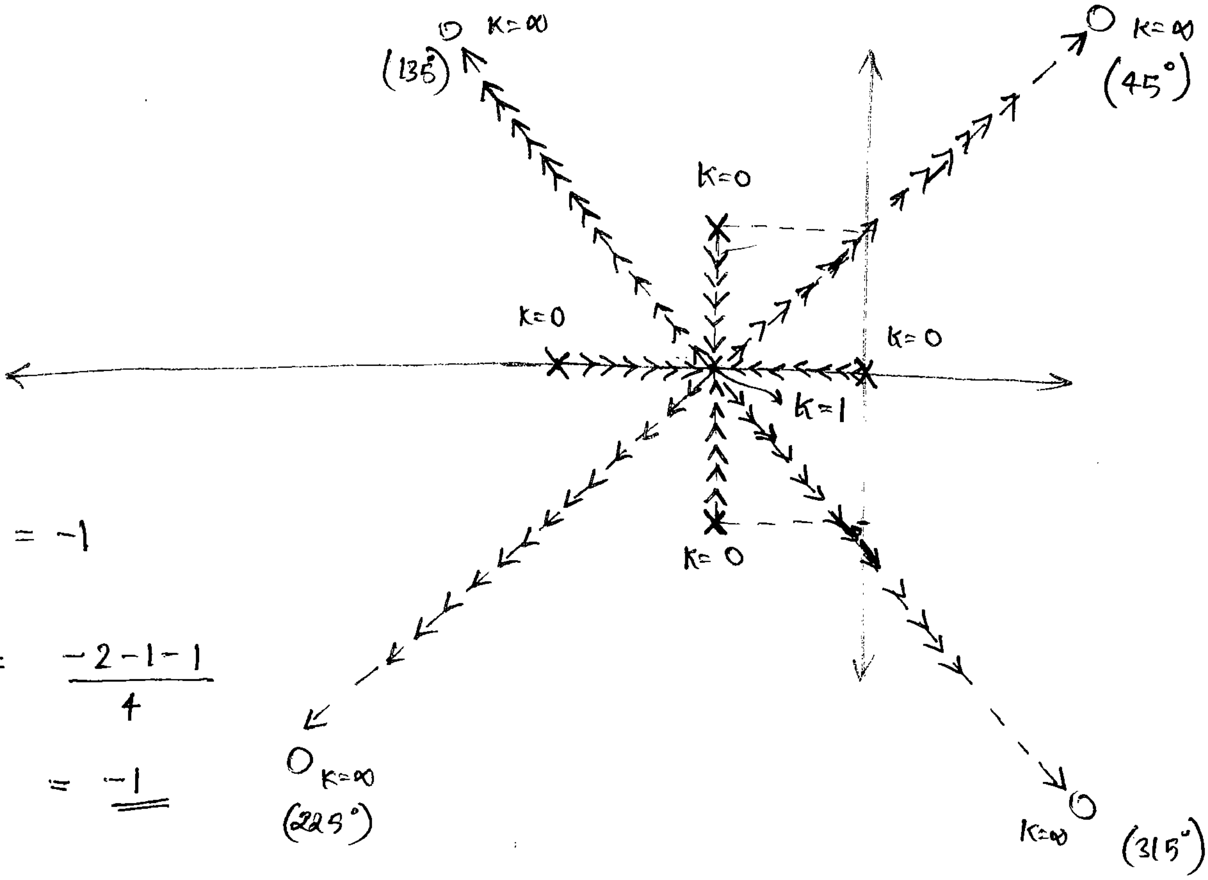
Assymptote = 4

$\theta = 45^\circ, 135^\circ, 225, 315.$

$\phi_d = \underline{\underline{-135^\circ}}$

Note whenever the Root Locus Diagram having complex conjugate pole and the magnitude of the Breakpoint is less than magnitude of the centroid, then the angle of departure at a complex pole is $> \mp 90^\circ$

13 $G(s)H(s) = \frac{K}{s(s^2+2s+2)(s+2)}$



$B \cdot P = -1$

$\sigma = \frac{-2-1-1}{4}$

$= \underline{\underline{-1}}$

$\theta = (45, 135, 225, 315^\circ)$

$BP = \sigma$, and all the poles symmetric to B·P. Then all the poles meet at the B·P

In the above diagram, 4 poles meet at the B·P. The K value at the B·P is 1.

$K = 1 \times 1 \times 1 \times 1 = \underline{\underline{1}}$

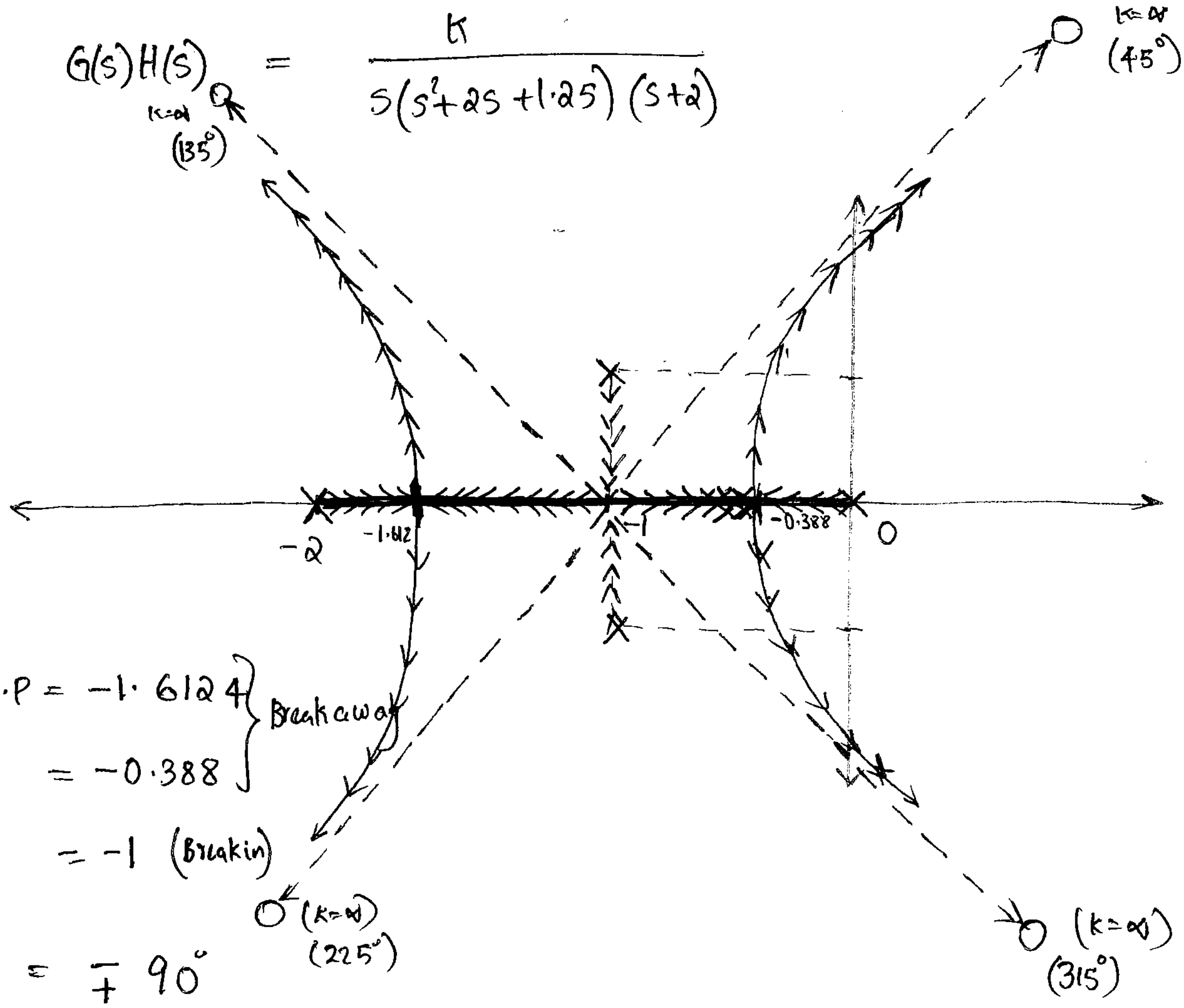
The CLTF at B·P is $\frac{1}{s(s^2+2s+2)(s+2)+1}$

$\frac{1}{(s+1)^4}$ (Directly writing, considering 4 poles at $s=-1$)

$\phi_d =$

Meet at Imaginary axis at $s = \pm j1$

14, $\quad G(s)H(s) = \dfrac{K}{s(s^2+2s+1.25)(s+2)}$



B.P = −1.6124 } Breakaway
= −0.388 }
= −1 (Breakin)

$\phi_d = \mp 90°$

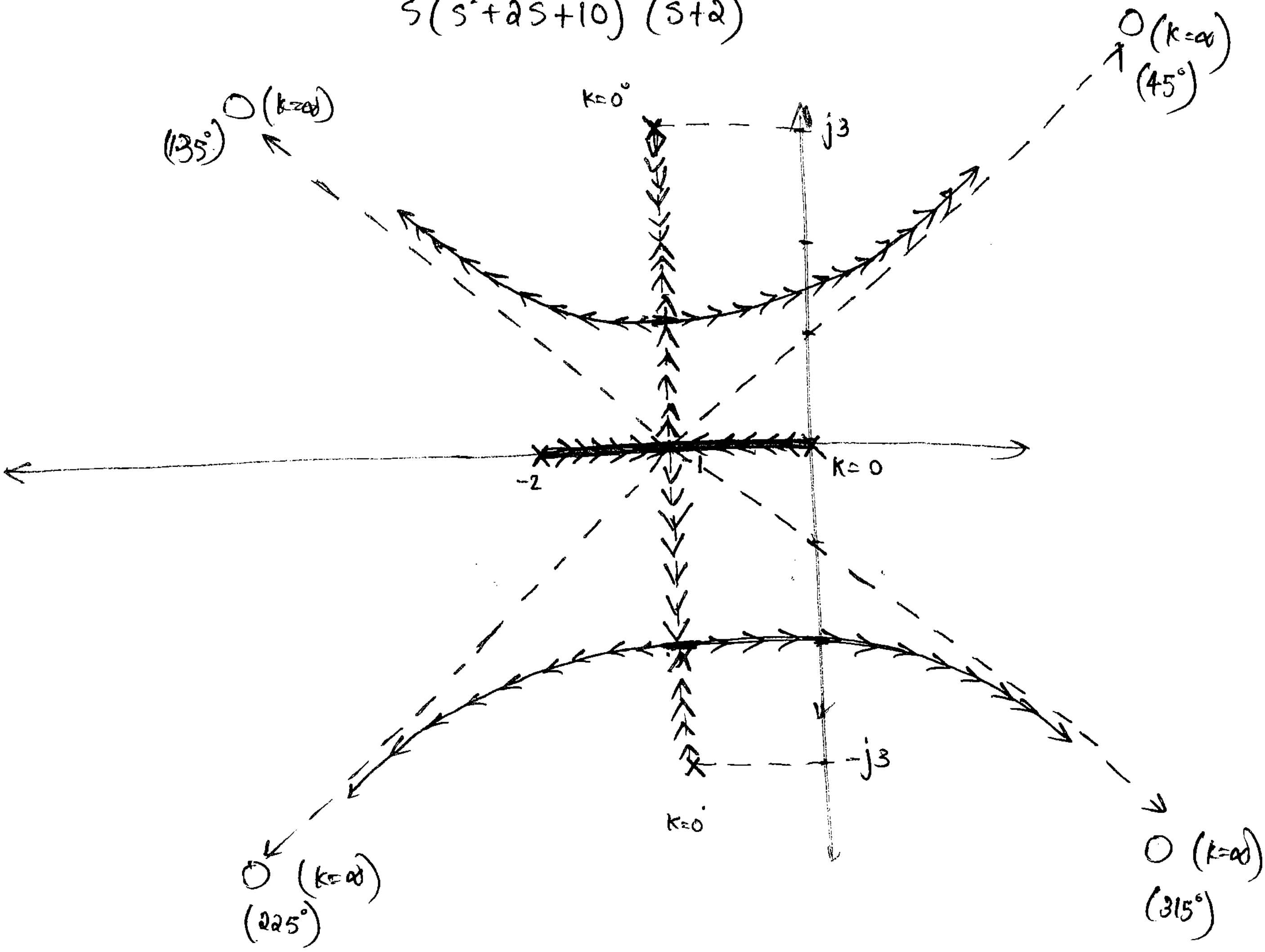
$\sigma = \dfrac{-2-1-1}{4} = \underline{\underline{-1}}$

⇒ whenever the B.P = Centroid and complex poles very close to real axis, then the number of B.Ps, on the real axis increases.

$BP = \sigma \Rightarrow \phi = \mp 90°$

15,

$$G(s)\,H(s) = \frac{K}{s(s^2+2s+10)(s+2)}$$

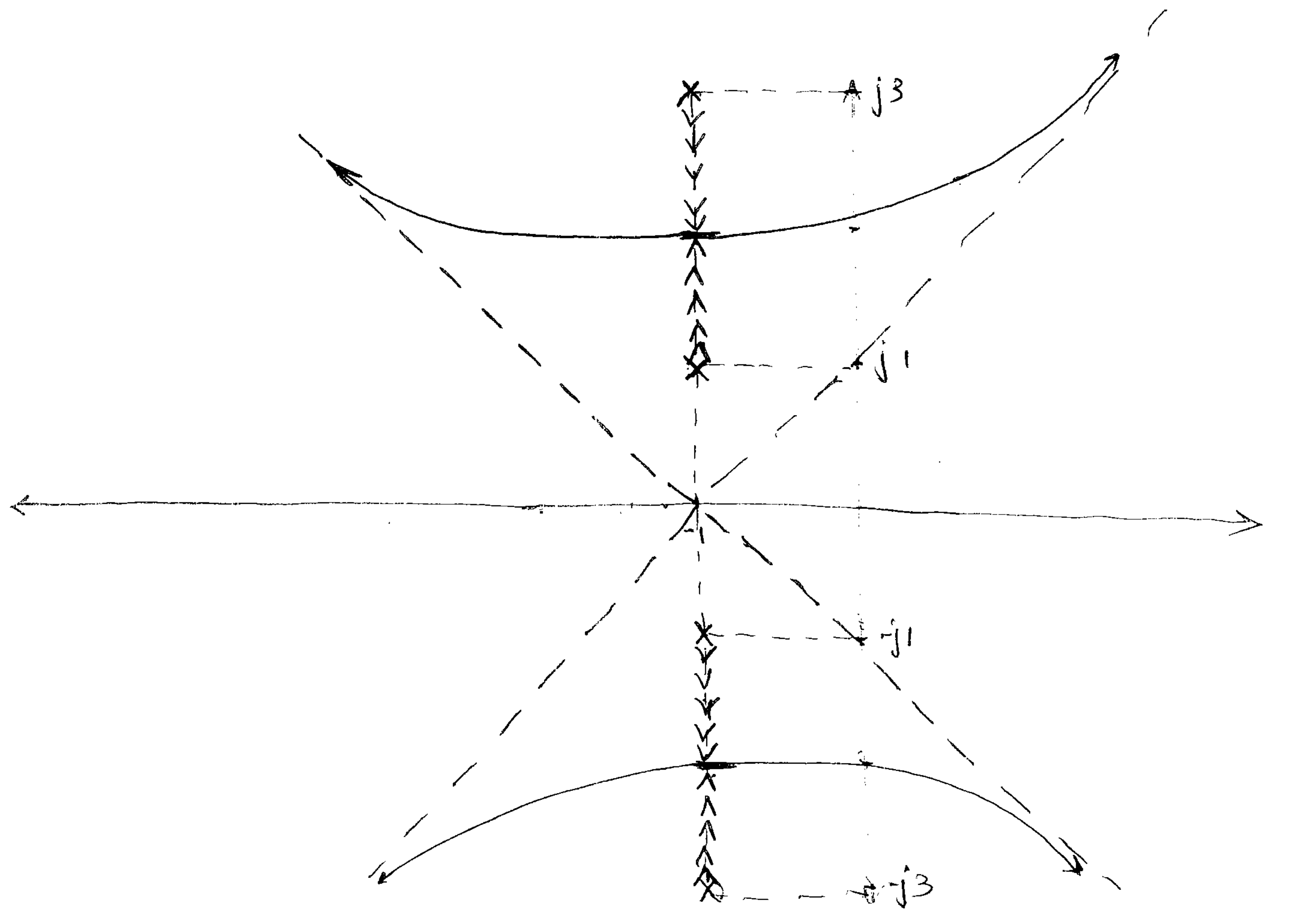


→ (Break point = Centroid and complex poles are far away from the real axis, 1 Real axis B.P and 1 pair of complex B.P will be existing)

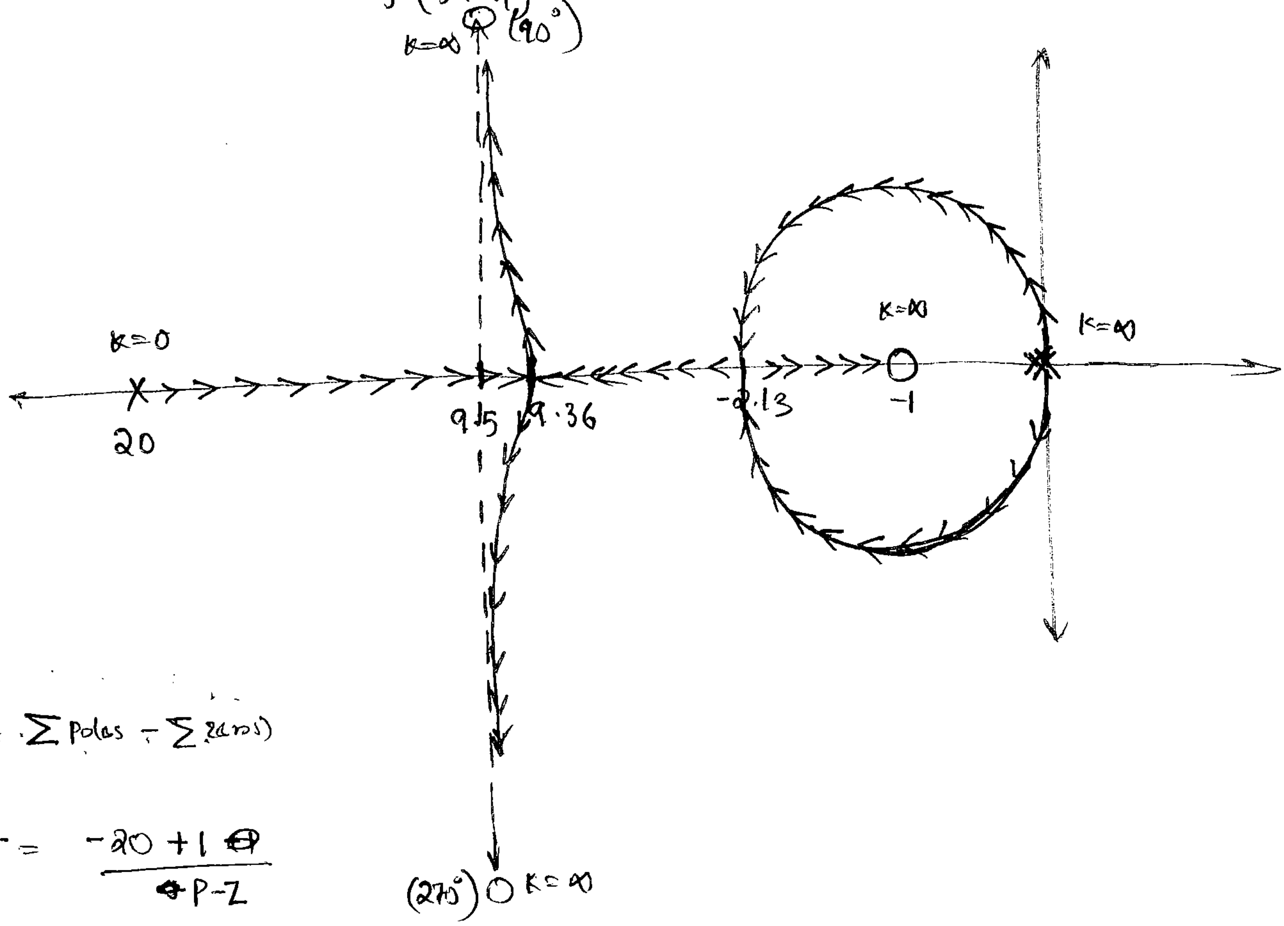
σ = Real part of a Complex pole = BP

- case (i) → Symmetrical poles.
- case (ii) → Complex poles very close to Real axis → 3 BPs
  - 1 BIP
  - 2 BAP
- case (iii) → Complex poles far away from Real axis. → 3 BPs
  - 1 BIP
  - 1 pair of complex BAP

$$\Rightarrow G(s)H(s) = \frac{K}{(s^2+2s+2)(s^2+2s+10)}$$



Q, $G(s)H(s) = \dfrac{K(s+1)}{s^2(s+k_1)}$ $\qquad k_1 = 20$

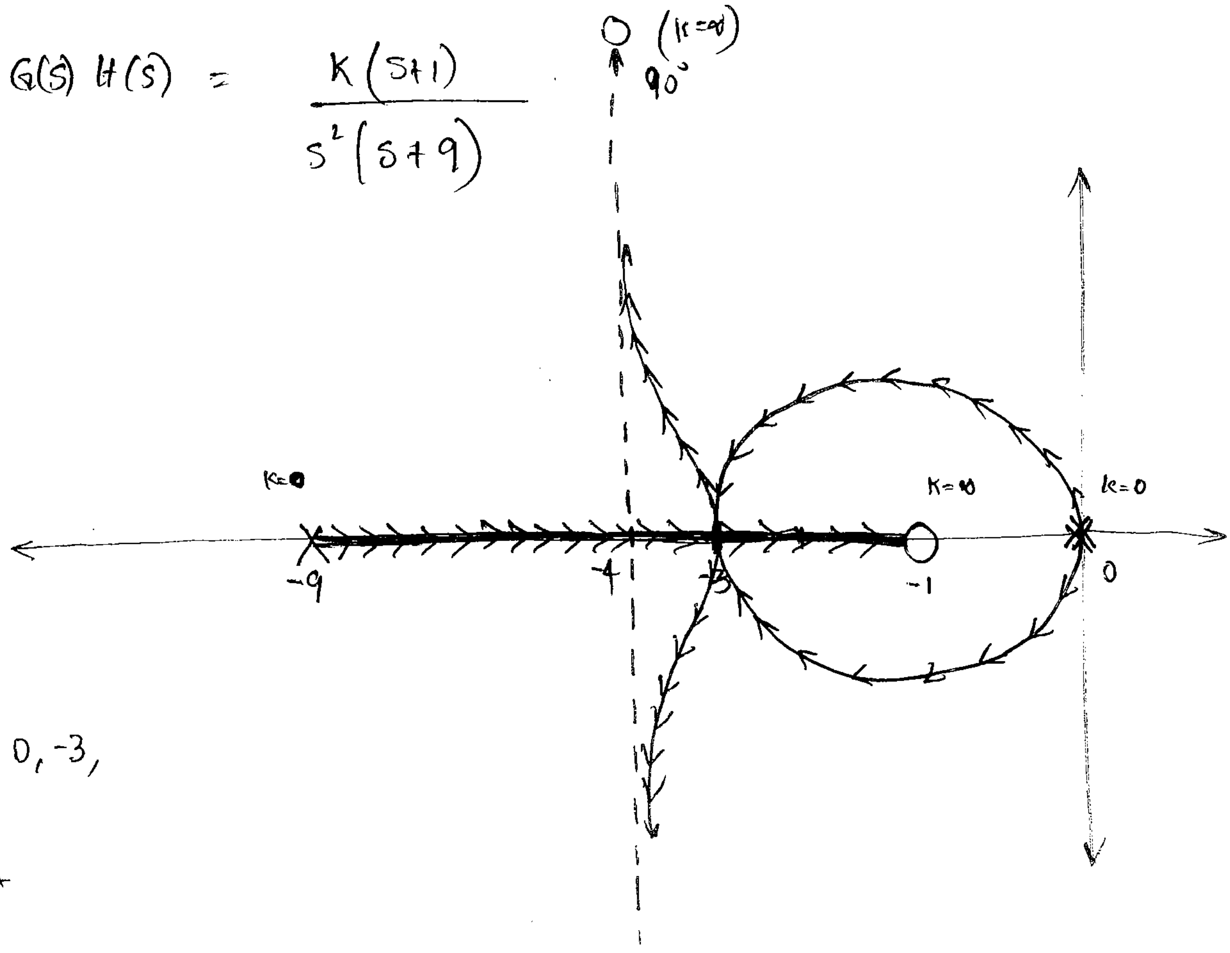


σ = ΣPoles − ΣZeros)

$$\sigma = \frac{-20 + 1}{P - Z}$$

$= \frac{-19}{2} = -9.5$

Q, $G(s)H(s) = \dfrac{K(s+1)}{s^2(s+9)}$



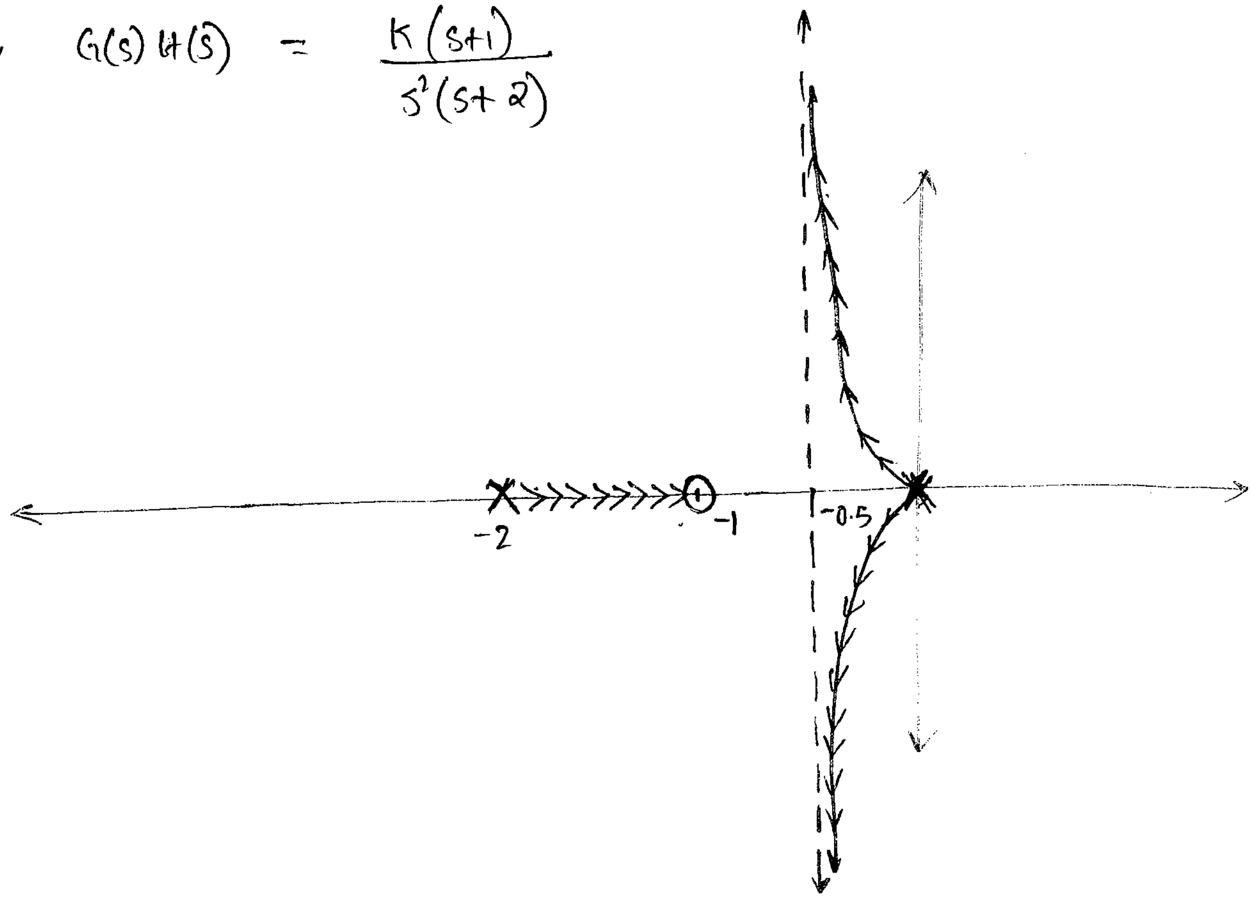
2B·P = 0, -3,

σ = -4

In the above problem, three poles meet at the B.P -3.

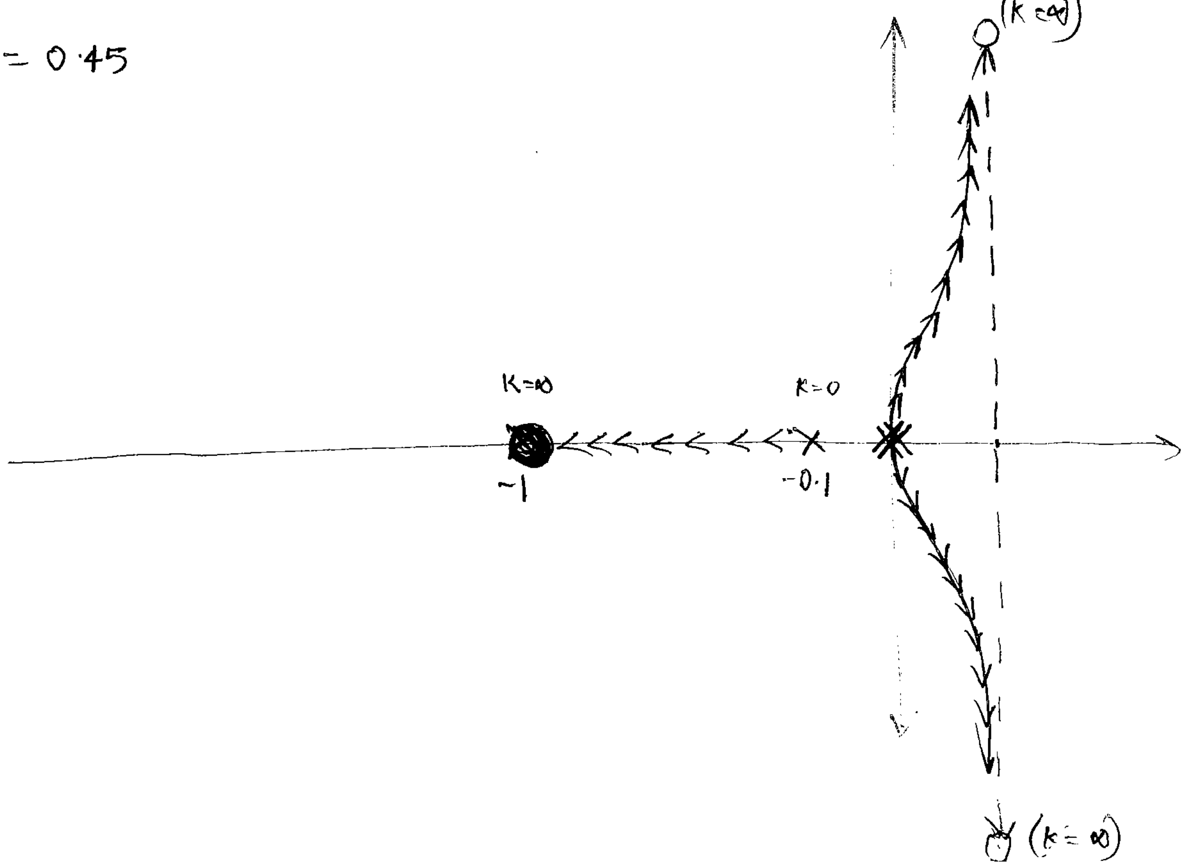
~~Now~~

Q, $G(s)H(s) = \dfrac{K(s+1)}{s^2(s+2)}$



Q, $G(s)H(s) = \dfrac{K(s+1)}{s^2(s+0.1)}$

$\sigma = 0.45$



ROOT CONTOUR

whenever the transfer function consists the multiple unknown parameters by varying all the parameters from zero to Infinity drawing a root locus diagram is called Root Contour.

Q. Draw the Root Contour to the given characteristic equn.

$$s^2 + as + K = 0 \longrightarrow (1)$$

Case (i)

"K" as a system Gain / parameter (X) vary from $0 \rightarrow \infty$

"a" as a Constant.

No of cases = No: of unknown parameters.

In one case, vary 1 unknown and take all others constant.

Finding CE in to the Form $1 + G(s)H(s) = 0$

For that divide $s^2 + as$ (remain terms other than K)

from (1)

$$1 + G(s)H(s) = 1 + \frac{K}{s^2 + as}.$$

To draw a Root locus diagram, the OLTF must be in the form of system gain.

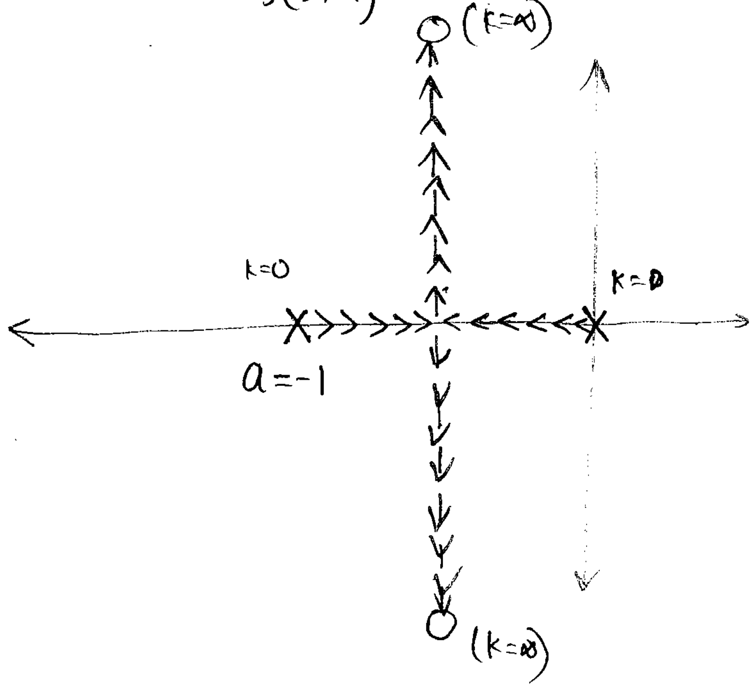
$$G(s)H(s) = \frac{K}{s^2 + as}$$

if it is $s^2 + as + K(s+1) = 0$ then $G(s)H(s) = \frac{K(s+1)}{s^2 + as}$

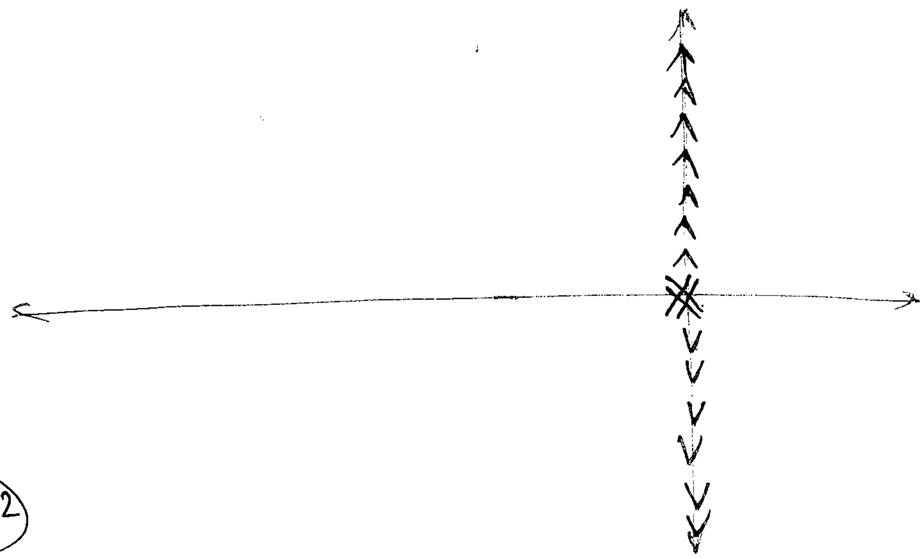
$$G(s)H(s) = \frac{k}{s(s+a)}$$

$\left.\begin{matrix} k>0 \\ a>0 \end{matrix}\right\}$ STABLE

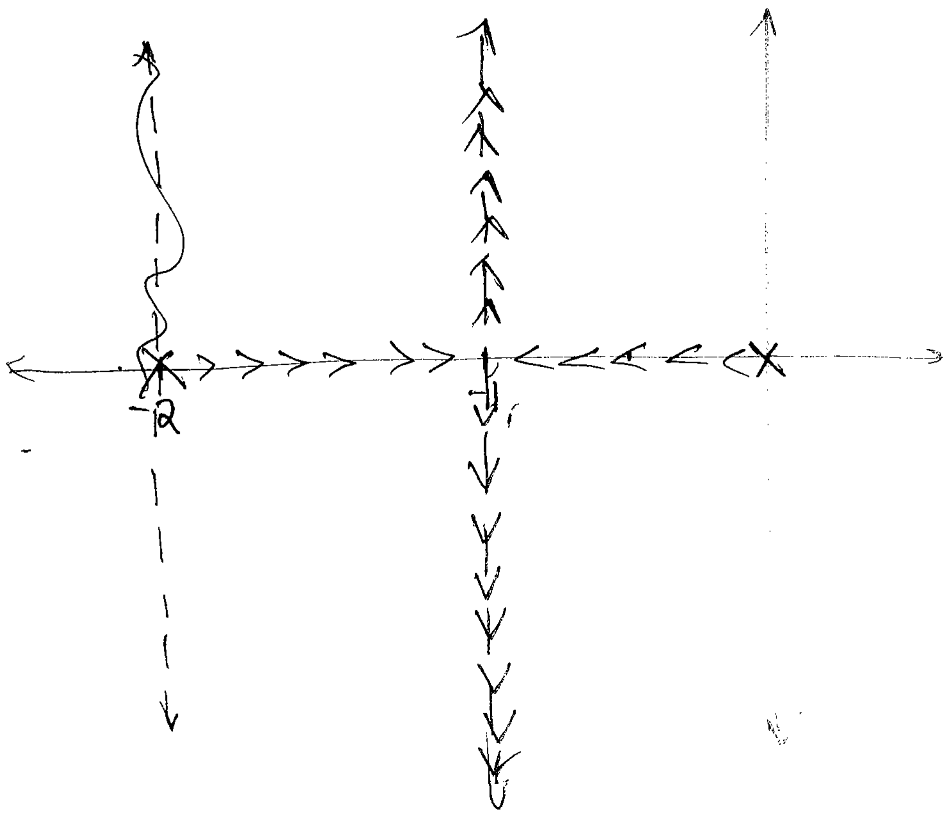
(a = -1)



(a = 0)



(a = -2)



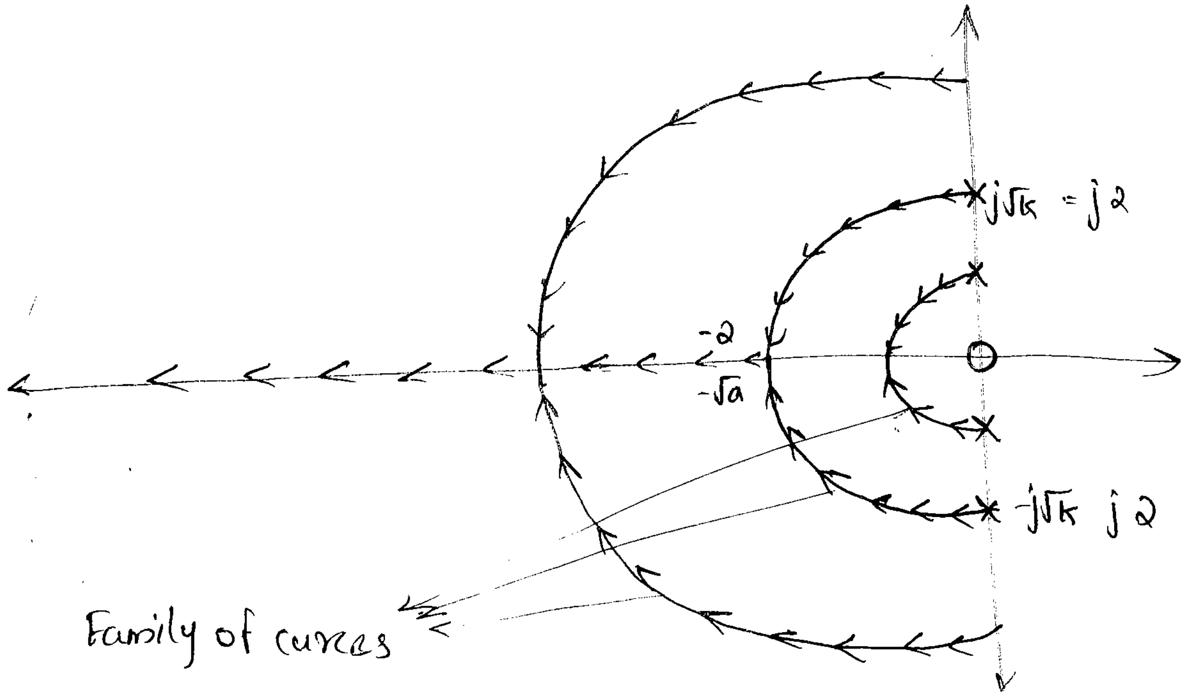
FAMILY OF CURVE with $0<a<\infty$

Case(ii)

a is the system paramter.

$K \Rightarrow$ constant

$$GH(s) = \frac{as}{s^2+K}$$



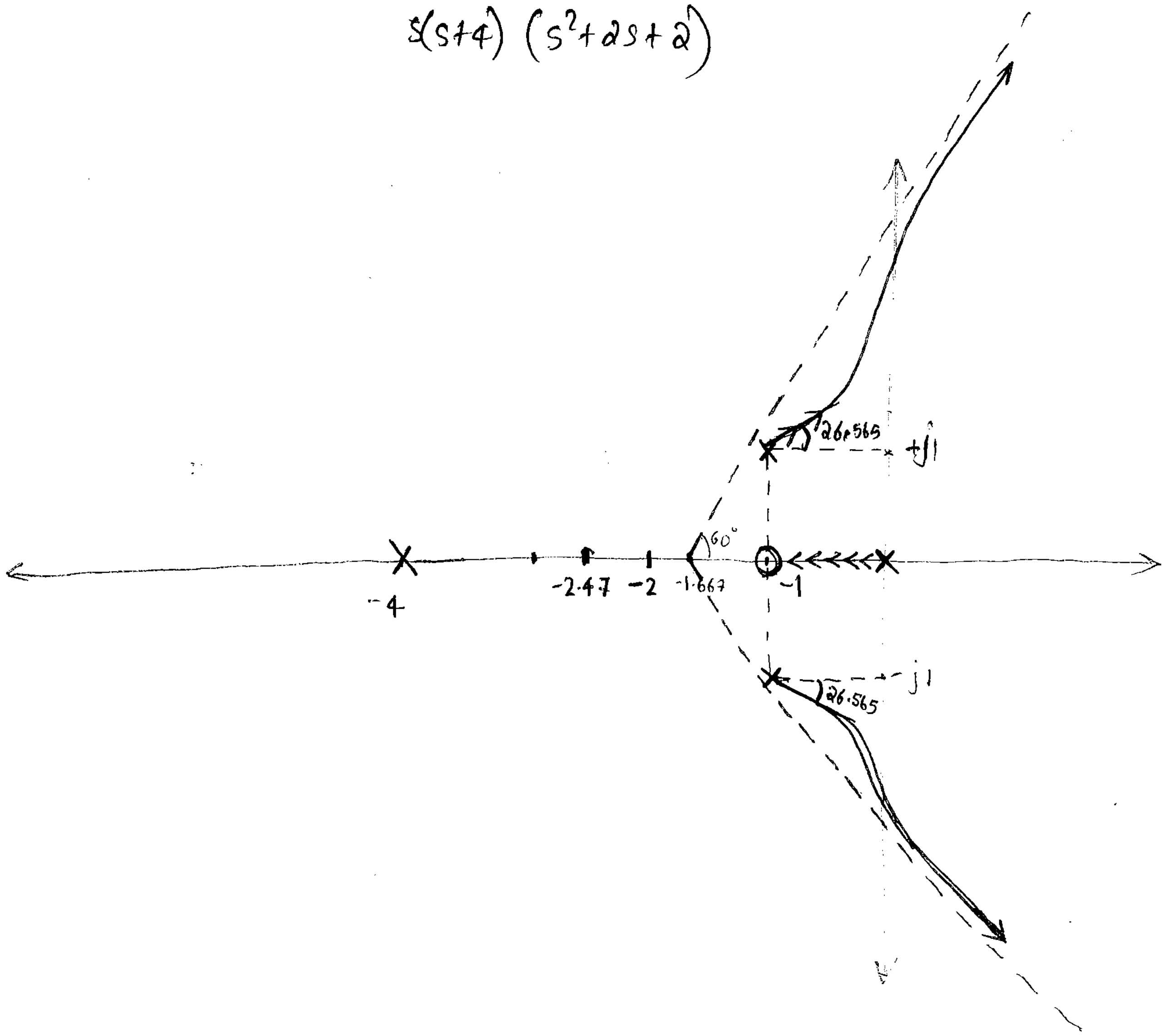
Consider a Control system with Characteristic equation.

$$s(s+4)(s^2+2s+2) + K(s+1) = 0$$

Draw the root locus diagram and label the important values and find angle of assymptotes angle of deparhure and intersection point with imaginary.

$$1 + G(s)H(s) = 1 + \frac{K(s+1)}{s(s+4)(s^2+2s+2)}$$

$$G(s)H(s) = \frac{K(s+1)}{s(s+4)(s^2+2s+2)}$$



$P-Z = 4-1$
$= \underline{\underline{3}}$

$\theta = \frac{(2q+1)\,180}{3}$
$= 60^\circ, 180^\circ, 300^\circ$

$\sigma = \frac{-1-1-4+1}{3} = \cancel{\frac{-6}{4}} \cancel{=-1.5}\ \ -5/3 = \underline{\underline{-1.667}}$

$\cancel{(s^2+4s)(s^2+}$

$(s^2+4s)(s^2+2s+2)+K(s+1)=0$

$K = \frac{-(s^2+4s)(s^2+2s+2)}{(s+1)}$

$\frac{dK}{ds} \Rightarrow 0 \qquad \frac{-(s+1)\left[(s^2+4s)(2s+2)+(s^2+2s+2)(2s+4)\right]+(s^2+4s)(s^2+2s+2)}{(s+1)^2}$

$\frac{-(s+1)\left[2s^3+2s^2+8s^2+8s+2s^3+4s^2+4s^2+8s+4s+8\right]+s^4+2s^3+2s^2+4s^3+8s^2+8s}{} = 0$

$-(s+1)\left[4s^3+18s^2+20s+8\right]+s^4+6s^3+10s^2+8s = 0$

$-\left[4s^4+18s^3+20s^2+8s+4s^3+18s^2+20s+8\right]+s^4+6s^3+10s^2+8s=0$

$-\left[4s^4+22s^3+38s^2+28s+8\right]+s^4+6s^3+10s^2+8s=0$

4 3 2

$$3s^4 + 16s^3 + 28s^2 + 20s + 8 = 0$$

~~$3 - 16 + 28 - 20 + 8 \Rightarrow$~~

$\therefore s = -2$ satisfies above eqn.

$$
\begin{array}{r|l}
 & 3s^3 + 10s^2 + 8s + 4 \\
\hline
s+2 & 3s^4 + 16s^3 + 28s^2 + 20s + 8 \\
 & 3s^4 \mp 6s^3 \\
\hline
 & 10s^3 + 28s^2 \\
 & 10s^3 \mp 20s^2 \\
\hline
 & 8s^2 + 20s \\
 & 8s^2 + 16s \\
\hline
 & 4s + 8 \\
 & 4s + 8 \\
\hline
 & 0
\end{array}
$$

~~$(s+2)(3s^3 + 10s^2 + 4) = 0$~~

$$3s^4 + 16s^3 + 28s^2 + 20s + 8 = 0$$

$$(s+2)(3s^3 + 10s^2 + 8s + 4) = 0$$

$s = -2$

$s = -2.473$

$s = -0.43 + 0.595j$

$s = -0.43 - 0.595j$

$$\angle G(s)H(s)\Big|_{-1+j} = \frac{\angle K \ \angle(s+1)}{\angle(s)\ \angle(s+4)\ \ \angle(s+1+j)\ \angle(s+1-j)}$$

$$= \frac{\angle K \ \angle \cancel{-1+j+1}}{\angle -1+j \ \ \angle -1+j+4 \ \ \angle \cancel{-1}+j+\cancel{1}+j \ \ \angle -1+j+1-j}$$

$$= \frac{0 + 90^\circ}{135^\circ + 18.435 + 90 + 0}$$

$$= \cancel{90^\circ - 135^\circ - 18.435 - 90^\circ}$$

$$= \underline{\underline{-153.435}}$$

$$\phi_d = 180 + (-153.435)$$

$$= \underline{\underline{+26.565}}$$

Angle of deparhure $= \underline{\underline{\pm 26.565}}$

$$s^4 + 4s^2 + 2s^2 + 4s^3 + 8s^2 + 8s + Ks + K = 0$$

$$s^4 + 4s^3 + 14s^2 + (8+K)s + K = 0$$

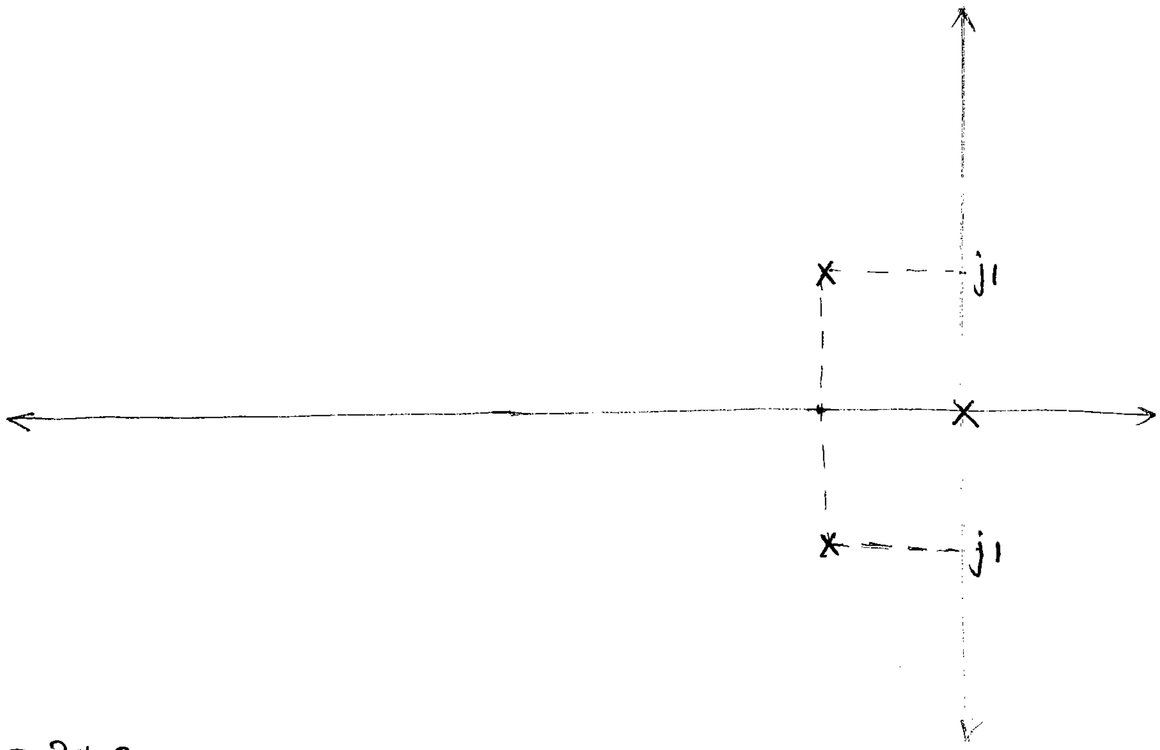
s^4	1	14	K
s^3	4	8+K	
s^2	$\frac{\cancel{56}\,48-K}{4}$	K	
s^1			
s^0	K		

$$\frac{48-K}{4}(8+K) - 4K = 0$$

$$\frac{48-K}{4}$$

$$(48-K)(8+K) - 16K = 0$$

$$384 - 8K + 48K - K^2 - 16K = 0$$



$K = 24.78 ms$.

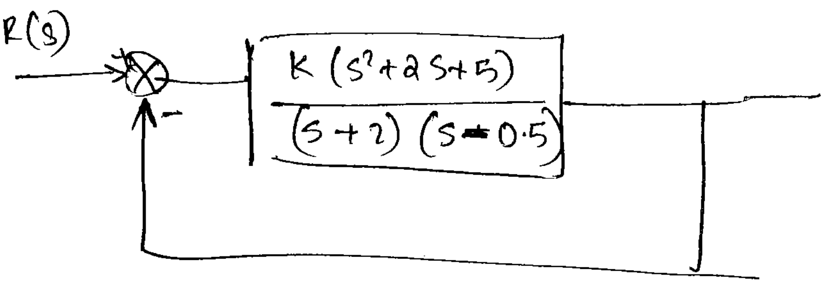
No B.P.

Check & do again.

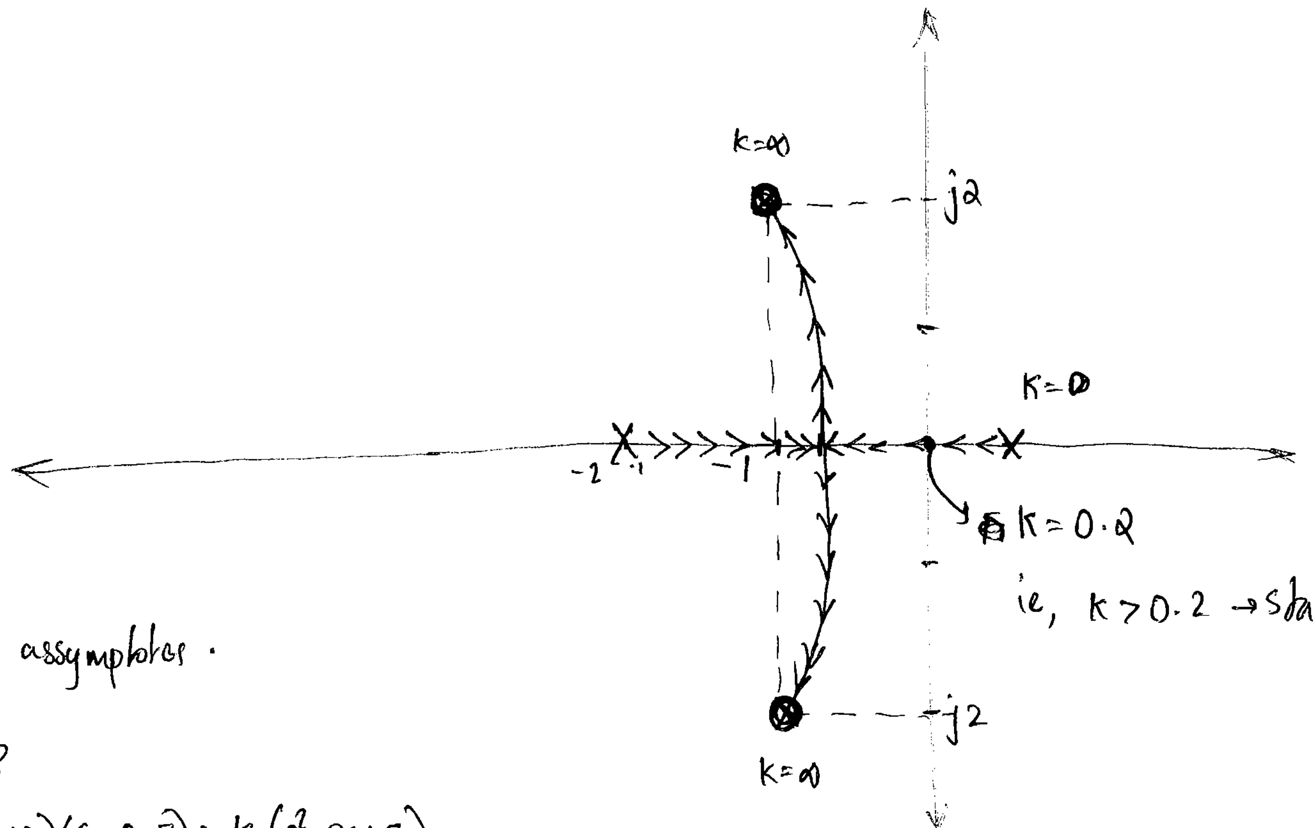
Q, consider the following control system. sketch the root locus diagram $0<k<\infty$. Determine the k value that gives the system CE of damping ratio 0.5.

$\xi = 0.5$

$\theta = 60^\circ$



$$G(s)H(s) = \frac{k(s^2+2s+5)}{(s+2)(s-0.5)}$$



No assymptotes.

B.P

$(s+2)(s-0.5) + K(s^2+2s+5)$

$$K = -\frac{(s+2)(s-0.5)}{s^2+2s+5}$$

$$K = -\frac{(s^2+1.5s-1)}{s^2+2s+5}$$

$$\frac{dK}{ds} \Rightarrow 0 \qquad -(s^2+2s+5)(2s+1.5) + (s^2+1.5s-1)(2s+2) = 0$$

$$-(2s^3 + \cancel{2s^3} + 1.5s^2 + 4s^2 + 3s + 10s + 7.5)$$

$$+(2s^3 + 2s^2 + 3s^2 + 3s - 2s - 2) = 0$$

$$\cancel{-2s^3} - \cancel{5.5s^2} - 13s - 7.5 + \cancel{2s^3} + \cancel{5s^2} + s - 2 = 0$$

$$-0.5s^2 - 12s - 9.5 = 0$$

$$0.5s^2 + 12s + 9.5 = 0$$

B.P $\quad s = -0.8197$ ✓

$\times \quad s = -23.180$

$$\angle G(s)H(s)\Big|_{-1+2j} = \frac{\angle K \ \angle s+1+2j \ \angle s+1-2j}{\angle s+2 \ \angle s-0.5}$$

$$= \frac{\angle K \ \angle -1+2j+1+2j \ \cancel{\angle -1+2j+1-2j}}{\angle -1+2j+2 \ \angle -1+2j-0.5} \qquad \frac{2}{3}$$

$$= 0 + 90 + 0 - \cancel{45}\, 63.435 - \angle 126.869 = -100.30$$

$$\phi_a = 180 + (100.304) = \underline{\underline{280.304}}$$

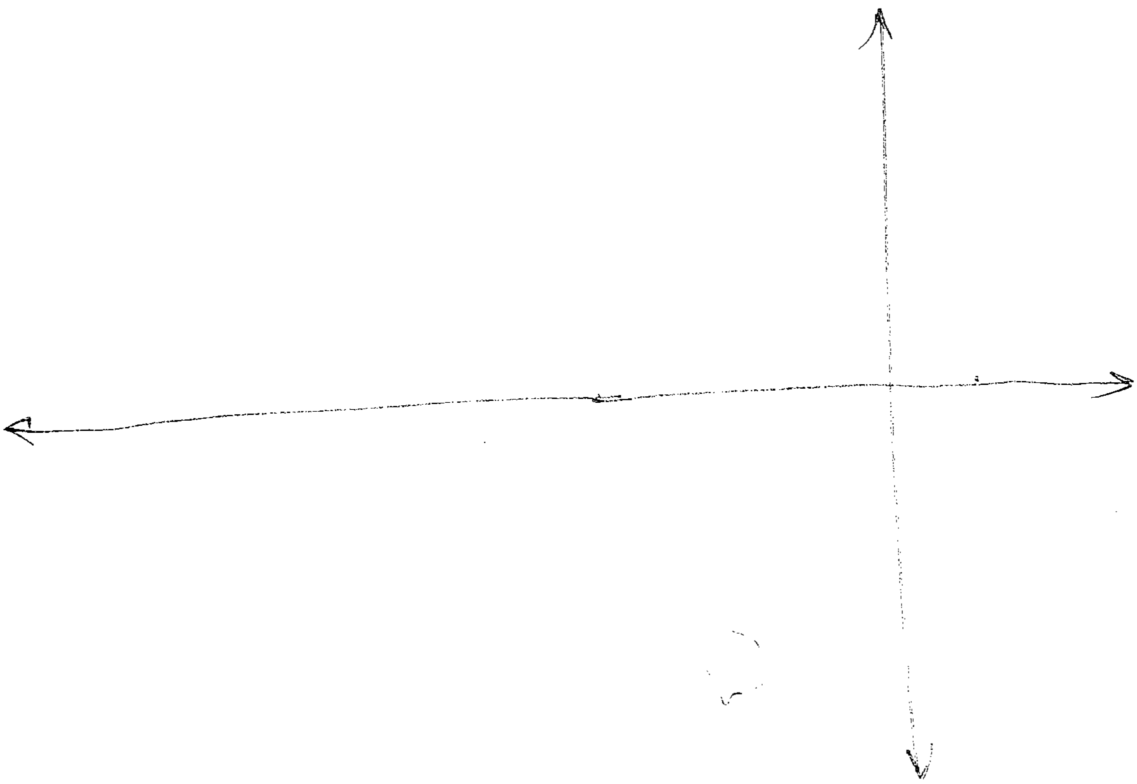
To get the stability condition, we require K at $s=0$

$$\left|\frac{K(5)}{2\times -0.5}\right| = 1$$

$$\left|\frac{K\times 5}{-2\times\frac{1}{2}}\right| = 1$$

$$K = \frac{1}{5} = 0.2$$

$K > 0.2 \rightarrow$ system stable.



(c) $\Rightarrow$

$$1 + G(s) = 0$$

$$(s+2)(s-0.5) + k(s^2+2s+5) = 0$$

$$s^2 - 0.5s + 2s - 1 + k(s^2+2s+5) = 0$$

$$s^2(k+1) + s(1.5+2k) + (5k-1) = 0$$

$$s^2 + s\left(\frac{1.5+2k}{k+1}\right) + \left(\frac{5k-1}{k+1}\right) = 0$$

compare it with std.

$$s^2 + 2\xi\omega_n + \omega_n^2$$

$$\omega_n = \sqrt{\frac{5k-1}{k+1}}$$

$$2\xi\omega_n = \frac{1.5+2k}{k+1}$$

$$\left(\frac{1.5+2k}{(k+1)}\right) = \sqrt{\frac{5k-1}{k+1}}$$

$$\frac{4k^2+6k+2.25}{\cancel{k^2+2k+1}\ \cancel{(k+1)^2}} = \frac{5k-1}{\cancel{k+1}}$$

$$\cancel{(4k^2+6k+2.25)(k+1) = (k^2+2k+1)(5k-1)}$$

$$\cancel{4k^3+6k^2+2.25k+4k^2+6k+2.25 = 5k^3+10k^2+5k-k^2-2k-1}$$

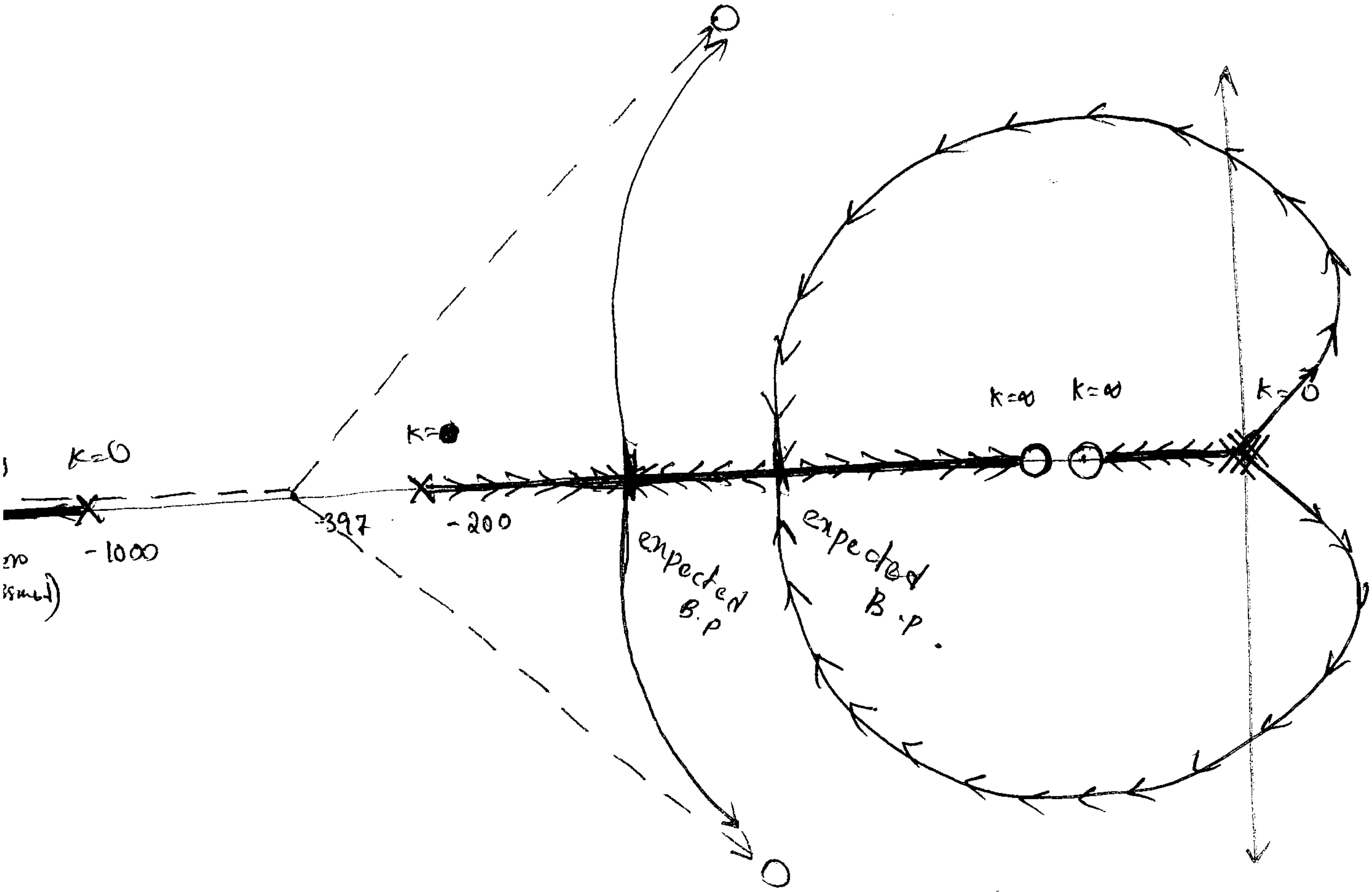
$$4k^2+6k+2.25 = (k+1)(5k-1) \qquad K = 3.066 \text{ get } \xi = 0.5$$

$$(4-5)k^2 + k(6-4) + 2.25 + 1 = 0$$

Q, The OLTF of a unity feedback system is

$$G(S) = \frac{K(s+4)(s+5)}{s^3(s+200)(s+1000)}.$$

Construct the root locus diagram and comment on stability



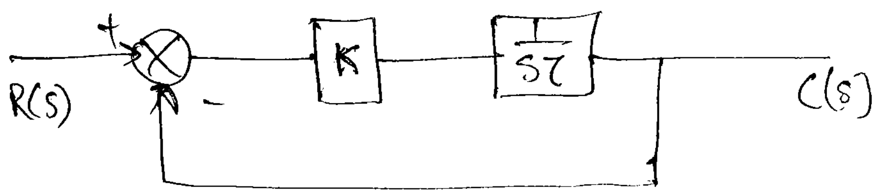
$$\sigma = \frac{-1000+9}{5-2} = -397$$

$$K(s+4)(s+5) + s^3(s+200)(s+1000) = 0$$

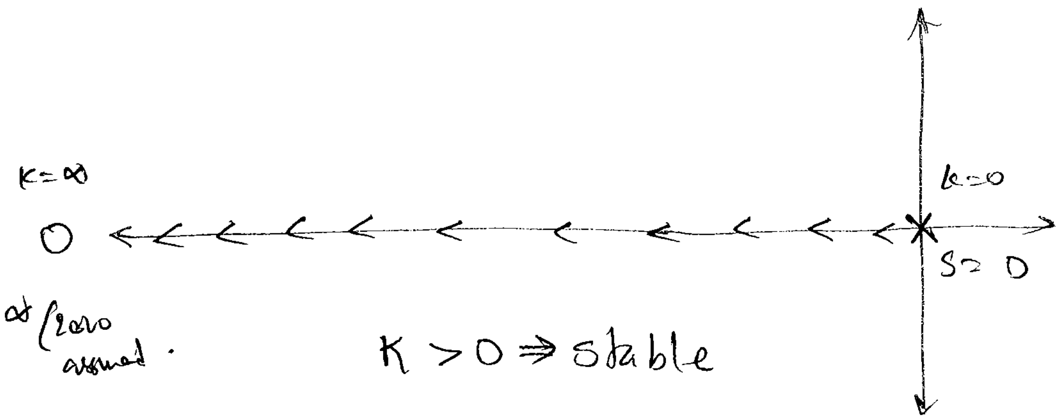
Q, Using the Root Locus technique, discuss the stability of the unity feedback control system of the first and 2^{nd} system of the system gain K.

- (i) ~~G(s)~~ 1^{st} order.

The standard form of 1^{st} order system is.



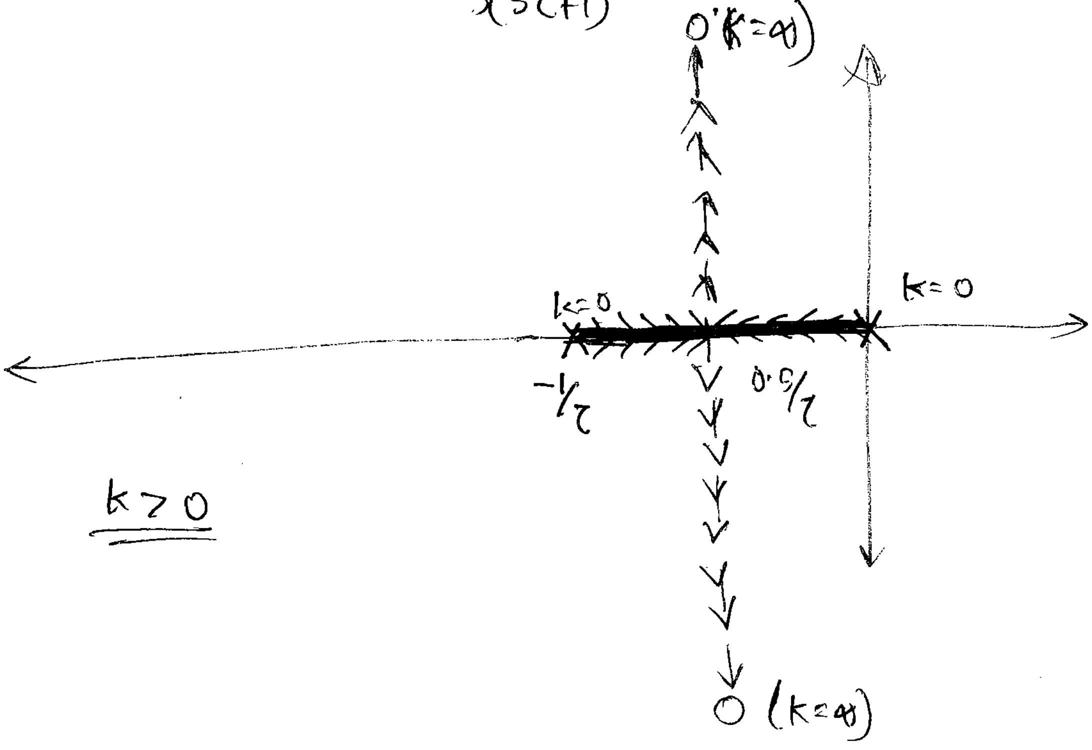
$$G(s) = \frac{K}{s\tau} \quad H(s) = 1$$



(ii) 2nd order system.

Consider the 2nd order system,

$$G(s)H(s) = \frac{k}{s(s\tau+1)}$$

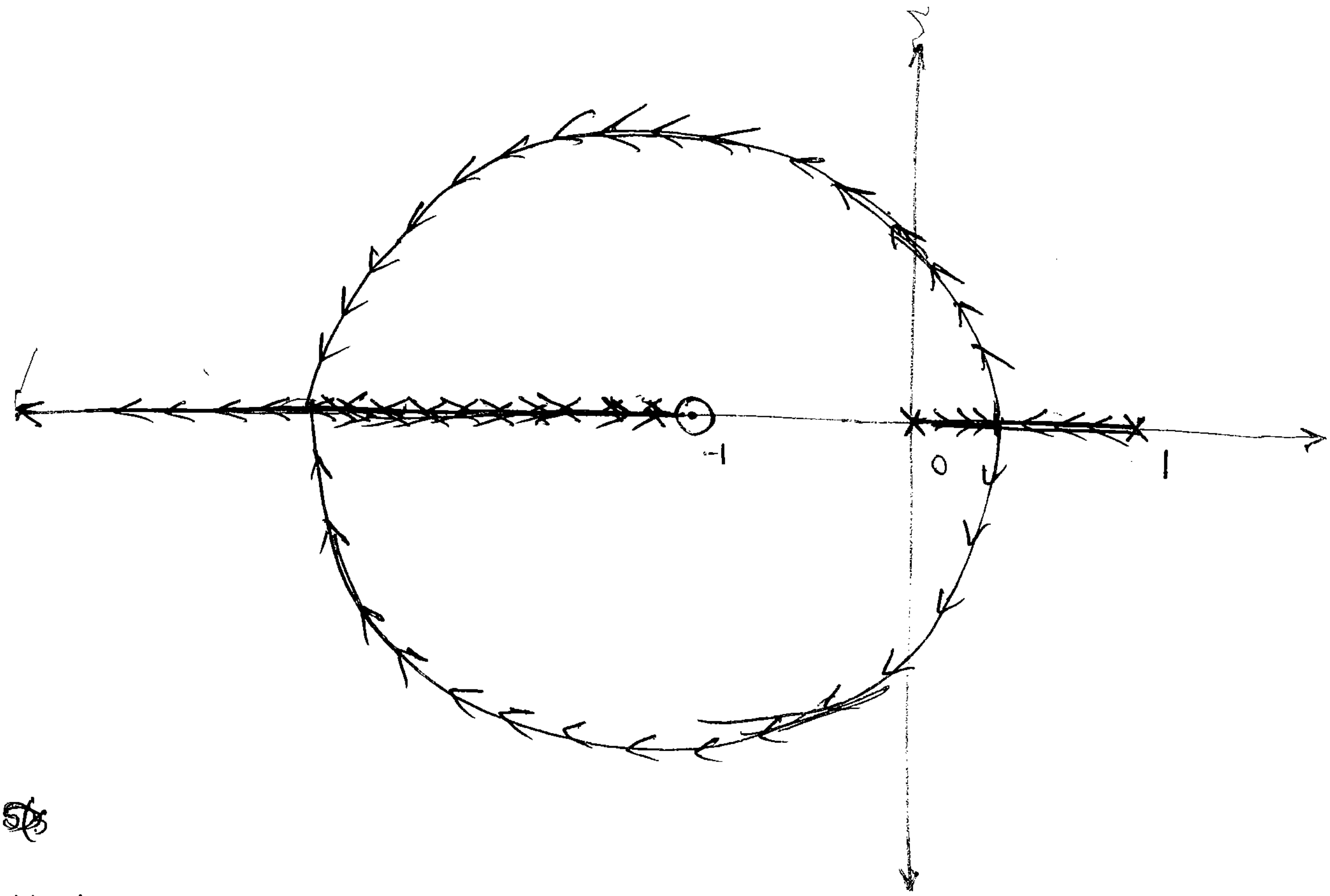


K > 0

Q, A unity feed back control system us an OLTF

$$G(s) = \frac{K(s+1)}{s(s-1)}$$

~~So that~~ show that a root locus of a complex roots are a part of the circle with centre (-1,0) and radius of $\sqrt{2}$ sketch the root locus diagram with K as a system gain parameter.



N = 1

Angle of assymtote = 180°.

$$s(s-1) + K(s+1) = 0$$

$$K = \frac{-s(s-1)}{s+1} = \frac{-s^2+s}{s+1}$$

$= (s+1)(-2s+1) + (s^2-s) = 0$

$-2s^2 - 2s + s + 1 + s^2 - s = 0 \qquad -s^2 - 2s + 1 = 0$

~~BP = +0.414, -2~~

BP = +0.414 , -2.414.

Converting the given G(s) in the circle form.

$1+G(s) = 0$

$\cancel{s^2 - s + k(s+1) = 0}$

$$1+\frac{k(s+1)}{s(s-1)} = 0$$

$$\frac{k(s+1)}{s(s-1)} = -1+j0$$

Take the phase angle on both sides.

$$\angle\frac{k(s+1)}{s(s-1)} = \angle(-1+j0)$$

Consider s as a complex number $s = \sigma + j\omega$.

$$\angle\frac{\angle k \;\angle(\sigma+j\omega+1)}{\angle\sigma+j\omega \;\angle\sigma+j\omega-1} = \pm 180^\circ$$

$$0+\tan^{-1}\frac{\omega}{(\sigma+1)} - \tan^{-1}\frac{\omega}{\sigma} - \tan^{-1}\frac{\omega}{(\sigma-1)} = \pm 180^\circ$$

$$\tan^{-1}\left(\frac{\omega}{\sigma}\right) + \tan^{-1}\left(\frac{\omega}{\sigma-1}\right) = \pm 180^\circ + \tan^{-1}\left(\frac{\omega}{\sigma+1}\right)$$

$$\tan^{-1}\left(\frac{\frac{\omega}{\sigma} - \frac{\omega}{\sigma-1}}{1-\frac{\omega^2}{\sigma(\sigma-1)}}\right) = \pm 180^\circ + \tan^{-1}\left(\frac{\omega}{\sigma+1}\right)$$

$$\frac{\frac{\omega}{\sigma}+\frac{\omega}{\sigma-1}}{1-\frac{\omega^2}{\sigma^2-\sigma}} = \tan\left(\pm 180^\circ + \tan^{-1}\left(\frac{\omega}{\sigma+1}\right)\right)$$

$$\frac{\omega}{\sigma}+\frac{\omega}{\sigma-1} = \frac{\omega}{\sigma+1}\left(1-\frac{\omega^2}{\sigma^2-\sigma}\right)$$

$$\frac{\omega}{\sigma}+\frac{\omega}{\sigma-1} = \frac{\omega}{\sigma+1} - \frac{\omega^3}{\sigma(\sigma-1)(\sigma+1)}$$

$$\frac{\omega(\sigma-1)+\omega\sigma}{\cancel{\sigma(\sigma-1)}} = \frac{\sigma\omega(\sigma-1)-\omega^3}{\cancel{\sigma(\sigma-1)}(\sigma+1)}$$

$$\omega(\sigma-1)(\sigma+1)+\omega\sigma(\sigma+1) = \omega\sigma(\sigma-1)-\omega^3$$

$$\cancel{\omega}(\sigma^2-1)+\cancel{\omega}(\sigma^2+\sigma) = \cancel{\omega}(\sigma^2-\sigma)-\omega^{\cancel{3}\,2}$$

$$\cancel{\omega\sigma^2-\omega+\omega\sigma}$$

$$\cancel{\sigma^2}-1+\sigma^2+\sigma = \cancel{\sigma^2}-\sigma-\omega^2$$

$$\sigma^2+\omega^2+2\sigma-1=0$$

$$\sigma^2+2\sigma+\omega^2=1$$

Add +1 on both side

$$\sigma^2+2\sigma+1+\omega^2=1+1$$

$(\sigma+1)^2+\omega^2=2$ it is of the form $(x+a)^2+y^2=r^2$

Then it represent circle with centre $a = -1$ and radius $r^2 = 2$ ie, $r = \sqrt{2}$

INVERSE ROOT LOCUS

DIRECT ROOT LOCUS

R_1 : Symmetrical

R_2 : No: of Loci

R_3 : Real axis Loci

A point exist on Root locus branch

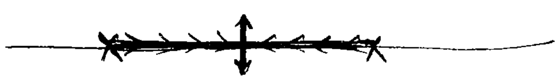
$\Sigma P+Z = ODD$

R_4: $\dfrac{(2q+1)180^\circ}{P-Z}$

R_5 : → Same ←

R_6 :

DRL



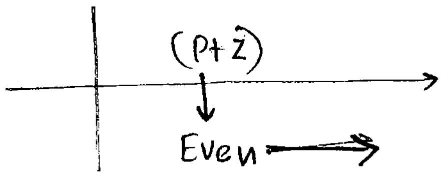
INVERSE ROOT LOCUS

R_1 : Symmetrical

R_2 : → Same ←

R_3 :

A point exist on Inverse root locus branch, the sum of the poles and zeros to the right hand side of that point should be even.



IRL

R_5 : $\dfrac{2q \times 180}{P-Z}$

R_5 : → Same ←

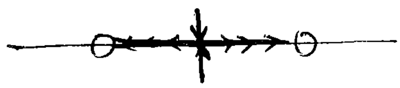
R_6 :

K increase from 0 to ∞	K incre (−∞ to 0)
IRL 	

Case(ii)

DRL

0 k<∞ IRL

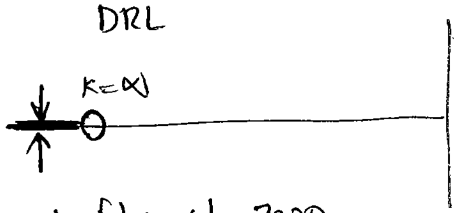


$-\infty < k < 0$

Case(iii) P>Z only.

~~Rightmost zero only.~~

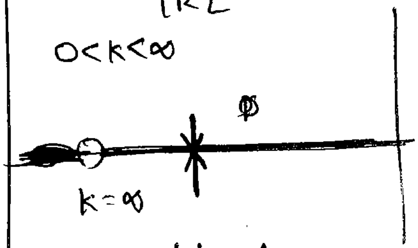
DRL



leftmost zero case.

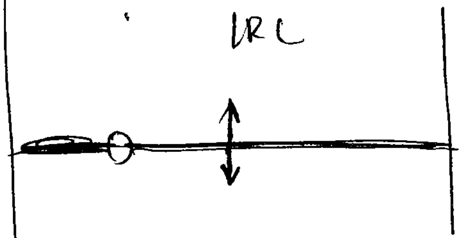
IRL

0<k<∞



Rightmost

IRL



(Rightmost)

⇓

⇓

whenever there exist the rightmost side 0, there exist a inverse root locus branch, then there should be the minimum one breakpoint to the rightmost of that zero.

RULE : 7 → Same ← | → Same ← | → Same ←

RULE : 8 $\phi_d = 180 + GH$

$\phi_a = 180 - GH$

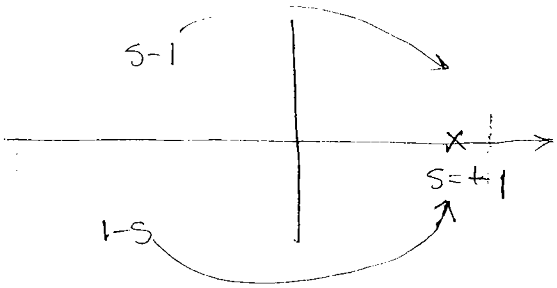
$\phi_d = 0 + \angle GH$

$\phi_d = 0 - \angle GH$.

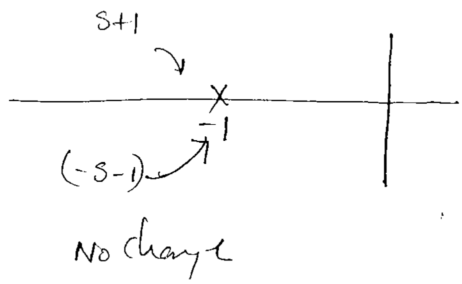
2) Draw the rootlocus for the system,

$$G(s)H(s) = \frac{K e^{-s}}{s(s+1)}$$

$$= \frac{K(1-s)}{s(s+1)}$$ Neglect the higher order terms.



No change.



Now $$G(s)H(s) = \frac{-K(s-1)}{s(s+1)}$$

NOTE : To draw a root locus diagram, the T.F should not have the -ve sign to S. (Take it out)

not mentioned where DRL or IRL

for that. we need to evaluate GH(s) & K, as follows :

GH = +K( )	
-ve F·B	+ve F·B
$1+GH(s)=0$	$1-GH(s)=0$
$1+K(\ )=0$	$1-K(\ )=0$
if + then DRL	if - IRL

GH = -K( )	
-ve F·B	+ve F·B
$1+GH(s)=0$	$1-GH(s)=0$
$1-K(\ )=0$	$1+K(\ )=0$
-ve so IRL	DRL

But if not specified about feedback & K,

By default, Negative feedback.

By default, K (0 to ∞)

In the above case

assume −ve feedback,

$CE \Rightarrow 1 + GH(s) = 0$

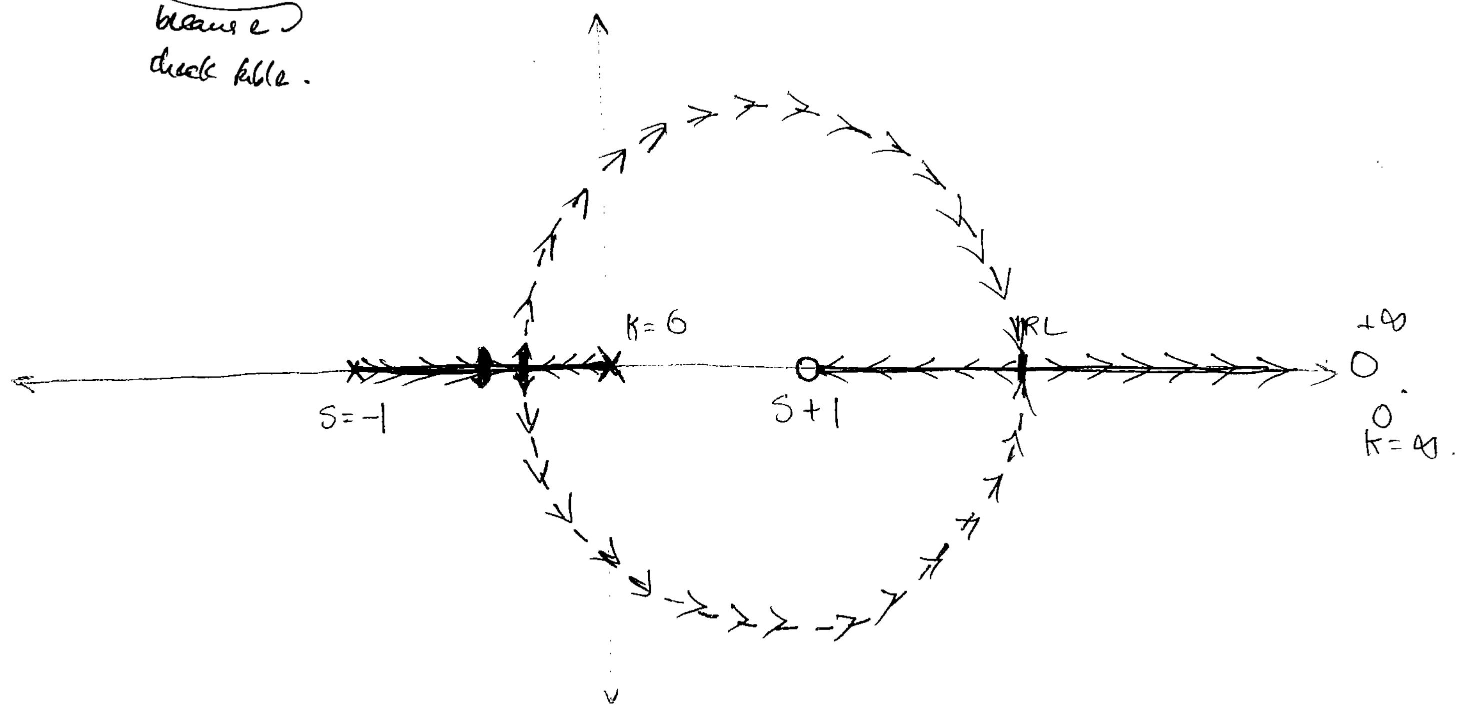
$$1 - \frac{K(s-1)}{s(s+1)} = 0$$

IRL

K varies from (0 to ∞)

$\times \rightarrow$ $\times \longrightarrow O$

because check table.



$s^2 + s + K(1-s) = 0$

$$K = -\frac{(s^2+s)}{1-s} = \frac{s^2+s}{s-1}$$

$$\frac{dK}{ds} = 0$$

$$\frac{(s-1)(2s+1) - (s^2+s)}{(s-1)^2} = 0$$

$2s^2 - 2s + s - 1 - s^2 - s = 0$

B.P $= -0.414, +2.414$

$s^2 - 2s + 1 = 0$

$\frac{dK}{ds} = 0 \qquad s = -0.414, +2.414$

For finding ~~such~~ such type of questions, with IRL, DRL, etc ~~Always for,~~ together, confused which is answer, then just substitute various values of $K = 0, 1,$ ~~etc~~ etc in CE and find points of s plane. It must ~~satisf~~ be there on root locus. It is a short even when we don't know Rules.

Q. $GH(s) = \frac{K}{s(s+2)}$ Draw the complete root locus.

For complete root locus, the $\boxed{K \text{ value is } -\infty < K < \infty}$

First draw $-\infty < K < 0 \longrightarrow$ IRL

draw $0 < K < \infty \longrightarrow$ DRL

IRL

Then the function changes to $GH(s) = \frac{-K}{s(s+2)}$

$K \rightarrow (-\infty < K < 0) \qquad CE \Rightarrow 1 + GH(s) = 1 - \frac{K}{s(s+2)} = 0$

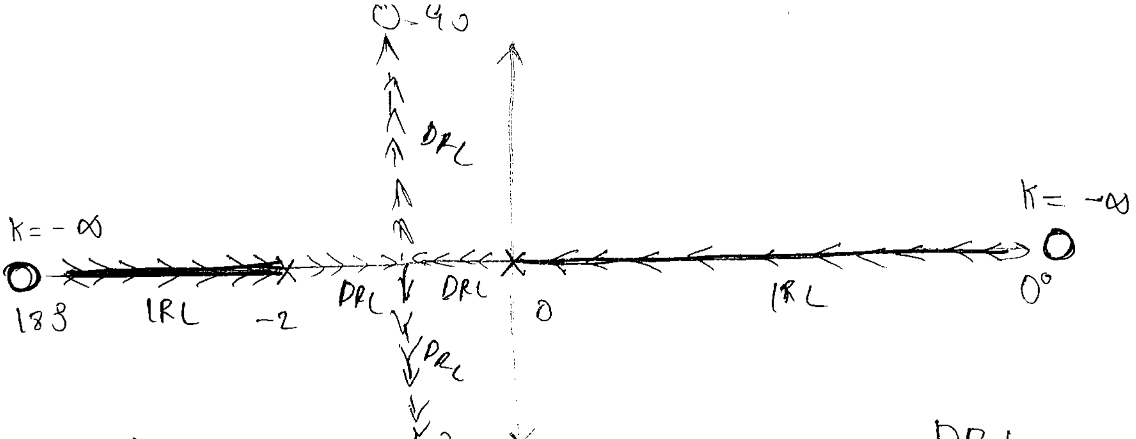
DRL

$GH(s) = \frac{K}{s(s+1)} \qquad CE \Rightarrow 1 + GH = 0$

$1 + K\frac{s}{s(s+2)} = 0$

$K\uparrow (0 \text{ to } +\infty)$

$K \rightarrow 0$



IRL

$N = P - Z = 2$

$$\theta = \frac{(2q)180^\circ}{2} = 0, 180^\circ$$

$$\cancel{\frac{dK}{ds}} \quad \frac{K}{s(s+2)} = 1$$

$$K = s(s+2)$$

$$\frac{dK}{ds} = s(1) + s+2 = 0$$

$$2s+2 = 0$$

B.P $\quad s = -1$

It is a invalid B·P

This B·P corresponds to DRL.

DRL

$\sigma = -1$

$$\theta = \frac{(2q+1)180}{2}$$

$$= 90^\circ, 270$$

BODE PLOTS

PURPUSE : 1. To draw the frequency response, of the opeloop transfer function.

2. To find the closed loop system stability.

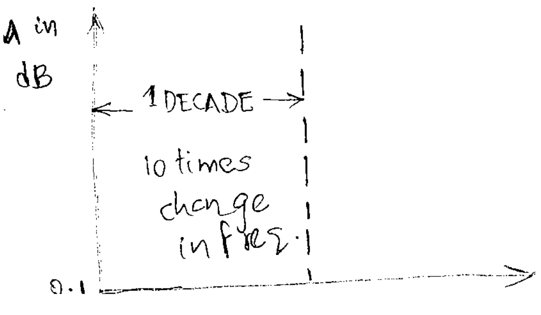
3. To find the gain Margin, phase margin, gain crossover frequency and phase crossover frequency.

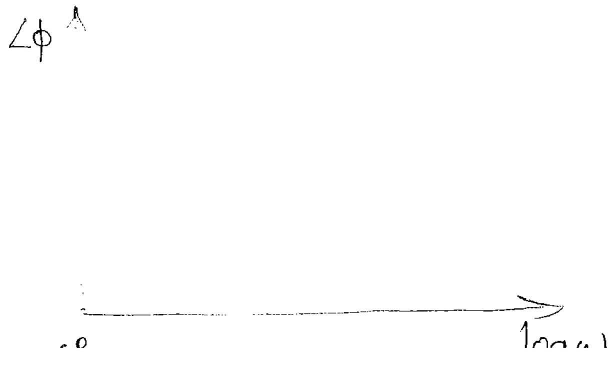
4. To find the relative stability by using Gain margin, phase margin. ① If the Gain Margin, phase Margin is very large, then the system is more relatively stable. But the system response become slow.
② If the Gain Margin, phase Margin is very small, then the system is less relatively stable as well as More Oscillatory.
The optimum value of Gain margin is 5 – 10 dB and Gain margin (30° – 40°).

The Bode Plot consists of two plots.

(i) Magnitude plot

(ii) phase plot.





$$\boxed{\omega_2 = 10\,\omega_1} \longrightarrow \text{DECADE}$$

$$\boxed{\omega_2 = 2\,\omega_1} \longrightarrow \text{OCTAVE}$$

$$20\log 2 = 2\log 10$$

$$\boxed{6\,dB\Big|_{octave} = 20\,dB\Big|_{decade}}$$

PROCEDURE TO DRAW BODE PLOT.

S1 : S replace by $j\omega$ to convert into frequency domain.

S2 : Write the magnitude and convert into dB.

The magnitude in dB is

$$\boxed{M \text{ in dB} = 20\log|GH(j\omega)|}$$

$$\boxed{\phi = \tan^{-1}\left(\frac{\text{Imaginary part}}{\text{Real part}}\right)}$$

S3: Vary the ω from minimum and maximum value and draw the approximated magnitude and phase plot.

Q. Draw bode plots for $GH(s) = K$

$M = K$

$M_{in\,dB} = 20\log K$

$\text{Find slope} = \dfrac{dM}{d(\log\omega)}$

$= 0\ dB/dec.$

$Y(M)$

$\delta = \dfrac{dy}{dx} = \dfrac{dM}{d\log\omega}$.

$X(\log\omega)$

$\angle\phi = \angle(K+j0) = 0^\circ$ (irrespective of value of k).

$\tan^{-1}\left(\frac{0}{K}\right)$

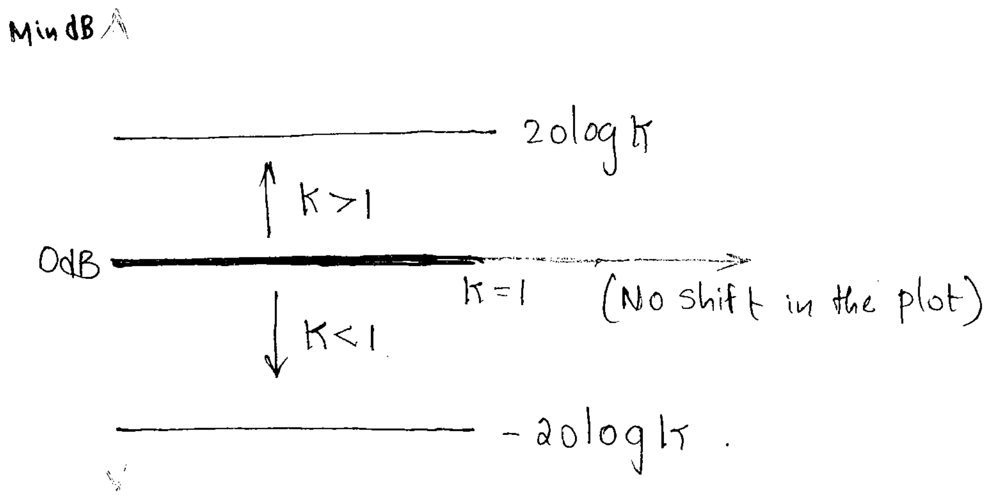
- M plot is straight line $\parallel$ to horizontal axis and height depends on value of.

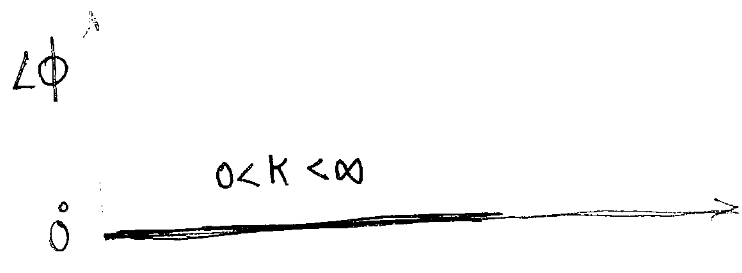
- (i) $K = 1 \longrightarrow M$ in dB $= 0$ dB.

  (ii) $K > 1$ $(10) \longrightarrow M_{in\,dB} = +20$ dB.

  (iii) $K < 1$ $(0.1) \longrightarrow M$ in dB $= -20$ dB.

MAGNITUDE PLOT





N POLES AT ORIGIN

$$GH(s) = \frac{1}{s^n}$$

$s - j\omega$ $\quad GH(j\omega) = \frac{1}{(j\omega)^n}$

$$\left|\frac{1}{(j\omega)^n}\right| = \frac{1}{\omega^n}$$

$$M \text{ in dB} = 20 \log \frac{1}{\omega^n}$$

$$M \text{ in dB} = -20n \log \omega .$$

$$\text{slope} = \frac{dM}{d\log\omega}$$

$$\text{slope} = -20n \ \text{dB/dec}.$$

$$\angle\phi = \frac{\angle 1}{\angle j\omega \ldots \text{ntimes}} = -90^\circ n$$

$$\boxed{\begin{aligned} \text{Magnitude}/_{\omega=0.1} &= -\text{slope} \\ \text{Magnitude}/_{\omega=1} &= 0 \end{aligned}}$$

Every plot must start from $\omega = 0.1$, passes throug $\omega = 0.1$ (0 dB line) and entented upto corner frequency. If no corner frequency existed, then entend upto ∞.

N ZEROS AT ORIGIN

$$GH(s) = s^n$$

$s - j\omega$ $\quad GH(j\omega) = (j\omega)^n$

$$M \text{ in dB} = 20 \log \omega^n$$

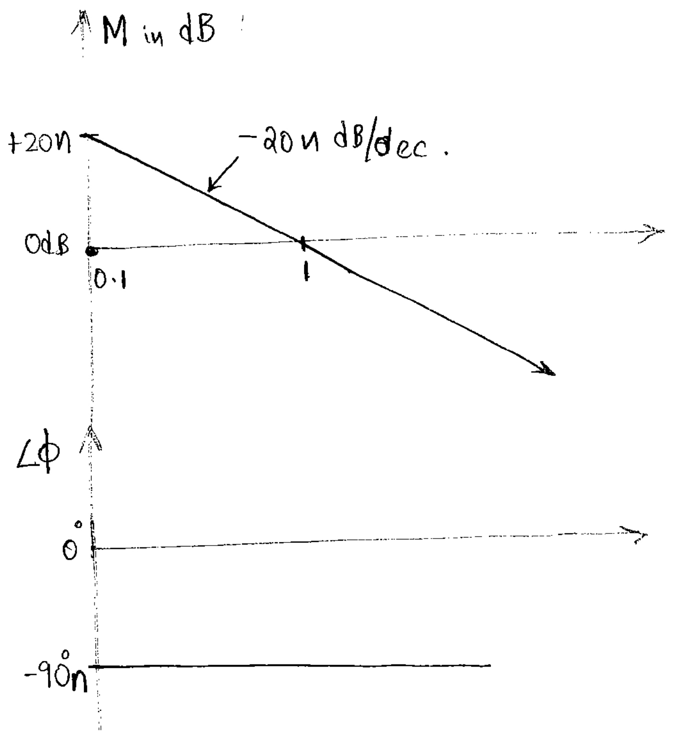
$$M \text{ in dB} = 20n \log \omega .$$

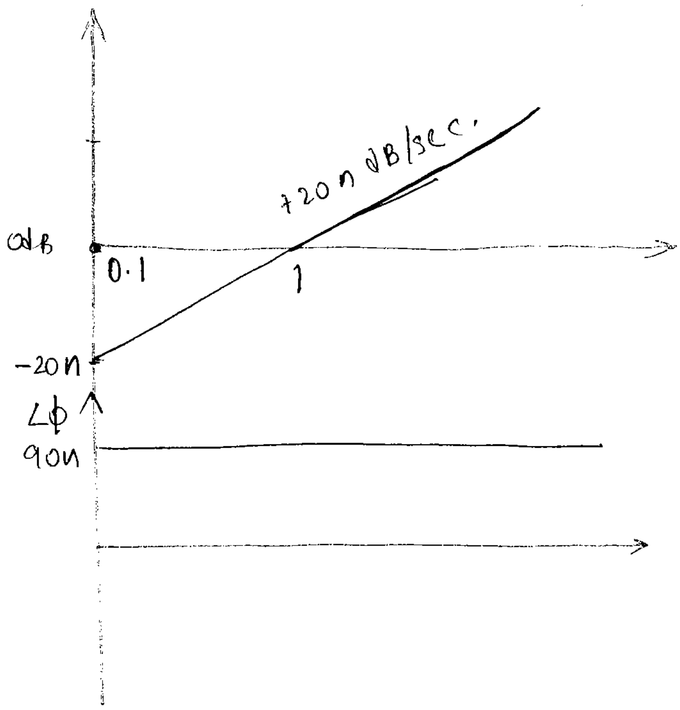
$$\text{Slope} = \frac{dM}{d\log\omega} = +20n \ \text{dB/dec}.$$

$$\angle\phi = \frac{\angle j\omega \ldots \text{ntimes}}{\angle 1} = +90^\circ n .$$

$$M\Big|_{\omega=0.1} = -20n \ \text{dB}.$$

$$M\Big|_{\omega=1} = 0$$





NOTE : whenever the transfer functions consists poles or Zeros at origin, then the plot starts with a magnitude of opposite sign of ~~opposite~~ slope at a frequency of 0.1 . And it should be passes through 0 dB line and intersect at $\omega = 1$ and entended upto first corner frequency, if existed otherwise extend upto infinity.

Q, Draw Bode Blot of $G(s)H(s) = \frac{100}{s^5}$

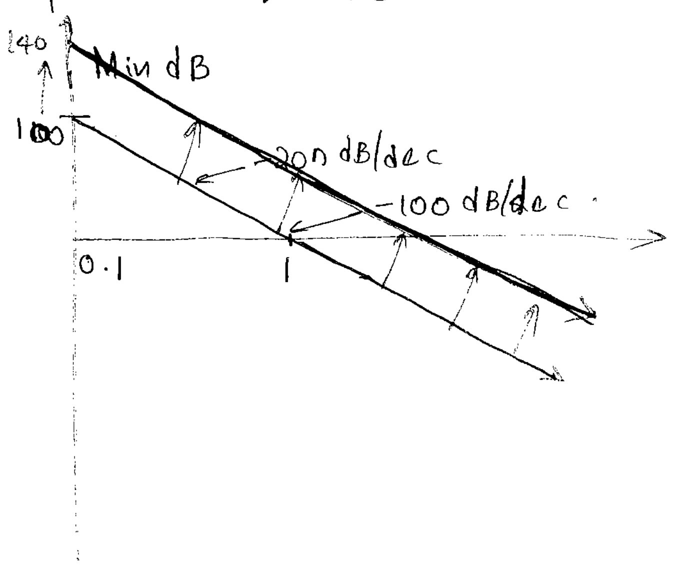
Sol $GH(j\omega) = \frac{100}{(j\omega)^5}$.

② M in dB = $20\log \frac{100}{\omega^5}$

Slope = 5 poles at origin → $-20n$ ⇒ slope = $\underline{\underline{-100\ dB/sec}}$.

$\angle\phi = -90^\circ n = \underline{\underline{-450^\circ}}$

$M|_{\omega_{0.1}}$ = − slope = 100 dB.



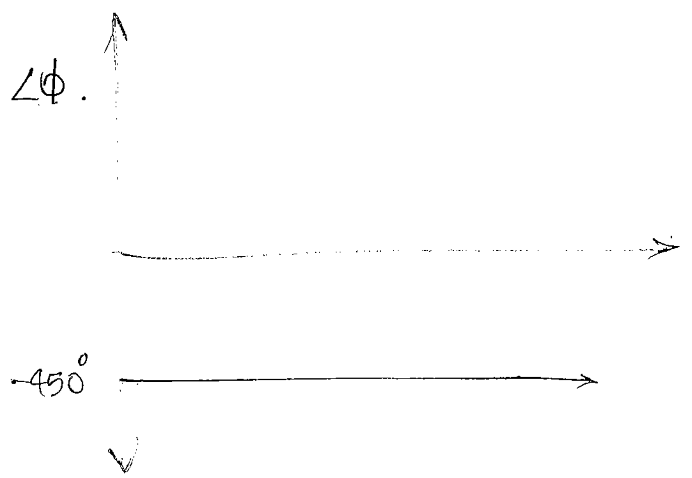
Now the effect of K

K > 1

∴ upward shift

shift = 20 log 100

= +40 dB.



N FINITE POLES

$$GH(s) = \frac{1}{(s\tau+1)^n}$$

$s \to j\omega$.

$$GH(j\omega) = \frac{1}{(j\omega\tau+1)^n}$$

$$M = \left(\frac{1}{\sqrt{(\omega\tau)^2+1}}\right)^n$$

$$M_{\text{in dB Actual}} = -20n \log\left(\sqrt{(\omega\tau)^2+1}\right)$$

$$\boxed{\phi_{actual} = \frac{\angle 1}{\angle(j\omega+1)\ldots n\text{ times}} = -n\tan^{-1}(\omega\tau)}$$

Asymptotic / Approximate Analysis.

Case (i) $\omega\tau < 1$, Neglect $\omega\tau$ $\boxed{\omega < \frac{1}{\tau}}$

$$M_{assympto} = 0 \Rightarrow S = 0\ dB/dec$$

$$\phi_{asymp} = 0°$$

Case (ii) $\omega\tau > 1$, Neglect 1 $\boxed{\omega > \frac{1}{\tau}}$ corner freq

$$M_{assymptotic} = -20n\log(\omega\tau)$$

$$M_{assymptotic} = -20n\log(\omega) - 20n\log(\tau)$$

$$S = \frac{dM}{\cdot} = -20n\ dB/dec$$

N FINITE ZEROS

$$GH(s) = (s\tau+1)^n$$

$s \to j\omega$.

$$GH(j\omega) = (j\omega\tau+1)^n$$

$$M = \left(\sqrt{(\omega\tau)^2+1}\right)^n$$

$$M_{\text{in dB Actual}} = +20n \log\left(\sqrt{(\omega\tau)^2+1}\right)$$

$$\phi_{actual} = \angle(j\omega+1)\ldots n\text{ times} = n\tan^{-1}(\omega\tau)$$

Assymptotic / Approximate Analysis

Cas (i) $\omega\tau < 1$ Neglect $\omega\tau$ $\omega < \frac{1}{\tau}$

$$M_{assymptote} = 0 \Rightarrow S = 0\ dB/dec$$

$$\phi_{assymp} = 0°$$

Case (ii) $\omega\tau > 1$, Neglect 1 $\omega > \frac{1}{\tau}$

$$M_{assymptotic} = 20n\log(\omega\tau)$$

$$M_{assymptotic} = 20n\log\omega + 20n\log\tau$$

$$S = \frac{dM}{d\log\omega} = +20n\ dB/dec$$

$$\phi_{assymptotic} = \frac{\angle 1}{\angle(j\omega\tau) \ldots n\,times}$$

$$\phi_{assymptotic} = -90^\circ n$$

Corner frequency is the frequency at which ~~ϕ~~. slope changes.

CORNER FREQUENCY

The frequency at which slopes changes from one level to another level.

The corner frequencies are nothing but finite poles and finite zeros location in the form of Magnitude.

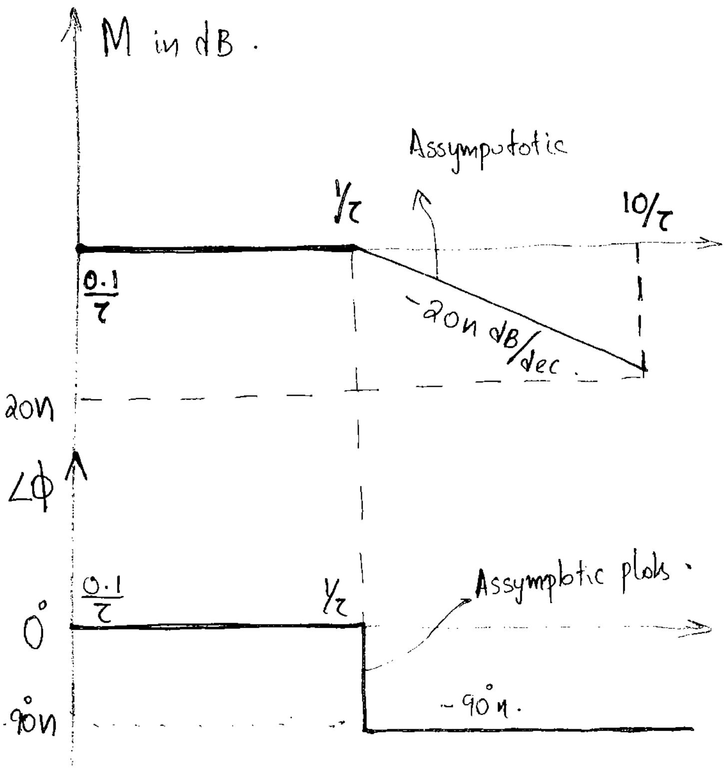
$$s = \left|-\tfrac{1}{\tau}\right|$$

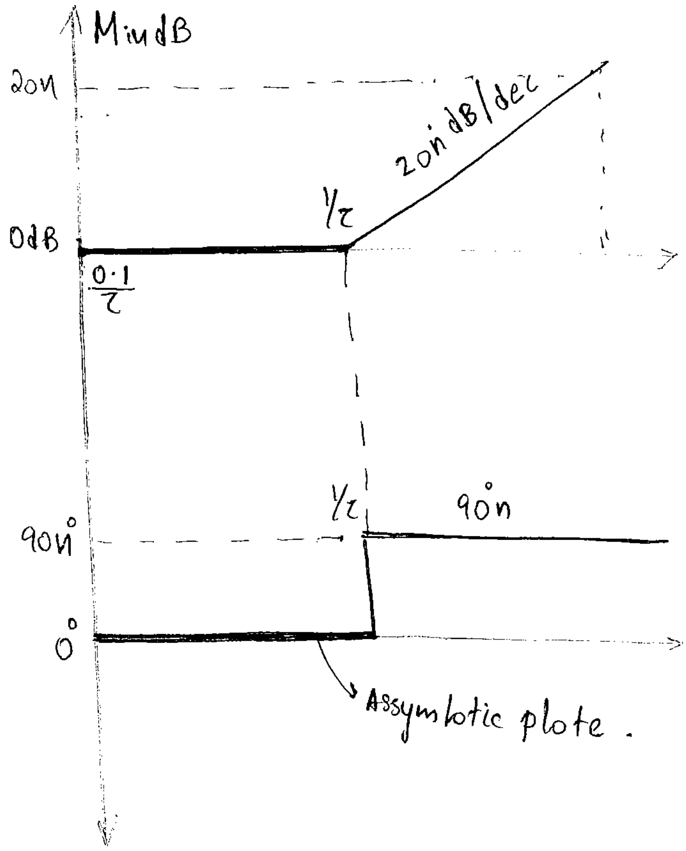
	SLOPE	$\angle\phi$
Below ~~Above~~ corner frequency	0 dB/dec	0°
Above Corner frequency	-20n dB/dec	$-90^\circ n$.

$$\phi_{assymptotic} = \frac{\angle(j\omega\tau) \ldots n\,times.}{\angle 1}$$

$$\phi_{assymptotic} = 90^\circ n.$$

	SLOPE	$\angle\phi$
Below corner frequency.	0 dB/dec	0°
Above corner frequency.	20n dB/dec	$+90^\circ n$





ERRORS IN MAGNITUDE PLOTS

$$\left\{ \text{ERROR} \right\} = \left\{ \begin{matrix} \text{Actual} \\ \text{Value} \end{matrix} \right\} - \left\{ \begin{matrix} \text{Assymptotic} \\ \text{Value} \end{matrix} \right\}$$

Error same eqn

But values will be +ve.

E at CF $\omega = \frac{1}{\tau}$ = M_{actual} $\omega = 1/\tau$ − Assymptotic $\omega = 1/\tau$

The actual value is obtained from the given transfer function and assymptotic value is obtained from the given plot.

$E_{at\ \omega = 1/\tau} = -3n - 0 = -3n\,dB$,

$E_{at\ \omega = 1/\tau} = 3n - 0 = \underline{3n\,dB}$

eg: Error at $\frac{0.1}{\tau} = \omega$

$M_{actual} = -20n \log \sqrt{\left(\frac{0.1}{\tau} \times \tau\right)^2 + 1}$

$= -20n\log$ same at $\frac{10}{\tau}$

error at $\frac{0.1}{\tau} = \omega$.

$M_{actual} = +20n \log \sqrt{\left(\frac{0.1}{\tau} \times \tau\right)^2 + 1}$

$= \underline{+0.043n}$ at $\omega = \frac{10}{\tau}$

eg: error at $\omega = \frac{0.5}{\tau}$

$E_{\omega=\frac{0.5}{\tau}} = -0.96n$ Same at $\omega = \frac{5}{\tau}$.

$E_{\omega = \frac{0.5}{\tau}} = 0.96n$

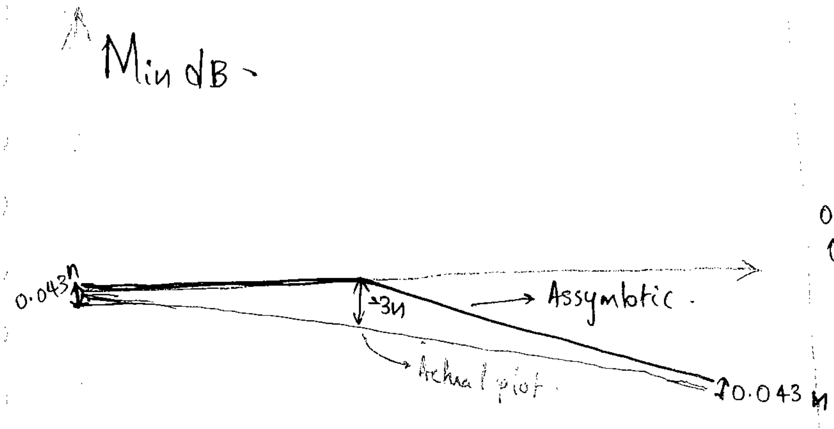
NOTE: The error is maximum at corner frequency. The error decreases symmetrically either above or below corner frequencies.

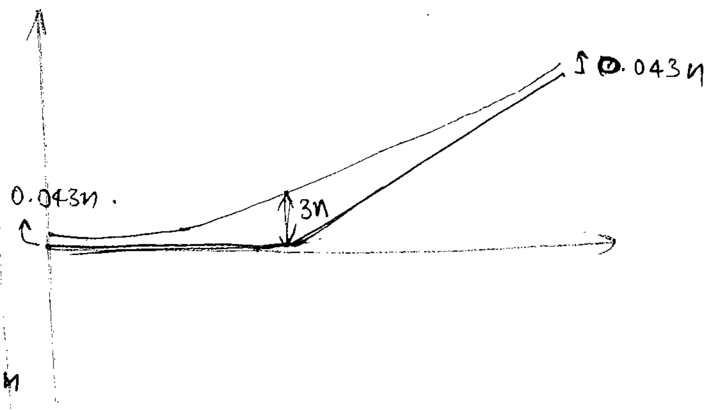
Error in PHASE PLOT

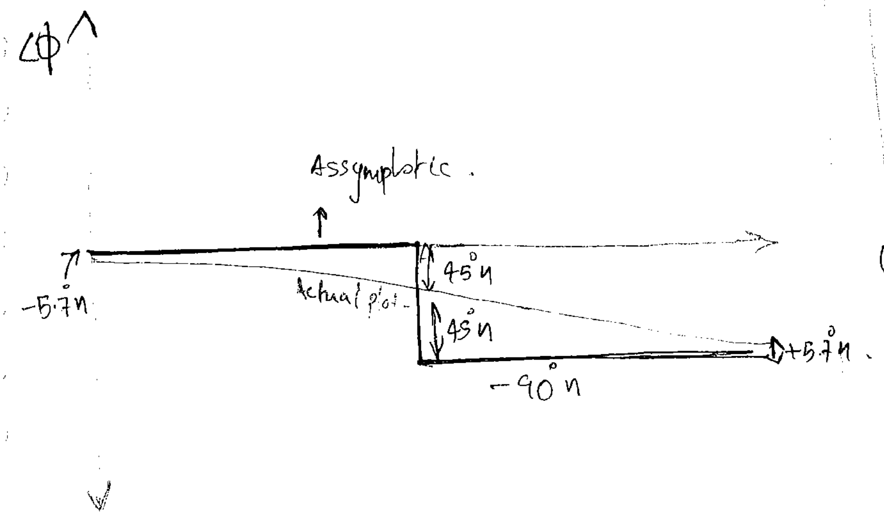
$$\begin{Bmatrix} E \text{ at CF} \\ \omega = 1/\tau \end{Bmatrix} = \begin{Bmatrix} \phi_{actual} \\ \omega = 1/\tau \end{Bmatrix} - \begin{Bmatrix} \phi_{assymd} \\ \omega = 1/\tau \end{Bmatrix} \Longrightarrow \text{Same Error equation.}$$

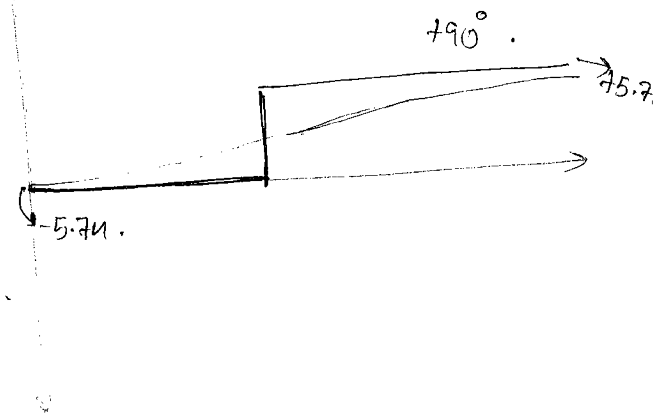
$E_{\omega=1/\tau} = -45^\circ n - 0^\circ$
$= -45^\circ n.$

→ But variation in sign
$E_{\omega = 1/\tau} = 45^\circ n - 0^\circ = 45^\circ n.$









Q, $GH(s) = \frac{10(s+5)^2}{s(s+2)(s+10)}$

$$GH(j\omega) = \frac{10(j\omega+5)^2}{j\omega(j\omega+2)(j\omega+10)}$$

$$= \frac{10\left(\sqrt{\omega^2+5^2}\right)^2}{\omega\sqrt{\omega^2+4}\ \sqrt{\omega^2+100}}$$

Magnitude.

$20\log|GH(j\omega)|$

$$= 20\log\left(\frac{10\left(\sqrt{\omega^2+5^2}\right)^2}{\omega\sqrt{\omega^2+4}\ \sqrt{\omega^2+100}}\right)$$

$$\phi = \angle \frac{10(j\omega+5)^2}{j\omega(j\omega+2)(j\omega+10)}$$

$$\cancel{= 0 + 90^\circ \times 2 - 90^\circ - \tan^{-1}}$$

$$= -90^\circ - \tan^{-1}\left(\frac{\omega}{2}\right) - 2\tan^{-1}\left(\frac{\omega}{10}\right) - \tan^{-1}\left(\frac{\omega}{5}\right)$$

Such qus will asked to find magnitude and phase for a particular ω. ie, ω will be given.

$M|_{\omega=1} = ?$

$M|_{\omega=10} = ?$

$\phi|_{\omega=10} = ?$

$\phi|_{\omega=100} = ?$

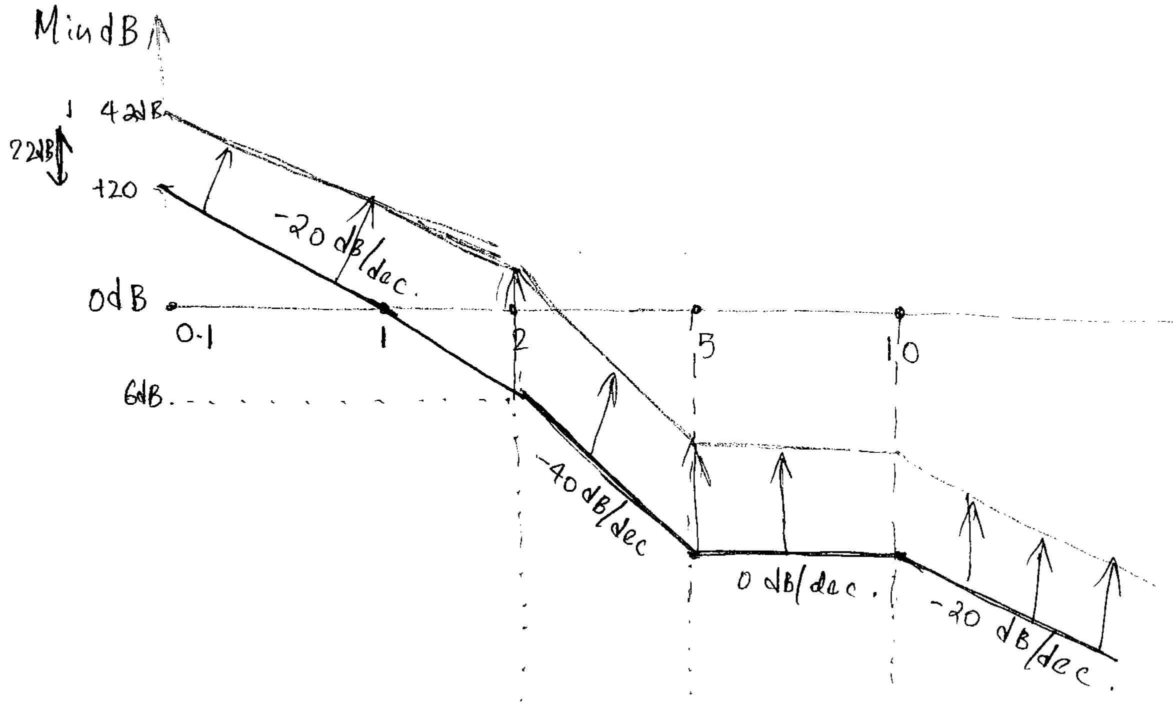
Now go for drawing Bode plot.

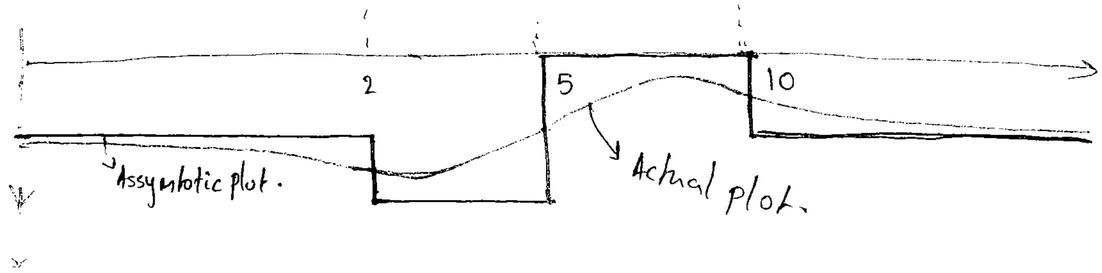
S1 First T.F must be written in the time constant form,

$$GH(s) = \frac{\cancel{10}\times 5^2\left(1+s/5\right)^2}{2\times\cancel{10}\, s\left(1+s/2\right)\left(1+s/10\right)}$$

$$= \frac{12.5\left(1+s/5\right)^2}{\ldots}$$

Denominators of s terms are called corner frequencies.





Irrespective of corner frequency always mark 0.1 and 1 for getting initial sloped.

The Initial slope of plot is given by poles and Zeros located at origin.

Here pole at origin initial slope = -20 dB/dec

$$M\Big|_{0.1} = -\text{slope} = -(-20) = \underline{\underline{20\,dB}}$$

∴ plot starts at 20dB with slope -20dB/dec passes through 0 dB and extended upto next corner frequency ie, upto 2.

For finding Magnitude at 2, we need to us slope Equation.

$$\boxed{S = \frac{M_2 - M_1}{\log\omega_2 - \log\omega_1}}$$

where $(M_1, \log\omega_1)$ and $(M_2, \log\omega_2)$ are two points of line.

Here $$-20 = \frac{M_2 - 20}{\log 2 - \log 1}$$

$$-20 = \frac{M_2 - 20}{1.3}$$

$$M_2 = \underline{\underline{-6\,dB}}$$

At 2, another pole is present, so additional $-20\,dB/dec$ slope.

Hence net slope $= -20\,dB/dec - 20\,dB/dec$

$= -40\,dB/dec$

At 5, 2 zeros are present, additional slope of $+40\,dB/dec$

~~net slo~~

Net slope $= -40\,dB/dec + 40\,dB/dec$

$= \underline{\underline{0}}\ dB/dec$.

At 10, one pole is there, additional slope of $-20\,dB/dec$

Net slope $= 0 - 20\,dB/dec = -20\,dB/dec$.

Now consider K value

$K = 12.5$. $K > 1$

Shift $= 20\log(12.5)$

$= \underline{22\, dB}$

→ So entire plot shifted upwards ($K>1$) by 22 dB.

→ phase plot must be drawn only below magnitude plot.

Direct conversion b/w slope & phase angle.

$$\boxed{\pm 20n \Longleftrightarrow \pm 90^\circ n}$$

$-20 \longrightarrow -90$

$-40 \longrightarrow -180$.

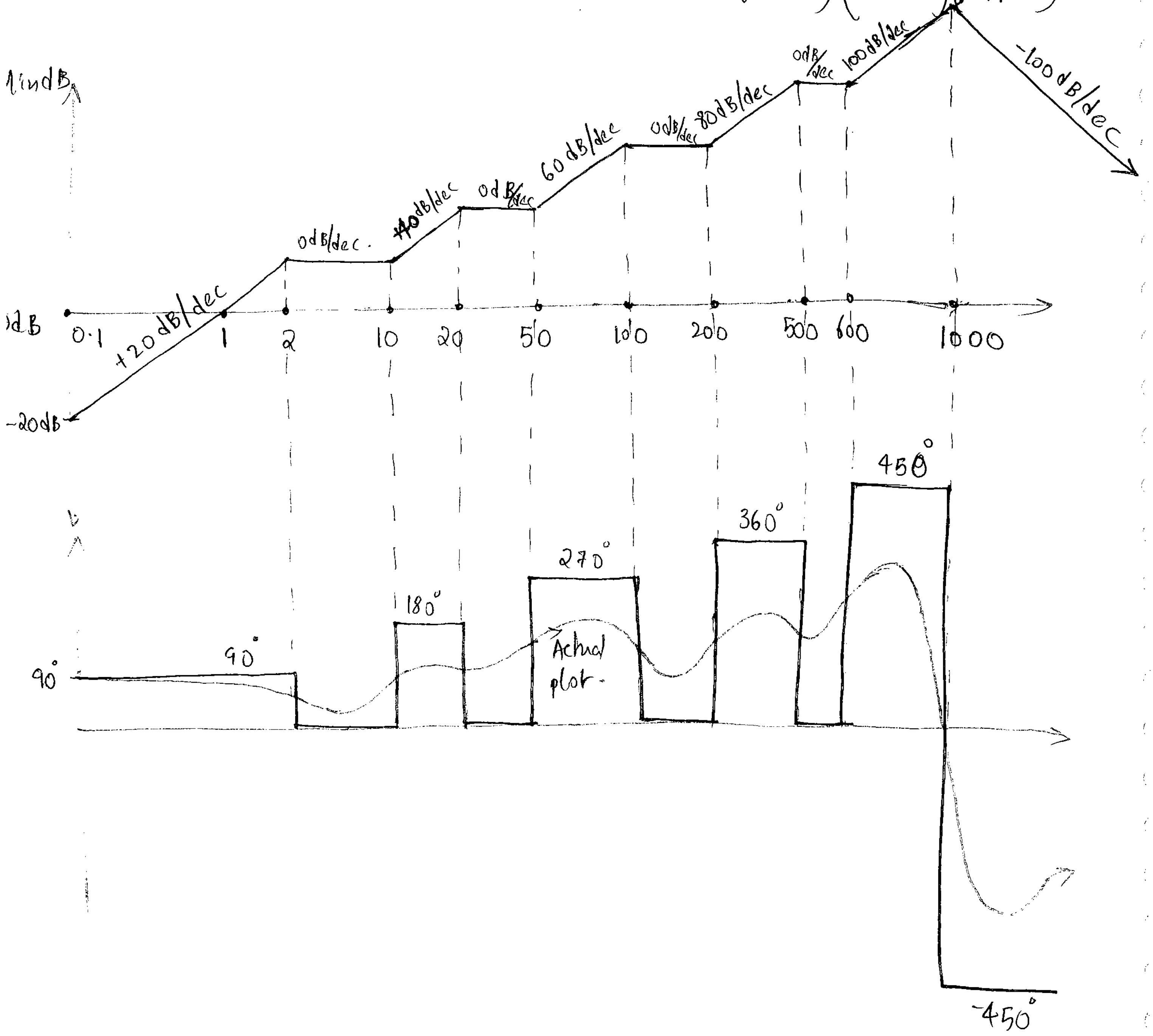
$+0 \longrightarrow 0$

$-20 \longrightarrow -90$.

With above values draw assymplotic plot. Using assymplotic, with errors, and half concept at corner frequencies, we can draw actual phase plot.

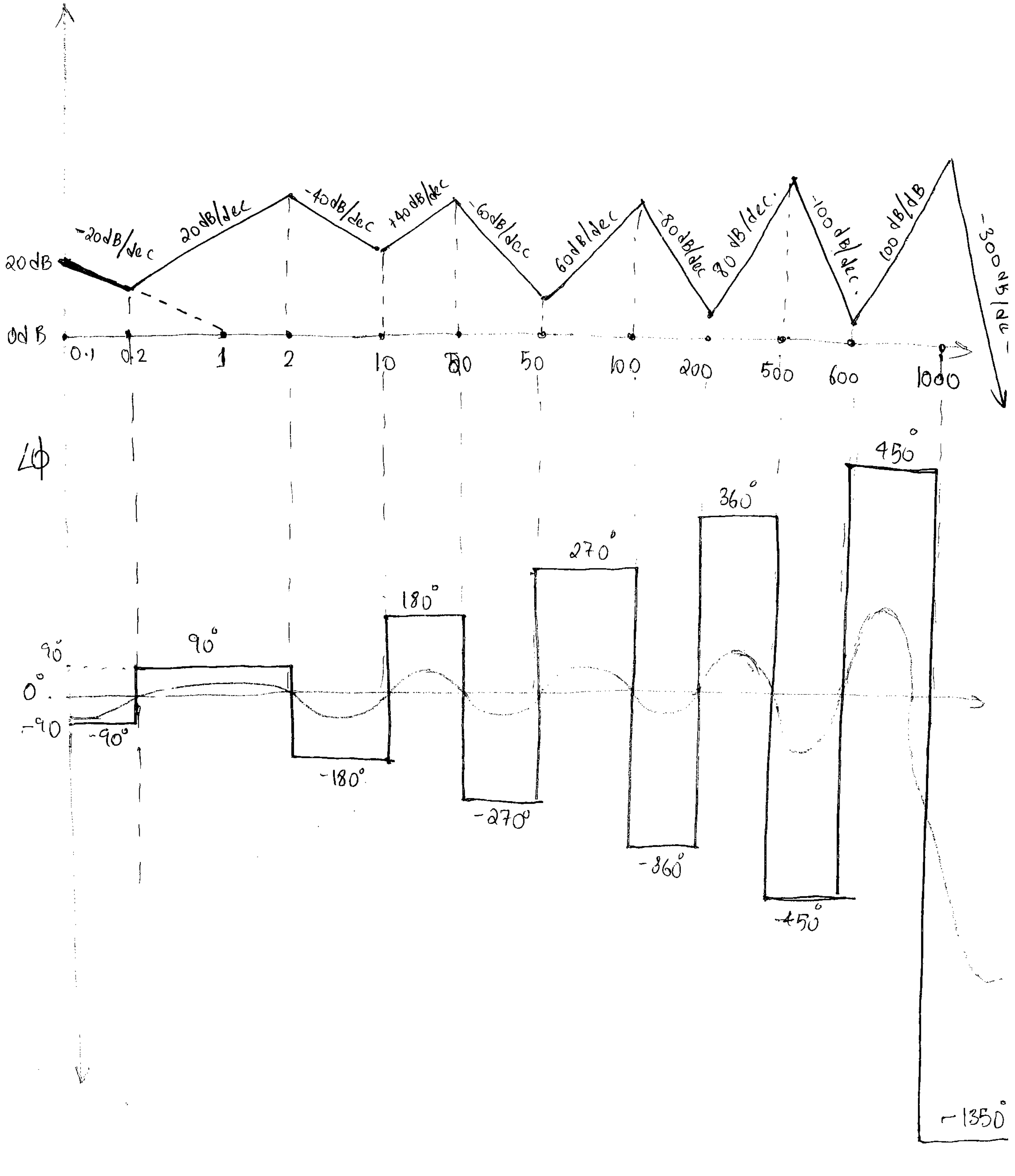
Q. Draw the Bode plot

$$GH(s) = \frac{s\left(1+s/10\right)^2\left(1+s/50\right)^3\left(1+s/200\right)^4\left(1+s/600\right)^5}{\left(1+s/2\right)\left(1+s/20\right)^2\left(1+s/100\right)^3\left(1+s/500\right)^4\left(1+s/1000\right)^{10}}$$



Q, Draw bode plot.

$$GH(s) = \frac{\left(1+\frac{s}{0.2}\right)^{2}\left(1+\frac{s}{10}\right)^{4}\left(1+\frac{s}{50}\right)^{6}\left(1+\frac{s}{200}\right)^{8}\left(1+\frac{s}{600}\right)^{10}}{s\left(1+\frac{s}{2}\right)^{3}\left(1+\frac{s}{20}\right)^{5}\left(1+\frac{s}{100}\right)^{7}\left(1+\frac{s}{500}\right)^{9}\left(1+\frac{s}{1000}\right)^{20}}$$



Q, Find the change in slopes at the following corner (i) frequencies.

(i) $\omega = 2$
(ii) $\omega = 10$
(iii) $\omega = 20$
(iv) $\omega = 50$
(v) $\omega = 100$
(vi) $\omega = 200$
(vii) $\omega = 500$
(viii) $\omega = 600$
(ix) $\omega = 1000$

Q, Find the slope of the line between two corner (ii) frequencies.

(i) 2 to 10
(ii) 10 to 20
(iii) 20 to 50
(iv) 50 to 100
~~(v) 100 to 200~~

(iv) At frequency assymptote.

Q, Find the slopes around the ~~cor~~ following corner (iii) frequency.

(i) $\omega = 2$
(ii) $\omega = 20$
(iii) $\omega = 200$
(iv) $\omega = 1000$

For
$$G(s)H(s) = \frac{s^3\left(1+\frac{s}{10}\right)^7\left(1+\frac{s}{50}\right)^{20}\left(1+\frac{s}{200}\right)^{50}\left(1+\frac{s}{600}\right)^{200}}{\left(1+\frac{s}{2}\right)^4\left(1+\frac{s}{20}\right)^{16}\left(1+\frac{s}{100}\right)^{40}\left(1+\frac{s}{500}\right)^{100}\left(1+\frac{s}{1K}\right)^{300}}$$

change in slope = New slope − Earlier slope.

$$\boxed{\text{Number of poles/zeros} = \pm \frac{\text{change in slope}}{20}}$$

+ → Zeros
− → poles.

The change in slope is slope of finite poles, finite zeros, at that particular corner frequency.

(i)

$$\frac{s^3 \left(1+s/10\right)^{7} \left(1+s/50\right)^{20} \left(1+s/200\right)^{50} \left(1+s/600\right)^{200}}{\left(1+s/2\right)^4 \left(1+s/20\right)^{16} \left(1+s/100\right)^{40} \left(1+s/500\right)^{100} \left(1+s/1k\right)^{300}}$$

Corner freq	P/Z	Change in frequency
2 →	4P →	−80
10 →	7Z →	140
20 →	16P →	−320
50 →	20Z →	400
100 →	40P →	−800
200 →	50Z →	1000
500 →	100P →	−2000
600 →	200Z →	+4000
1000 →	300P →	−6000

(ii)

(a) slope b/w 10 & 20

>10   < 20

↓   ↓

IN   OUT

10. include   20 excluded.

Now consider all corner frequencies, up to 10,

Take total Poles and zero.

~~Poles 7~~

To get the slope of the line between two corner frequencies from ω_1 to ω_2

Consider all the terms ~~upto~~ in the transfer function upto ω_1 only.

~~For eg:~~ (a) 2 to 10

>2 $\downarrow$ in <10 $\downarrow$ out.

$$\frac{s^3}{(1+s/2)^4} \rightarrow \text{4 poles, 3 zeros.}$$

Net 1 pole $\Longrightarrow$ -20 dB/dec.

(b) 10 to 20

>10 <20

$$\frac{s^3 (1+s/10)^7}{(1+s/2)^4}$$

10 zeros 4 poles

$\Rightarrow$ 6 zeros net $\Rightarrow$ $+120$ dB/dec.

(c) 200 to 500

>200 $\downarrow$ in <500 $\downarrow$ out.

80 zeros
60 poles $\longrightarrow$ 20 zeros $\Rightarrow$ $+400$ dB/dec

(d) High frequency assymptote
consider all poles & zeros.

280 Z
460 Pole $\}$ 180 poles

(iii) around : from which to which :

<10 → out   >10 → in

ie, find without 10 and with 10 ~~find~~ find separately.

<10 ~~~~ without 10

$\left.\begin{matrix} 4P \\ 2Z \end{matrix}\right\} 2P \rightarrow -40$

>10 with 10

$\left.\begin{matrix} 6Z \\ 4P \\ \hline 2Z \end{matrix}\right\} \rightarrow +40$

(a) $\omega = 2$

without 2

3 zeros

$\Rightarrow +60$ dB/dec

with 2

$\left.\begin{matrix} 3 \text{ zeros} \\ 4 \text{ poles} \end{matrix}\right\} \Rightarrow 1 \text{ pole} \Rightarrow -20$ dB/dec

(b) $\omega = 20$

without 20

$\begin{matrix} 10Z \\ 4P \\ \hline 6Z \end{matrix}$

$\Rightarrow +120$ dB/dec.

with 20

$\begin{matrix} 10Z \\ 20P \\ \hline 10P \end{matrix}$

$\rightarrow -100$ dB/dec.

(c) $\omega = 200$

without 200

$\left.\begin{matrix} 30Z \\ 60P \end{matrix}\right\}$ 30 poles $\rightarrow -600$ dB/dec

with 200

$\left.\begin{matrix} 80Z \\ 60P \end{matrix}\right\}$ 20 zeros $\rightarrow$ 400 dB/dec

(d) $\omega = 1K$,

<u>without 1K</u>

$\left.\begin{matrix} 280Z \\ 160P \end{matrix}\right\}$ 120Z

↓

2400 dB/dec

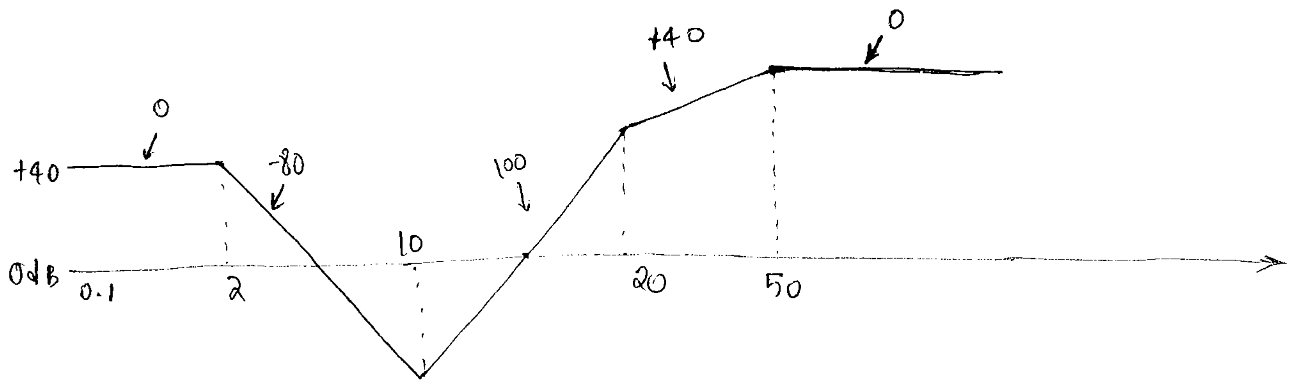
<u>with 1K</u>

$\left.\begin{matrix} 280Z \\ 460P \end{matrix}\right\}$ 180P

↘ 3600 dB/dec.

TRANSFER FUNCTION FROM BODE PLOT

Q, ~~Find~~ write the T.F to the given assymptotic magnitude plot.



S1: Observe the initial slope.

which gives the number of poles and Zeros at origin.

S2: Find the change in slope at each and every corner frequency.

If the change in slope is +ve, then consider the finite zeros.

If the change in slope is -ve, then consider the finite poles.

S3: Find the K value by using the known magnitude at known frequency.

Here, initial slope = 0 $\therefore$ No poles and Zeros at origin.

$$GH(s) = \frac{K\left(1+s/10\right)^9}{\left(1+s/2\right)^4\left(1+s/20\right)^3\left(1+s/50\right)^2}$$

At 2, change in slope = -80 dB/dec.

At 10, change in slope = -80 $\rightarrow$ +100

= +180 $\rightarrow$ 9 zeros at 10

At 20, change in slope = +100 $\rightarrow$ +40 $\rightarrow$ -60

3 poles at 20.

At 50, change in slope = +40 $\rightarrow$ 0 $\rightarrow$ -40

2 poles at 50.

$$20\log(\;) = 40$$

$$\log_{10}(\;) = 2$$

$\underline{K = 100}$

$$40\Big|_{0.1} = 20\log\left(\frac{K\,(1+s/10)^9}{(1+s/2)^4\,(1+s/20)^3\,(1+s/50)^2}\right)$$

$$\cancel{= 20\log\,(K\,(}$$

$$\cancel{= 2\log}$$

$$= 20\log K - 80\log\sqrt{\cancel{1+\left(\tfrac{\omega}{2}\right)^2}} - 60\log\sqrt{\cancel{1+\left(\tfrac{\omega}{20}\right)^2}}$$

Neglect   Neglect.

$$-40\log\sqrt{\cancel{1+\left(\tfrac{\omega}{50}\right)^2}} + 180\log\sqrt{\cancel{1+\left(\tfrac{\omega}{10}\right)^2}}$$

Neglect   Neglect.

$$\cancel{= 20\log K - 0.0433}$$

$$40\Big|_{0.1} = 20\log K$$

$$\underline{\underline{K = 100}}$$

$\therefore$

$$GH(s) = \frac{100\,(1+s/10)^9}{(1+s/2)^4\,(1+s/20)^3\,(1+s/50)^2}.$$

For Conventional, make a tabular form.

Corner frequency	change in slope.	Terms (factors)	
I.S	0	No pole/zero at origin.	
2	−80	4 poles $(1+s/2)^4$	$\rightarrow \dfrac{1}{(1+s/2)^4}$
10	+180	9 zeros $(1+s/10)^9$	$\rightarrow (1+s/10)^9$
20	−60	3 poles $(1+s/20)^3$	$\rightarrow \dfrac{1}{(1+s/20)^3}$
50	−40	2 poles $(1+s/50)^2$	$\rightarrow \dfrac{1}{(1+s/50)^2}$

In the above problem, for $\omega = 2$ also, we can find K value with in that

in that $-80 \log \sqrt{1 + \left(\frac{\omega}{2}\right)^2}$ at $\omega = 2$

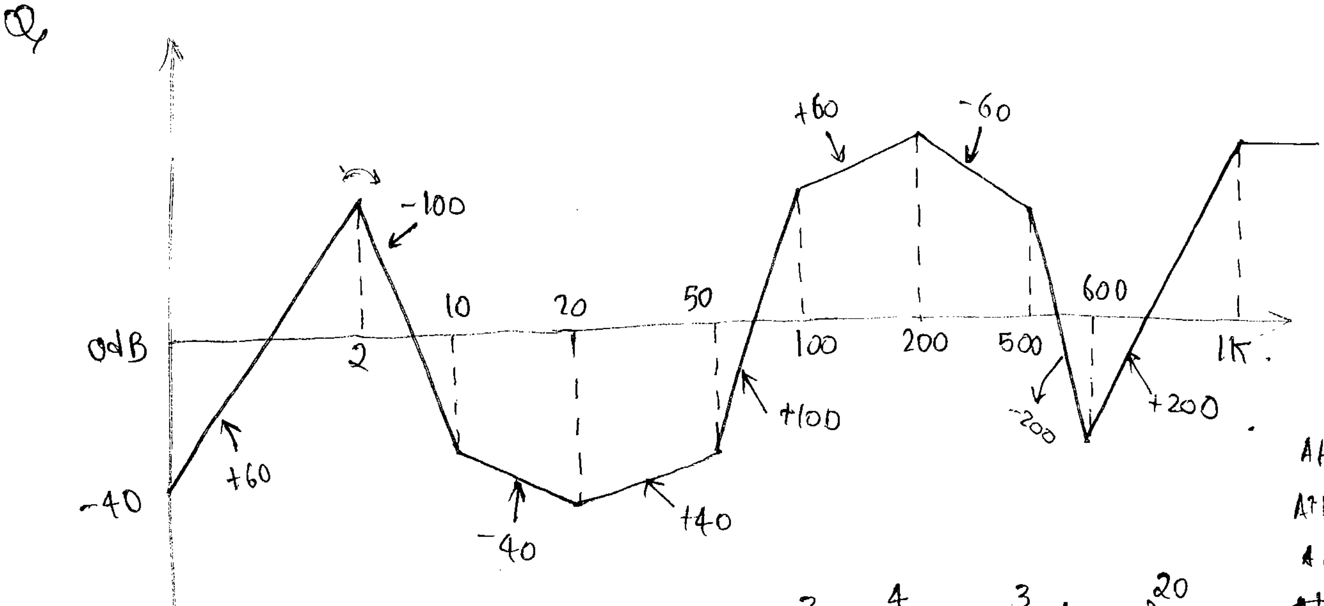
$= -80 \log \sqrt{2}$ can also be neglected because, at the corner frequency, there the error is the maximum so that much can be neglected.

Note that the above eqn is giving actual magnitude and given is only assymptotical.

NOTE: while finding the k value compare the corner frequency with the frequency where we knowns the magnitude.

If the corner frequency is greater than or equal to the frequency where the magnitude is known, then that corner frequency term is neglected.

Q



At f = 20
At + 80
At = 50 → +60

$$GH(s) = \frac{K s^3 \left(1+\frac{s}{10}\right)^3 \left(1+\frac{s}{20}\right)^4 \left(1+\frac{s}{50}\right)^3 \left(1+\frac{s}{600}\right)^{20}}{\left(1+\frac{s}{2}\right)^8 \left(1+\frac{s}{100}\right)^2 \left(1+\frac{s}{200}\right)^6 \left(1+\frac{s}{500}\right)^7 \left(1+\frac{s}{1000}\right)^{10}}$$

At 2 −160

At 10 +60

$$-40\Big|_{\omega=0.1} = +20\log k + \overset{60\log 0.1}{\cancel{60}} + \cdots + $$

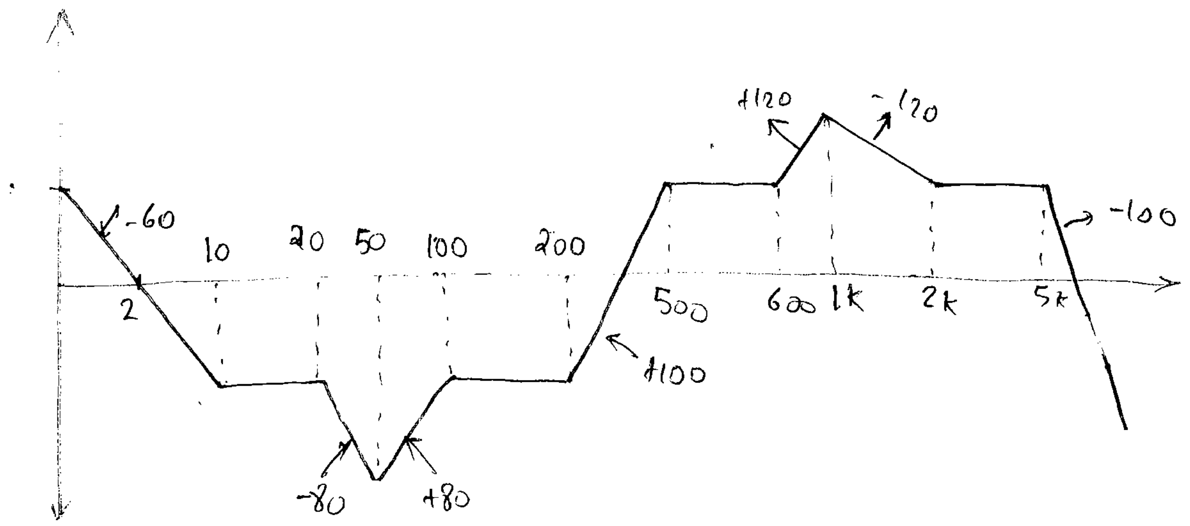
$$-40 = 20\log k - 60$$

$$20\log k = 20$$

$$\log k = 1$$

$$\underline{\underline{k = 10}}$$

Q)

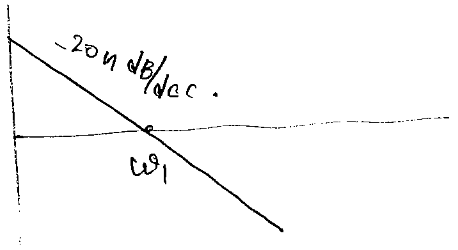


$$GH(s) = \frac{K\left(1+\frac{s}{10}\right)^3\left(1+\frac{s}{50}\right)^8\left(1+\frac{s}{200}\right)^5\left(1+\frac{s}{600}\right)^6\left(1+\frac{s}{2000}\right)^6}{s^3\left(1+\frac{s}{20}\right)^4\left(1+\frac{s}{100}\right)^4\left(1+\frac{s}{500}\right)^5\left(1+\frac{s}{1000}\right)^{12}\left(1+\frac{s}{5000}\right)^5}$$

~~At to~~ $\omega =$

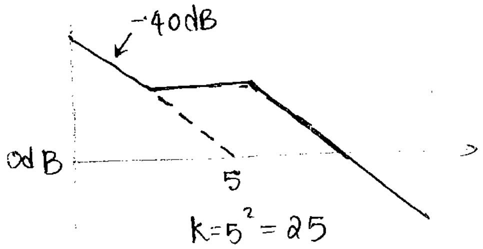
$$0\Big|_{\omega=2} = 20\log k - 60\log 2$$

$$0 = \log\frac{k}{2^3} \qquad \frac{k}{2^3} = 1 \qquad \underline{\underline{k = 8}}$$

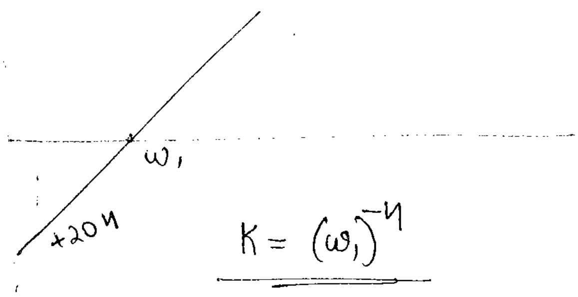


If we know ω_1

then $K = (\omega_1)^n$



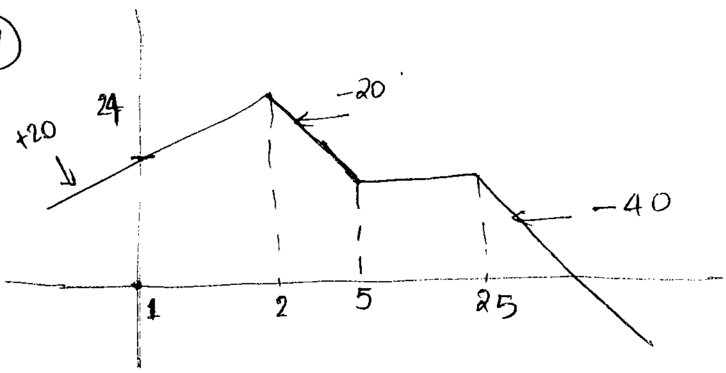
$K = 5^2 = 25$



$K = (\omega_1)^{-n}$

Q. Find the Transfer function

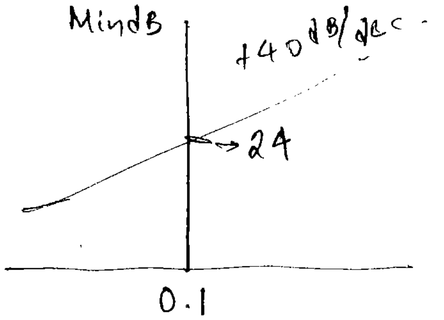
(1)



$$GH(s) = \frac{Ks\left(1+s/5\right)}{\left(1+s/2\right)^2\left(1+s/25\right)^2}$$

$$24 = 20\log K + 20\log 1$$

$$K = 15.849$$



$$GH(s) = K s^2$$

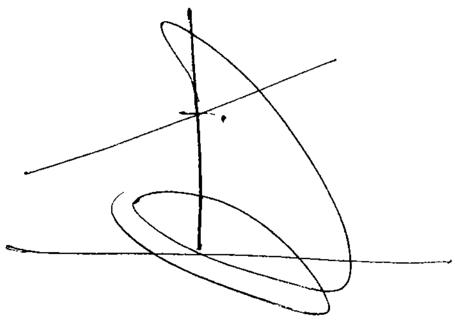
$$24 = 20 \log K + 40 \log 0.1$$

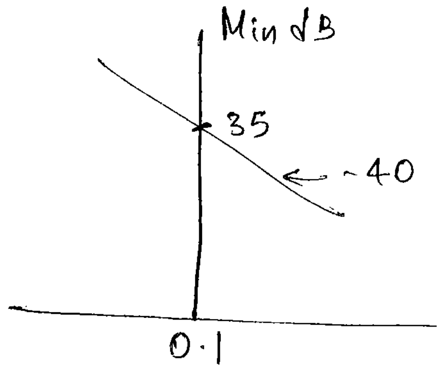
$$= 20 \log K\, 0.1^2$$

$$K\, 0.1^2 = 15.849$$

$$= \underline{\underline{1584.9}}$$

(3)





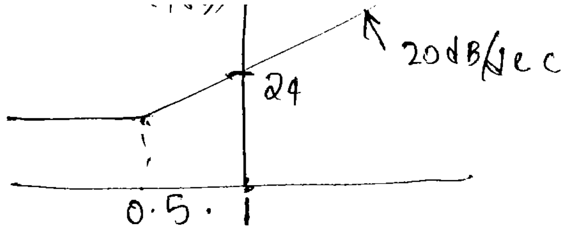
$$GH(s) = \frac{K}{s^2}$$

$$35 = 20 \log K - 40 \log 0.1$$

$$35 = \log \frac{K}{0.1^2}$$

$$\cancel{\frac{K}{0.1^2} =} \quad K = \underline{\underline{0.5623}}$$

Q



$$GH(s) = K\left(1+ s/_{0.5}\right)$$

$$24 = 20\log K + 20\log\sqrt{1+\left(1/_{0.5}\right)^2}$$

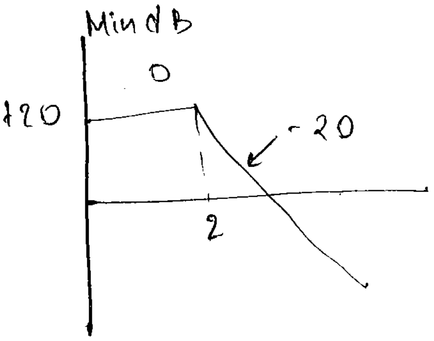
Here $\omega > 1/\tau$

1 must be neglected.

$$= K\sqrt{\cancel{1}+4}$$

~~K = 7.088~~ $\underline{K = 7.9245}$

Q. The steady state error to the unit step i/p is to the given plot is.



$$GH(s) = \frac{K}{\left(1+ s/_{2}\right)}$$

$$20\Big|_{\omega=2} = 20\log K$$

$$K = \underline{10}$$

Error constants are system Gain only.

$$GH = \frac{10}{\left(1+ s/_{2}\right)}$$

This is Type 2

Here unit step

so $K = K_p$

Here given unit step.

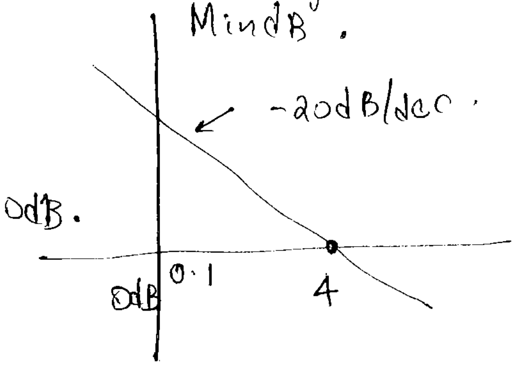
Take K value directly as Error constants

$$\therefore \boxed{e_{ss} = \frac{1}{1+K_p}} = \frac{1}{1+10} = \underline{\underline{\frac{1}{11}}}$$

$K_v = 0$ $\quad e_{ss} = \infty$

$K_a = 0$ $\quad e_{ss} = \infty$.

Q Find the steady state error for unit ramp.



GH(s)

Initial slope gives Type.

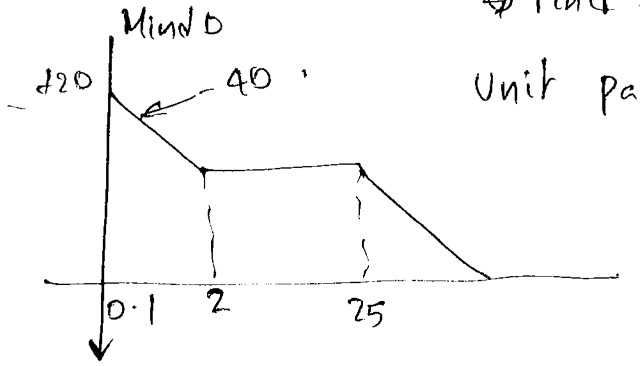
Type 1

$K = 4^1 = 4$

$K_v = 4$

$e_{ss} = \frac{1}{4} = 0.25$

Q



Find steadystate error to unit parabolic i/p.

$$GH(s) = \frac{K(1+s/2)^2}{s^2(1+s/25)}$$

$$20 = 20\log K - 20\log(0.1^2)$$

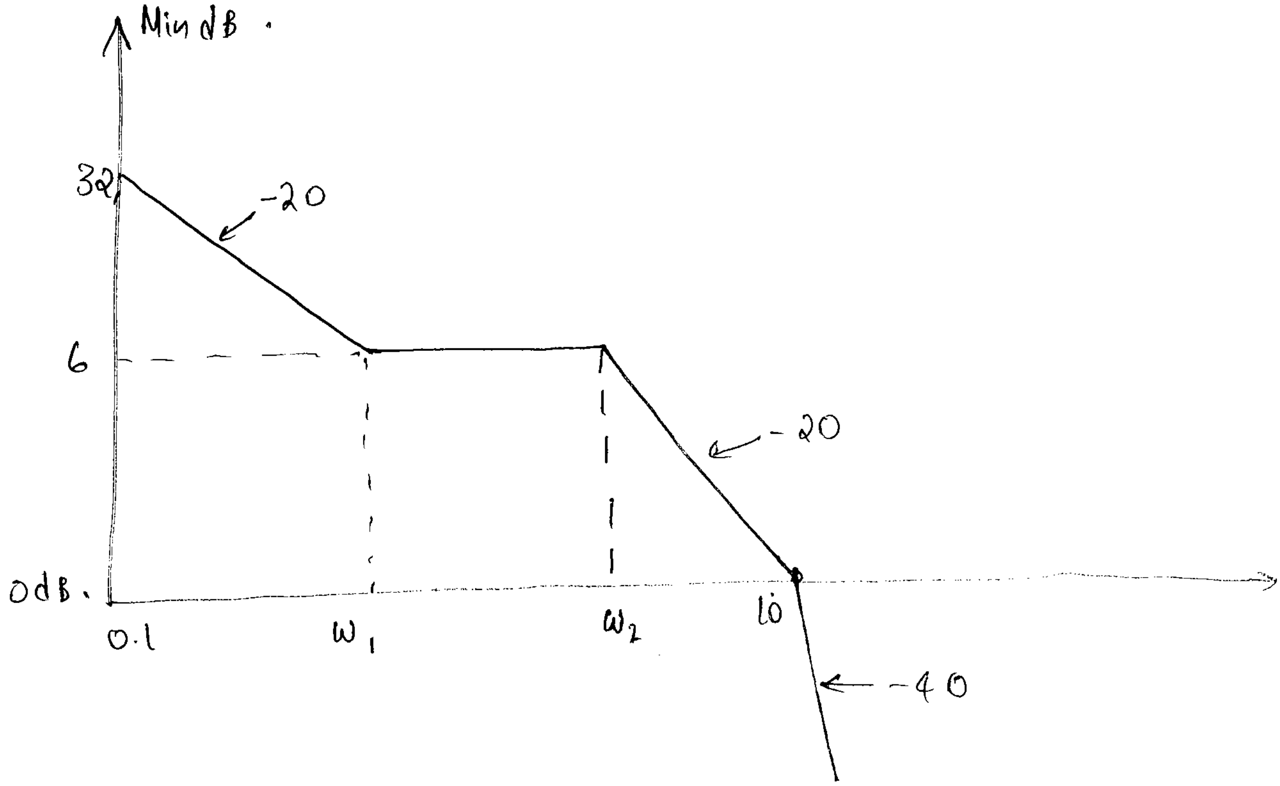
$$10 = \frac{K}{0.1^2}$$

$K_a = \cancel{100}\ 0.1$

$e_{ss} = \frac{1}{0.1} = 10$

$\cancel{S = M_2 - M_1}$

* The initial -ve slope represents the type of the system.



Find ω_1, ω_2, and T.F .

$$GH(s) = \frac{K\left(1+ s/\omega_1\right)}{s\left(1+ s/\omega_2\right)\left(1+ s/10\right)}$$

$$32 = 20\log K - 20\log 0.1$$

$$\cancel{32} = \frac{K}{0.1}$$

$$\underline{\underline{K = 3.981}}$$

$$S = \frac{M_2 - M_1}{\log\omega_2 - \log\omega_1}$$

$$-20 = \frac{32-6}{\log 0.1 - \log\omega_1}$$

$$\omega_1 = 1.995$$
$$= \underline{\underline{2}}$$

$$\cancel{\frac{10}{13}} \quad -\frac{13}{10} = \frac{\cancel{\log 0.1}}{-1 - \log\omega_1}$$

$\cancel{\log\omega_1}$

$$-20 = \frac{6-0}{\log\omega_2 - \log 10}$$

$$= \frac{6}{\log\omega_2 - 1}$$

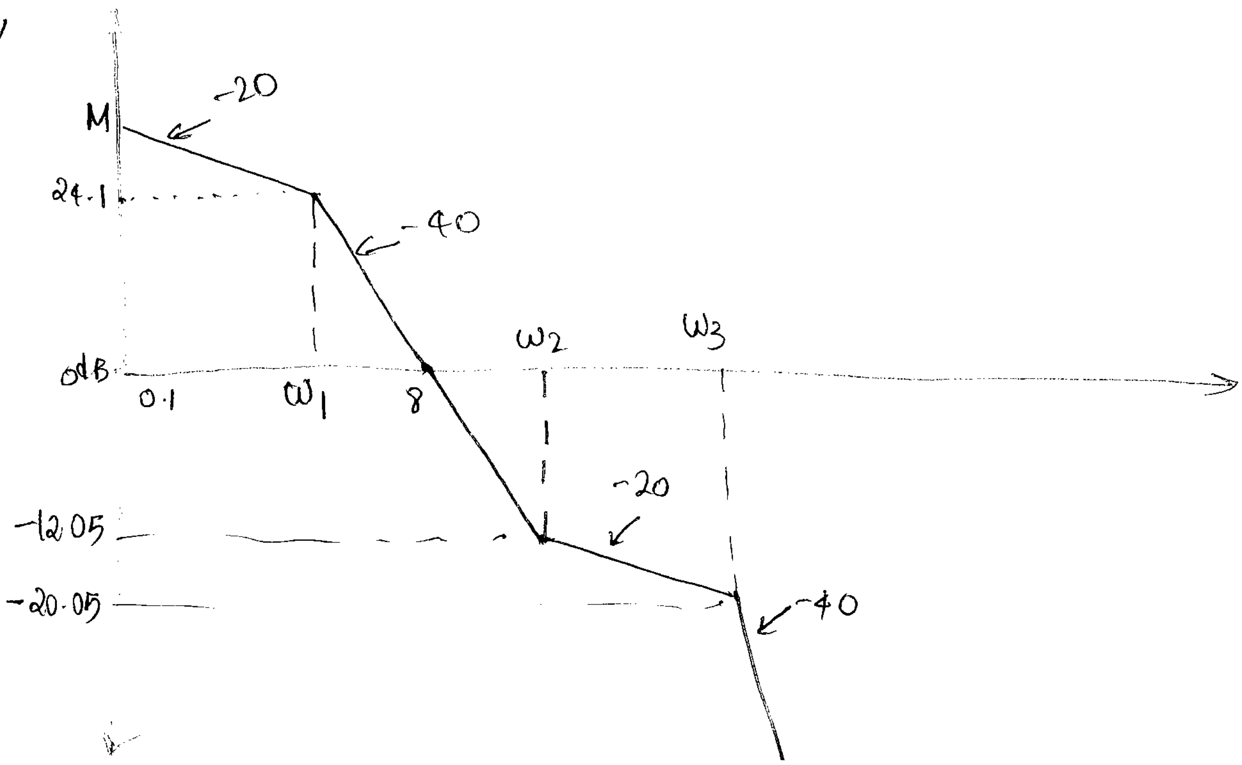
$$\log\omega_2 - 1 = -\frac{3}{10}$$

$$\underline{\underline{\omega_2 = 5}}$$

For finding K value, we can use slope equation at any point. All of them give the same value.

$$GH(s) = \frac{3.981\left(1+ s/_2\right)}{s\left(1+ s/_5\right)\left(1+ s/_{10}\right)}$$

D)



$$-40 = \frac{24.1 - 0}{\log \omega_1 - \log 8}$$

$$\log \omega_1 - \log 8 = -0.6025$$

$$\underline{\underline{\omega_1 = 2}}$$

$$\cancel{-40 = 0 - \log \omega \quad 0 -}$$

$$\cancel{-40 = \frac{24.1 - M_2}{\log 2 - \log \omega_2}} \qquad \cancel{-40 = \frac{\cancel{24.1}\ 0 - M_2}{\log 8 - \log \omega_2}}$$

$$\cancel{\log 2 - \log \omega_2 = \frac{24.1 - M_2}{-40}} \qquad \cancel{-40 = -M_2}$$

$$\cancel{40}$$

$$\cancel{\log \omega_2 - \log 8 = M_2}$$

$$\log 8 - \log \omega_2 = \frac{M_2}{40}$$

$$=$$

$$\cancel{M_2 + 10}$$

$$40 \log 8 - 40 \log \omega_2 = M_2$$

$$\cancel{M_2 + 40 \log \omega_2 = 40 \log}$$

$$-40 = \frac{0 + 12.5}{\log 8 - \log \omega_2}$$

$$\log 8 - \log \omega_2 = \frac{-5}{16}$$

$$\omega_2 = \underline{\underline{16.428}}$$

$$-20 = \frac{-12.05 + 20.05}{\log 16 - \log \omega_3}$$

$$\log \omega_3 - \log 16 =$$

$$\omega_3 = \underline{\underline{40.19}}$$

$$-20 = \frac{24.1 - M}{\log 2 - \log 1}$$

$$M = \underline{\underline{30.12 \text{ dB}}}$$

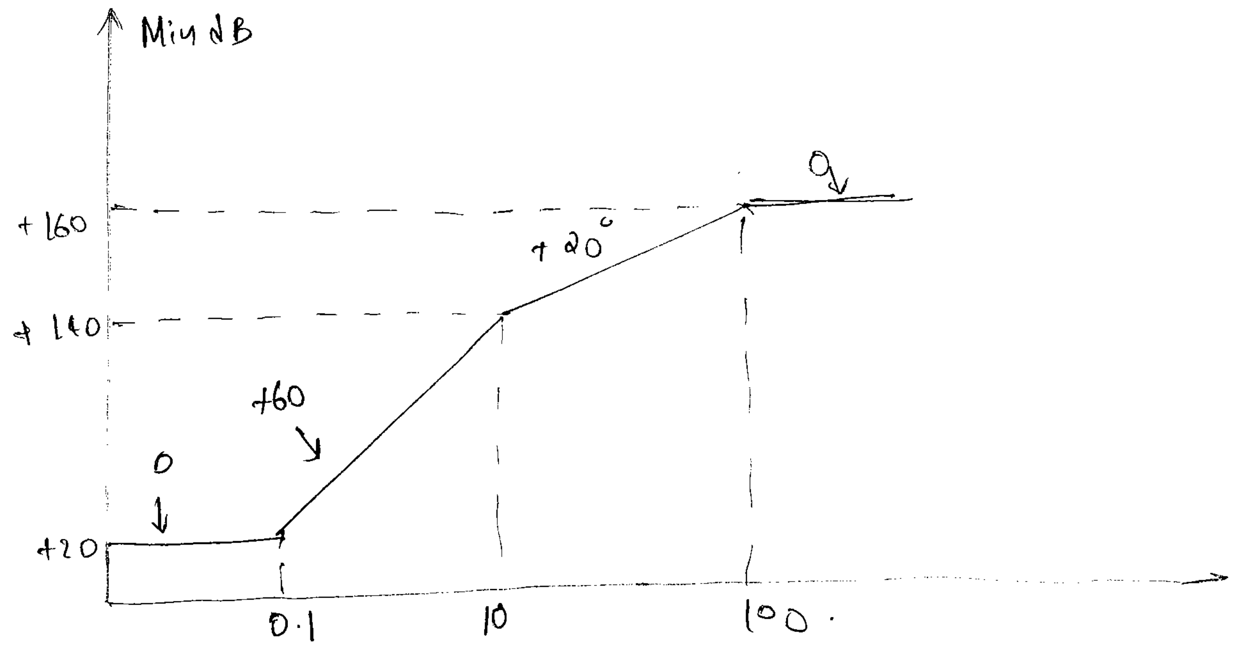
$$GH(s) = \cancel{30.12}\ \frac{K\left(1 + s/16\right)}{s\left(1 + s/2\right)\left(1 + s/40\right)}$$

$$30.12\Big|_{\omega=1} = 20\log K - 20\log 1$$

$$= 20\log \frac{K}{1}$$

$$K = \underline{\underline{32.06}}$$

Q, The assympotic approximation of the Bode Magnitude plot of a minimum phase system is shown in figure. Its transfer function is.



$20 = 20 \log K$

$\underline{\underline{K = 10}}$

$$\frac{140-20}{\log 10 - \log 0.1} = \frac{120}{1+1} = \underline{\underline{+60}}$$

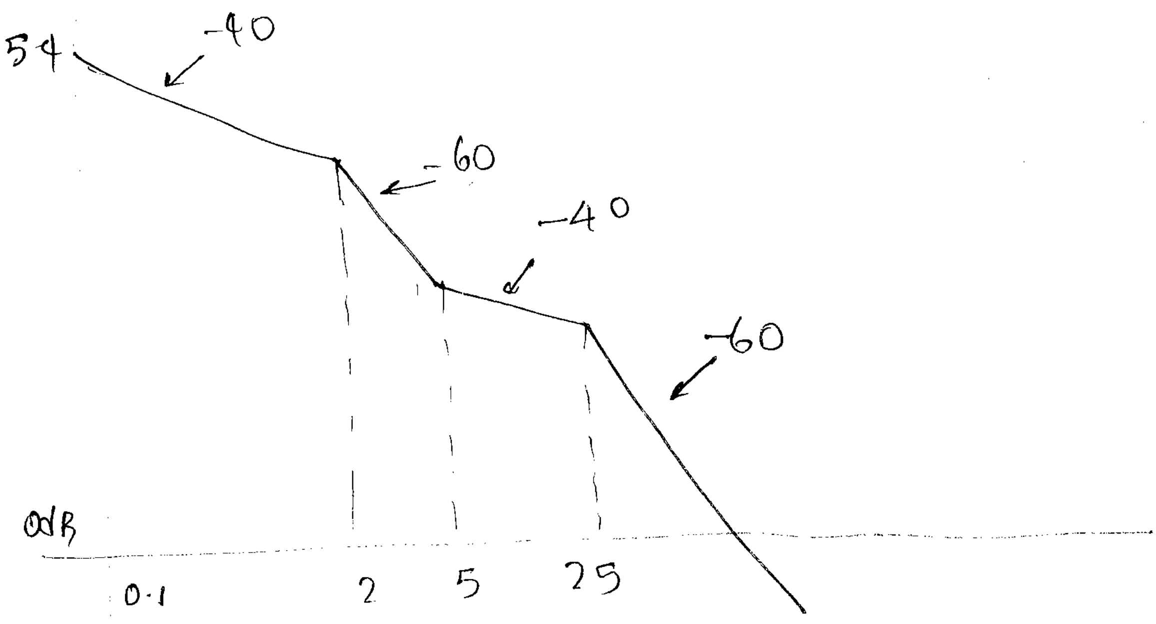
$$\frac{20}{2-1} = \underline{\underline{+20}}$$

$$\frac{10\left(1+s/0.1\right)^3}{\left(1+s/10\right)^2\left(1+s/100\right)}$$

$$\frac{10 \times 10^2 \times 100}{(0.1)^3} = \frac{10^5}{\left(\frac{1}{10}\right)^3} = \underline{\underline{10^8}}$$

$$GH(s) = \frac{10^8 \cancel{60} (s+0.1)^3}{(s+10)^2 (s+100)}.$$

Q, The assympotic approximation of the log magnitude v/s frequency plot of a minimum phase system is shown in fig. Its T.F is



$$GH(s) = \frac{K\left(1+\frac{s}{5}\right)}{s^2\left(1+\frac{s}{2}\right)\left(1+\frac{s}{25}\right)}$$

(d)

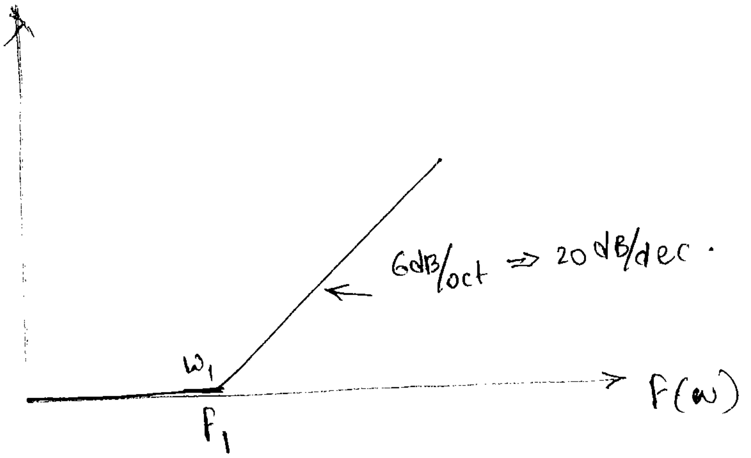
$$54 = 20\log\frac{K}{(0.1)^2}$$

$K = \underline{5.011}$ $\qquad \cancel{5\times} \frac{\cancel{5}}{\cancel{5}} \times 2 \times 25$

$= \underline{50}$

$$GH(s) = \frac{50(s+5)}{s^2(s+2)(s+25)}$$

Q. Find the T.F.



(a) $\frac{jf}{f_1}$. (b) $j\,f_1/f$ (c) $\frac{1}{(1+j\,f/f_1)}$ (d) $(1+j\,f/f_1)$

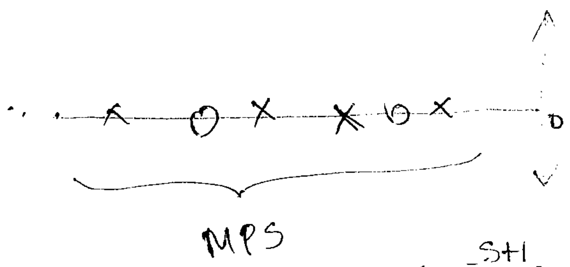
$$GH(s) = K\left(1+ s/\omega_1\right) = K\left(1+\frac{j\omega}{j\omega_1}\right)$$

$$= 1 + j\frac{2\pi f}{2\pi f_1} = \left(1+ jf/f_1\right)$$

Even for $\left(1-\frac{s}{\omega_1}\right)\underbrace{e^{\pm s}}_{1}$, the magnitude is $\sqrt{1+\left(\frac{\omega}{\omega_1}\right)^2}$

So to avoid confusions, we take Bode plot is valid for Minimum phase system.

Minimum phase system. (MPS)

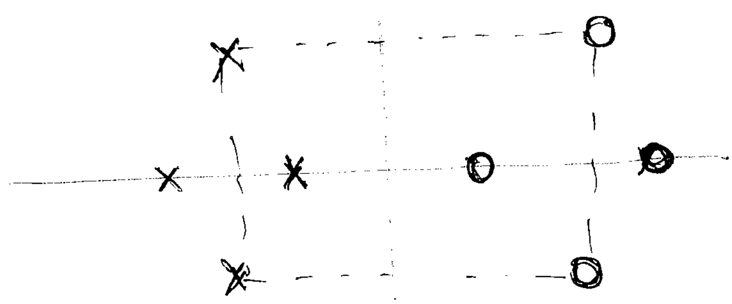


Always stable.

A system in which all the finite poles, finite zeros lies in the left of s plane. Then it is called Minimum phase system.

eg: $\frac{-s+1}{(s+2)(s+3)}$

All pass ~~phase~~ system.



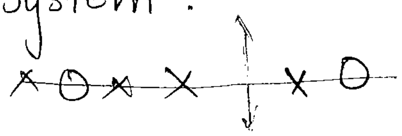
A system in which all the zeros lies in the right side, all the poles lies in the left side of s plane which are symmetrical about the imaginary axis, then the system is called All pass system.

All pass system gives magnitude 1 and phase angle varies b/w $\pm 180^\circ$

eg: $\frac{s-1}{s+1}$ $\left(\frac{s^2-2s+2}{s^2+2s+2}\right)$

Non-minimum phase system.

A system in which one or more poles or zeros lies in the right of s plane ~~or one or more zeros lies in the~~ then it is called Non-minimum phase system.



Note: In non minimum phase system, as ω increases from 0 to ∞ the phase angle should be more negative. The non minimum phase systems may be unstable system.

Consider the NMPS $= \dfrac{(S-1)(S+3)}{(S+2)(S+5)}$

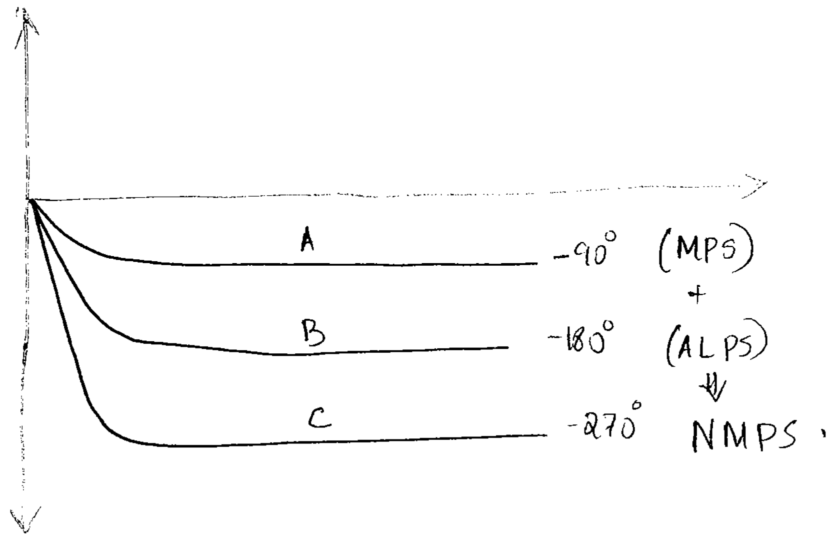
$$\text{NMPS} = \underbrace{\frac{(S+3)(S+1)}{(S+2)(S+5)}}_{\text{MPS}} \times \underbrace{\frac{S-1}{(S+1)}}_{\text{ALPS}}$$

ie, Non minimum phase system = Minimum phase system × ALPS

$$\boxed{\text{NMPS} = \text{MPS} \times \text{ALPS}}$$

$$\phi_{NMPS} = \phi_{MPS} + \phi_{ALPS}$$

Q, Identify the curves A, B, C



A → MPS

B → ALPS

C → ~~270~~ NMPS.

STABILITY CONDITION

The stability ~~of~~ conditions are to find the close loop system stability.

The closed loop system stability given by characteristic equation.

CE $\Rightarrow$ $1+G(S)H(S)=0$

$S \to j\omega$ $\quad 1+G(j\omega)H(j\omega) = 0 \longrightarrow (1)$

$G(j\omega)H(j\omega) = (-1+j0)$

$|G(j\omega)H(j\omega)| = 1 \rightarrow$ linear.

M in dB $= 20\log 1 = 0$

$\omega_{gc} \rightarrow$ Gain Cross over Frequency.

GAIN CROSSOVER ω

The Frequency at which Magnitude $=1$ in linear, 0 in dB is called Gain cross over frequency.

Consider phase of (2)

PHASE CROSSOVER ω

The frequency at which the phase angle $= -180^\circ$ is called the phase crossover frequency. $\angle GH(j\omega) = \angle -1+j0)$

Note: For phase crossover frequency selected -180°. Because the control systems are low pass filters which give the more negative angle than positive.

GAIN MARGIN : Gain Margin is defined as the reciprocal of the magnitude of the System, at the phase crossover frequency.

$$\text{GAIN MARGIN} = \frac{1}{|GH(j\omega)|_{\omega=\omega_{pc}}}$$

$$GM_{in\,dB} = -20\log |GH(j\omega)|_{\omega=\omega_{pc}}$$

Gain margin is the factor by which the system gain K is increased to drive the system to verge of instability.

PHASE MARGIN : phase Margin is defined as the amount of the additional phase lag at the Gain crossover frequency required to bring the system verge of instability. (~~As~~ from marginal stable).

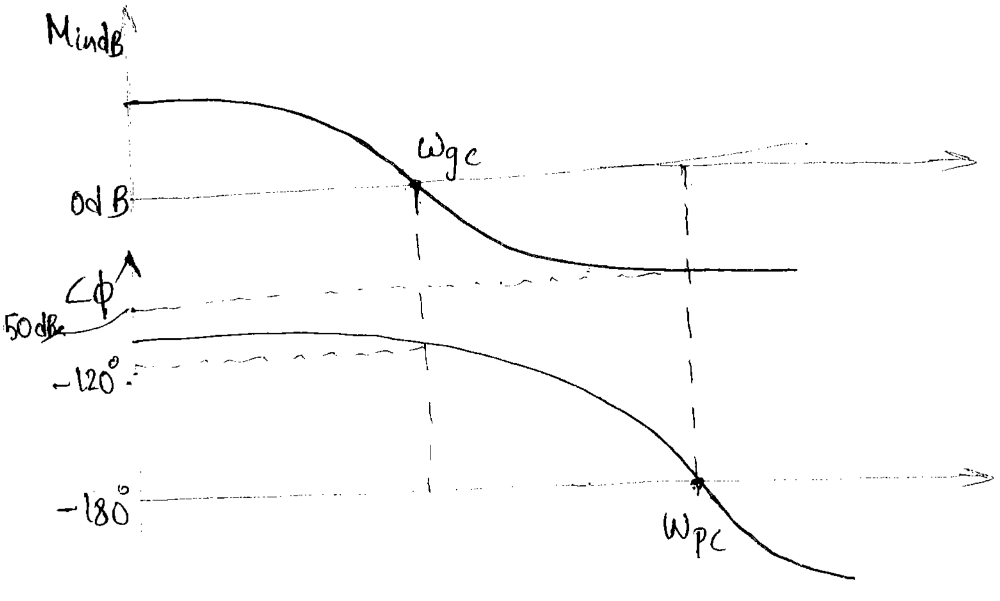
$$\text{PHASE MARGIN} = 180^\circ + \angle GH|_{\omega=\omega_{gc}}$$

~~$\omega_{pc} > \omega_{gc}$~~

Condition of stability.

$\omega_{pc} > \omega_{gc}$	Stable	Gain Margin >1 G.M +ve in dB	phase margin +ve.
$\omega_{pc} = \omega_{gc}$	Marginal stable.	Gain Margin $=1$ G.M = 0 dB	phase Margin 0°
$\omega_{pc} < \omega_{gc}$	Unstable	Gain Margin <1 G.M -ve in dB	phase Margin -ve.

Q, Identify the stability in the given Bodeplots.



$\omega_{pc} > \omega_{gc}$

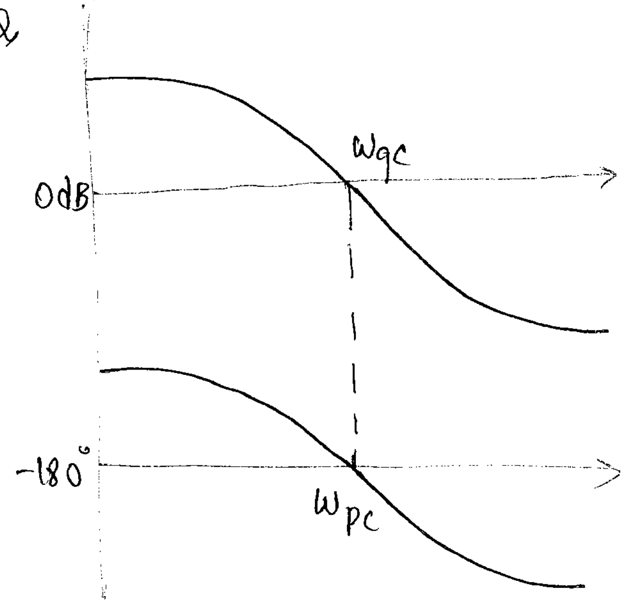
Hence Stable.

$$\text{Gain Margin} = -(M \text{ in dB})_{\omega_{pc}}$$

$$= -(-50\,dB) = \underline{+50\,dB}$$

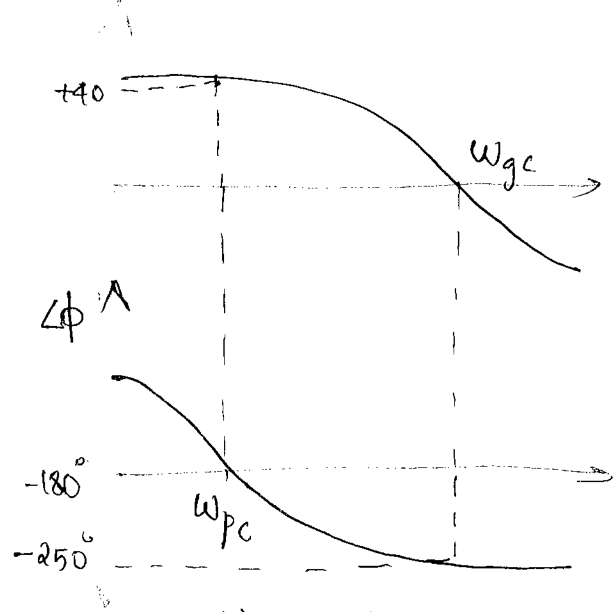
Phase Margin $= 180 + \angle GH(s) = 180 - 120 = +60^\circ$

Q



$\omega_{pc} = \omega_{gc}$ GM = + 0 dB)

Marginal stable . PM = 180 − 180 = $\underline{0}$



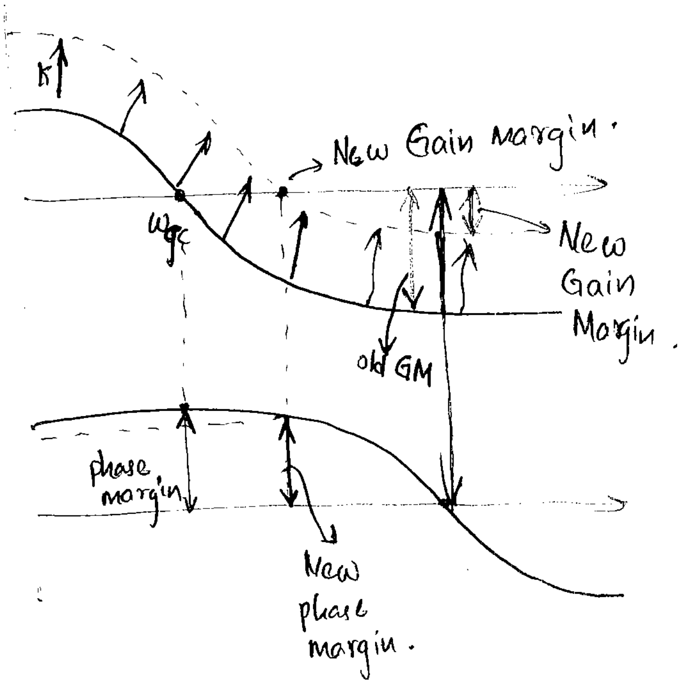
$\omega_{gc} > \omega_{pc}$ GM = −40dB

Unstable . PM = $180 - 250^\circ$

$= \underline{-70^\circ}$

Variation w.r.t to Variation in K

Note:



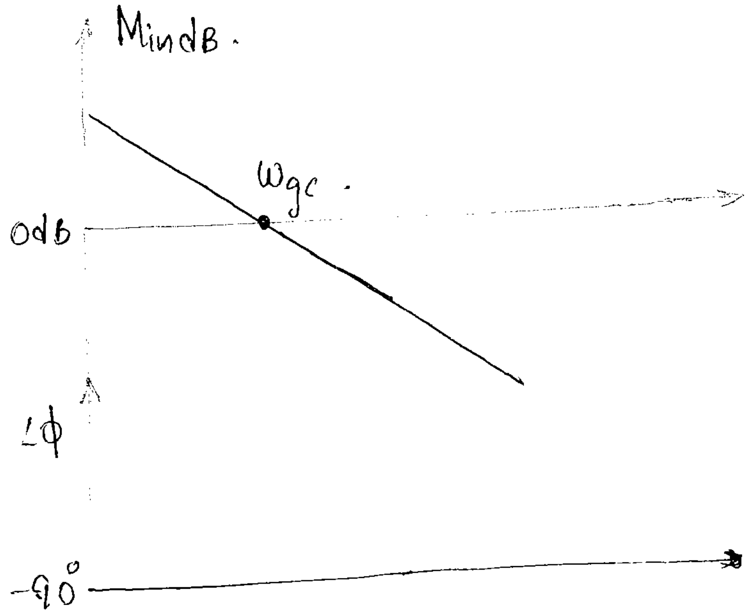
If the system Gain K is increases, then ω_{pc} does not changes. But ω_{gc} increases, The Gain margin and phase Margin decreases.

K ↑ ω_{pc} → constance

ω_{gc} ↑

GM, PM ↓

Q, Identify the stability to the following plots.



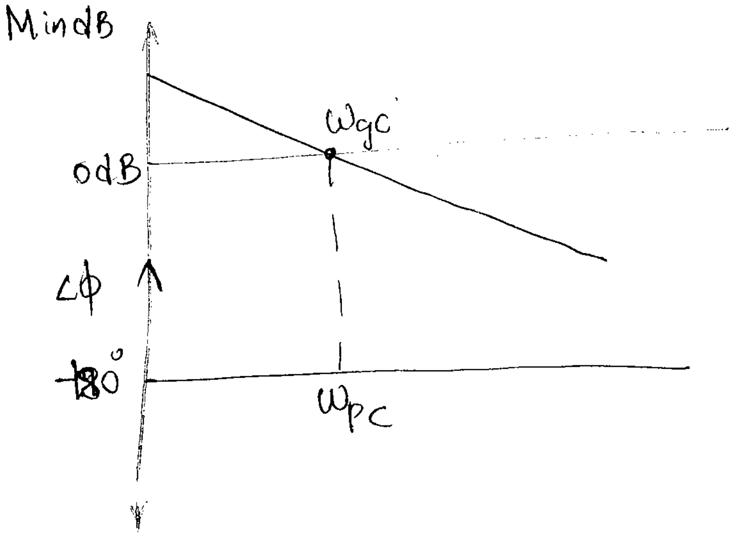
ω_{pc} does not exist.

For plots that never came down to 180, we approximate,

$$\boxed{\omega_{pc} \simeq \infty}$$

$\omega_{pc} >>> \omega_{gc}$

Hence STABLE.

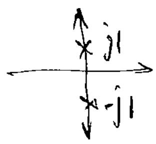


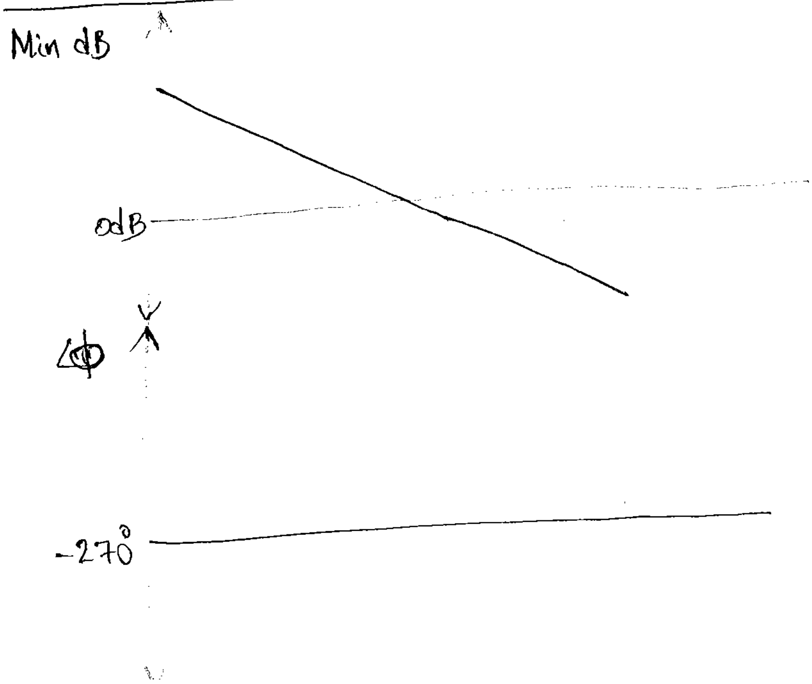
Actully $\omega_{pc} \Rightarrow 0$ to ∞

$\therefore$ For such system we take value $\boxed{\omega_{pc} \simeq \omega_{gc}}$

Hence MARGINAL STABLE

$GH(s) = \frac{1}{s^2}$ CLTF $= \frac{1}{s^2+1}$





Actully ω_{pc} does not exist.

we approximate

$$\boxed{\omega_{pc} \simeq 0}$$

Hence UNSTABLE

1. whenever the plot or transfer function maintains less negative than -180°, at all the frequency range then the system is Stable because $\boxed{\omega_{pc} > \omega_{gc}}$ (Actually ω_{pc} does not exist, but approximately ∞).

2. whenever the plot or transfer function maintains -180° at all the frequency range, then the system is marginal stable, because here $\boxed{\omega_{pc} = \omega_{gc}}$

3. whenever the plot or transfer function maintains more -ve than -180° at all the frequency range, then the system is unstable because here $\boxed{\omega_{pc} < \omega_{gc}}$ (Actually ω_{pc} does not exist, but approximately Zero).

Q. A unity feedback control system $G(s) = \dfrac{80}{s(s+2)(s+20)}$. Draw the Bode plot on semi-log sheet and determine ω_{gc}, ω_{pc}, G.M and P.M

$$G(s) = \frac{80}{s \cdot 2(1+s/2)\,20(1+s/20)} = \frac{2}{s(1+s/2)(1+s/20)}$$

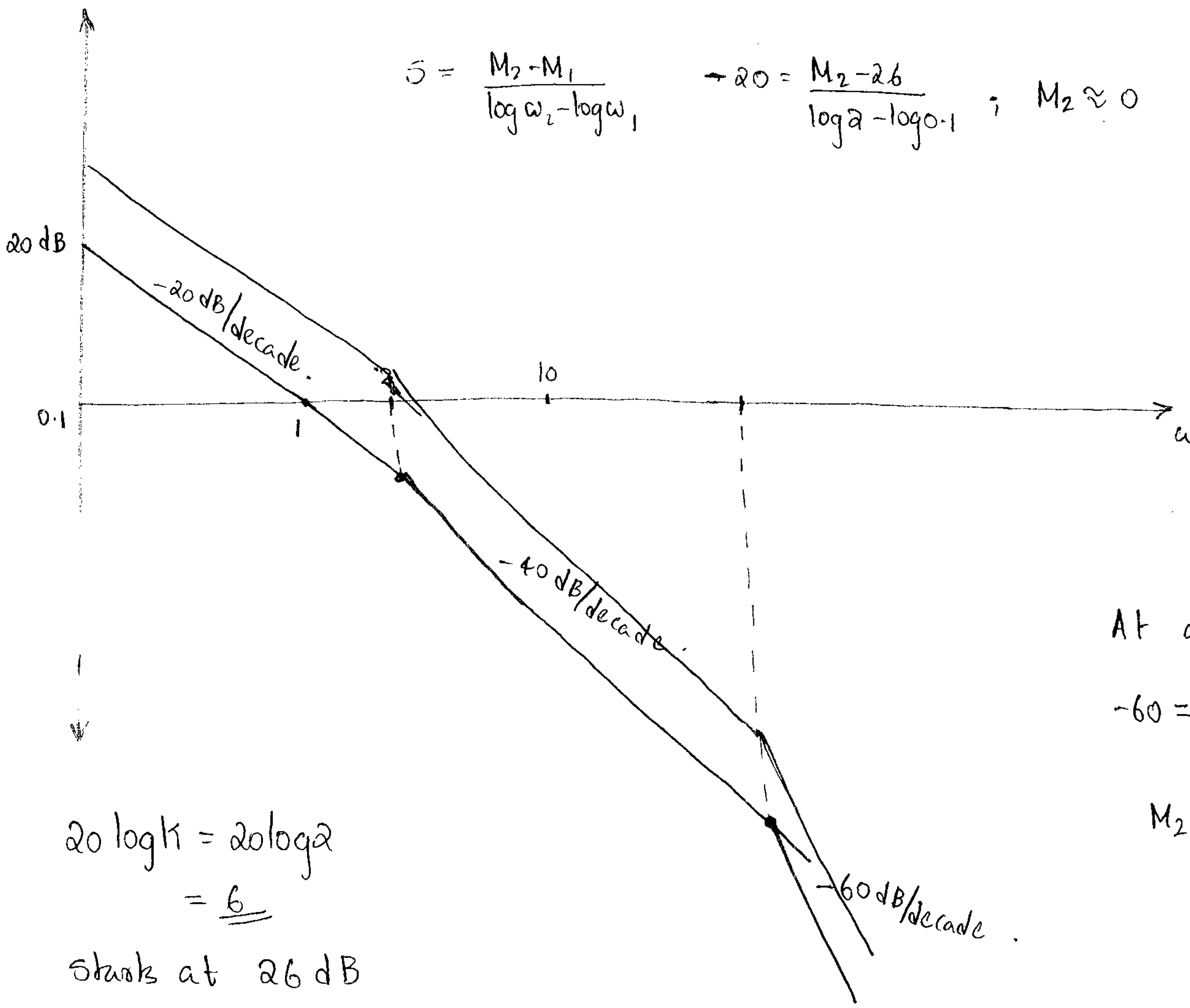
$$G(j\omega) = \frac{2}{j\omega(1+j\omega/2)(1+j\omega/20)}$$

$$|G(j\omega)|_{dB} = 20\log 2 - 20\log\omega - 20\log\sqrt{1+\left(\frac{\omega}{2}\right)^2} - 20\log\sqrt{\left(\frac{\omega}{20}\right)^2+1}$$

$$\angle\phi = 0 - 90^\circ - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{20}\right)$$

FACTOR	RESULTANT SLOPE	STARTING & ENDING FREQUENCY
1 pole at origin $\left(\frac{1}{s}\right)$	-20 dB/decade	$\omega = 0.1$ to 2
1 finite pole at $\omega=2$ $\left(1+\frac{s}{2}\right)$	-40 dB/decade	$\omega = 2$ to 20
1 finite pole $\left(1+\frac{s}{20}\right)$	-60 dB/decade	$\omega = 20$ to ∞

$$S = \frac{M_2 - M_1}{\log\omega_2 - \log\omega_1} \qquad -20 = \frac{M_2 - 26}{\log 2 - \log 0.1} \;; \quad M_2 \approx 0$$



At 200

$$-60 = \frac{M_2 + 40}{\log 200 - \log 20}$$

$M_2 = -100$ dB.

$20\log K = 20\log 2$

$= \underline{\underline{6}}$

Starts at 26 dB

$\angle\phi_{\omega=0.1} = -93.14^\circ$

$\angle\phi_{\omega=0.5} = -105.46^\circ$

$\angle\phi_{\omega=2} = -140.7^\circ$

$\angle\phi_{\omega=8} = -187.76^\circ$

$\angle\phi_{\omega=20} = -219.28^\circ$

$\angle\phi_{\omega=200} = -263.71^\circ$

$\omega_{gc} = 2 \text{ rad/s} \qquad \omega_{pc} = 6.0 \text{ rad/s}.$

$GM = -(-20\,dB) = 20\,dB$

$PM = 180 + \angle GH(j\omega)\Big|_{\omega=\omega_{gc}}$

$= 180 - 140 = \underline{\underline{40^\circ}}$

$\angle GH = -180^\circ$

$-180^\circ = -90^\circ - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{20}\right)$

$90^\circ = \tan^{-1}\left(\frac{\omega}{2}\right) + \tan^{-1}\left(\frac{\omega}{20}\right)$

$90^\circ = \tan^{-1}\left(\dfrac{\frac{\omega}{2} + \frac{\omega}{20}}{1 - \frac{\omega^2}{40}}\right)$

$\omega_{pc} = \sqrt{40} = 6.28 \text{ rad/s}.$

Q. For a unity feedback system, $G(s) = \dfrac{10}{s(s+1)(s+5)}$. sketch the Bode plot. Determine G.M and P.M & If the GM increases by 150%, then what is the value of new G.M and P.M

$G(s) = \dfrac{10}{s(1+s)5(1+s/5)} = \dfrac{2}{s(1+s)(1+s/5)}$

$G(j\omega) = \dfrac{2}{j\omega(1+j\omega)(1+j\frac{\omega}{5})}$

$|G(j\omega)|_{dB} = 20\log 2 - 20\log\omega - 20\log\sqrt{1+\omega^2} - 20\log\sqrt{1+(\omega/5)^2}$

$\angle\phi = -90^\circ - \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{5}\right)$

Factor	Resultant slope	starting and ending freq
1 pole at origin ($1/s$)	-20 dB/decade	$\omega = 0.1$ to $\omega = 1$
1 finite pole at $\omega=1$	-40 dB/decade	$\omega = 1$ to 5
1 finite pole at $\omega = 5$	-60 dB/decade	$\omega = 5$ to ∞

$20 \log 2 = 6.02$

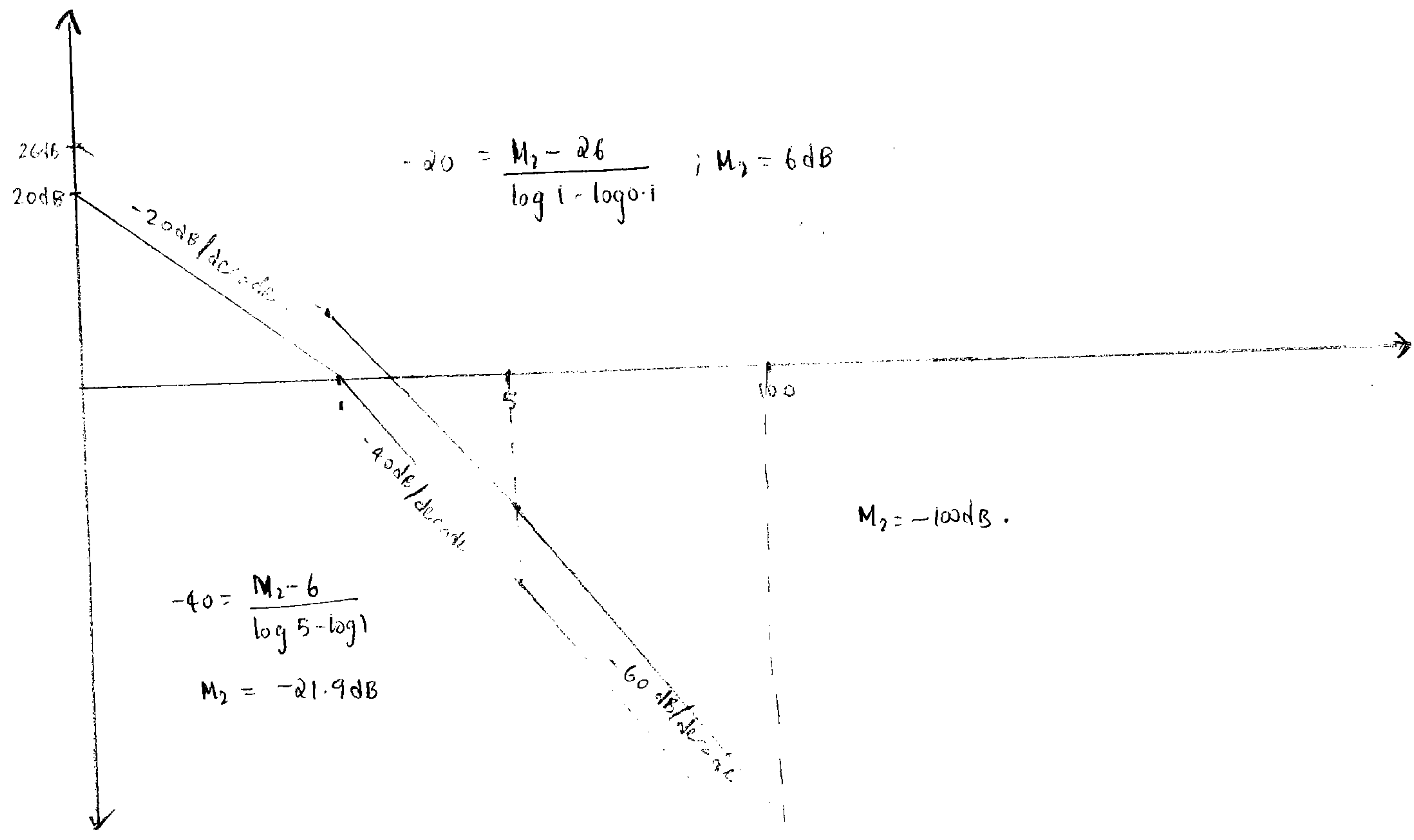
$\angle\phi_{0.1} = -96.85$ $\qquad \angle\phi_{\omega=1} = -146.3$ $\qquad \angle\phi_{\omega=5} = \underline{\underline{-216.6^\circ}}$

$\angle\phi_{\omega=0.5} = -122.2^\circ$ $\qquad \angle\phi_{\omega=2} = -175.2$

$\omega_{gc} = 1.5$ rad/s. $\qquad \omega_{pc} = 2.3$ rad/s.

$GM = -(-8)\,dB = 8\,dB$

$PM = 180 - 162 = \underline{\underline{18^\circ}}$



GM = 8dB

150 % of 8 dB = 1.5 × 8 dB.

New GM $= \frac{150}{100} \times$ old GM $= \underline{12\ dB}$

To get GM = 12 dB, at ω_{pc}, Magnitude should be = −12 dB.

To get GM of 12 dB, the should be shifted downwards by 4 dB.

ie, at ω_{pc} the magnitude should be −12.

$20 \log K = -4$ (shifted downwards).

$K = \underline{0.63}$

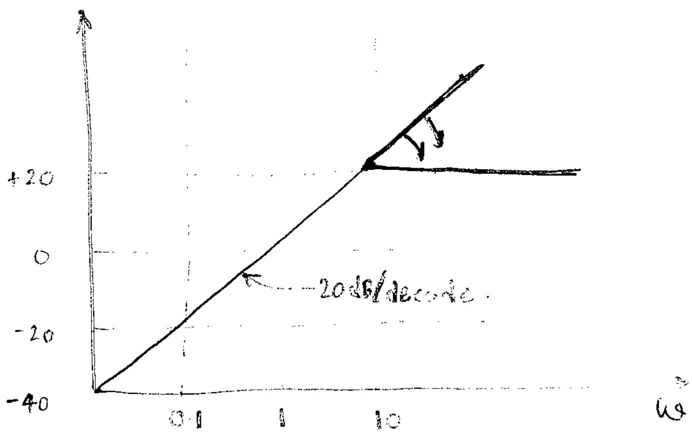
New value of K = Existing value × obtained value of K from plot.

$= 2 \times 0.63 = \underline{1.26}$

$\therefore G(s) = \frac{1.26}{s(s+1)(1+s/5)}$.

Q, The frequency response of a T.F given below.

i) Determine the T.F   ii) Step Response assuming zero initial energy stored.



i) $G(s)H(s) = \frac{Ks}{(1+s/10)}$

$0|_{\omega=1} = 20\log K$ $K = 10^0 = \underline{1}$

$$T.F = \frac{s}{(1+s/10)} \Rightarrow \frac{C(s)}{R(s)} = \frac{10s}{s+10}$$

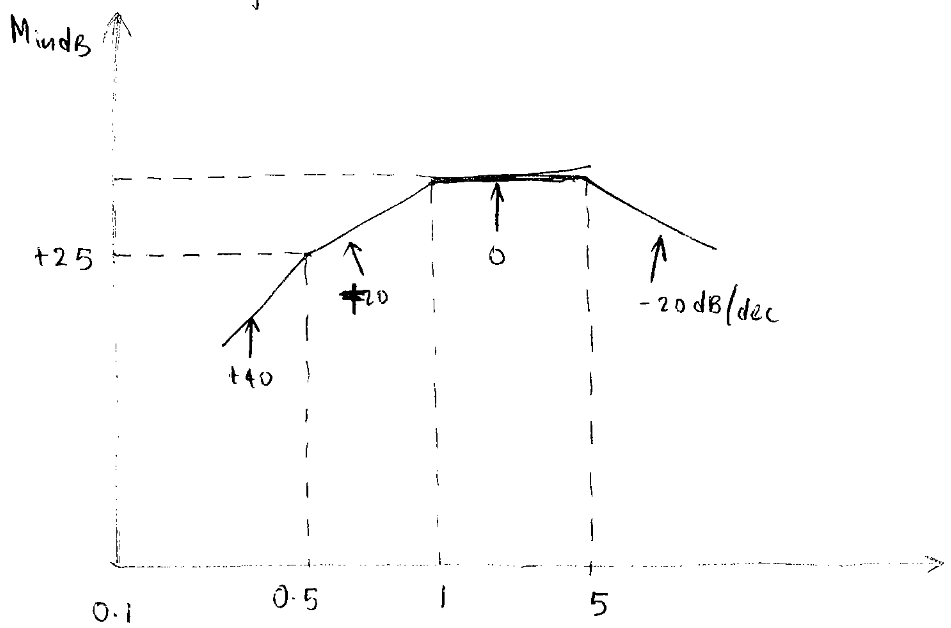
(ii) unit step input $r(t) = u(t)$

$R(s) = 1/s$.

$$c(s) = \frac{1}{s} \cdot \frac{10s}{(s+10)}$$

$$\therefore c(t) = 10e^{-10t}$$

Q. Determine the open loop T.F of a feedback control system of a Bode magnitude whose characteristics is shown in fig.



$$G(s)\,H(s) = \frac{K s^2 \cancel{(1+s)}}{(1+s)(1+s/0.5)^3 (1+s/5)}$$

$25 = 20\log 5 + 40\log\omega$

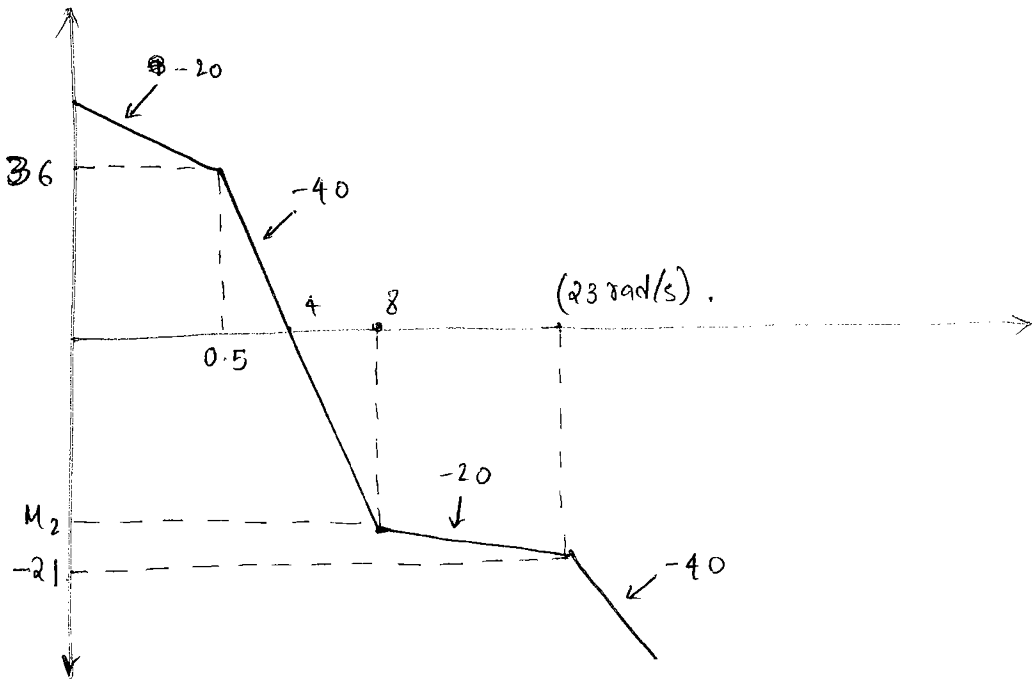
$25 = 20\log K + 40\log 0.5$

$= 20\log K \Rightarrow K = 71.13$

$$G(s) = \frac{71.13\, s^2 \times 0.5 \times 5}{(s+0.5)(s+1)(s+5)}$$

$$= \frac{178\, s^2}{(s+0.5)(s+1)(s+5)}$$

Q. The assymplotic log magnitude v/s frequency plot is show in fig. Determine T.F



$$-40 = \frac{36-0}{\log \omega_1 - \log 4}$$

$$-40 \log \omega_1 + 24.08 = 36$$

$$\underline{\omega_1 = 0.5 \text{ rad/s}}$$

$$-40 = \frac{M_2 - 36}{\log 8 - \log 0.5}, \quad \underline{M_2 = -12\,dB}$$

$$-20 = \frac{-12+21}{\log 8 - \log \omega_3} \qquad -18.06 + 20\log \omega_3 = 9$$

$$\omega_3 = 22.5 \approx \underline{23 \text{ rad/s}}$$

$$GH = \frac{K\left(1+s/8\right)}{s\left(1+s/0.5\right)\left(1+s/23\right)}$$

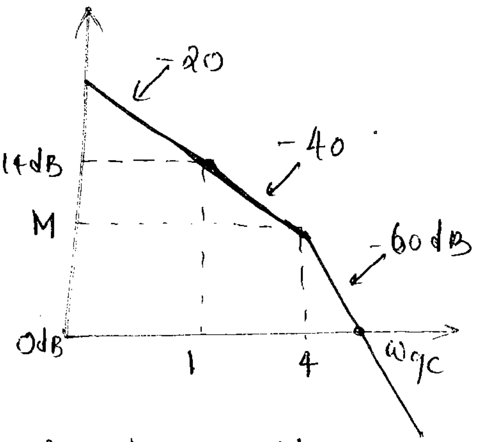
$$GH\big|_{\omega=0.5} = 36$$

$$36 = 20\log K - 20 \log \omega$$

$$\underline{K = 32}$$

$$GH = \frac{32\left(1+s/8\right)}{s\left(1+s/0.5\right)\left(1+s/23\right)}$$

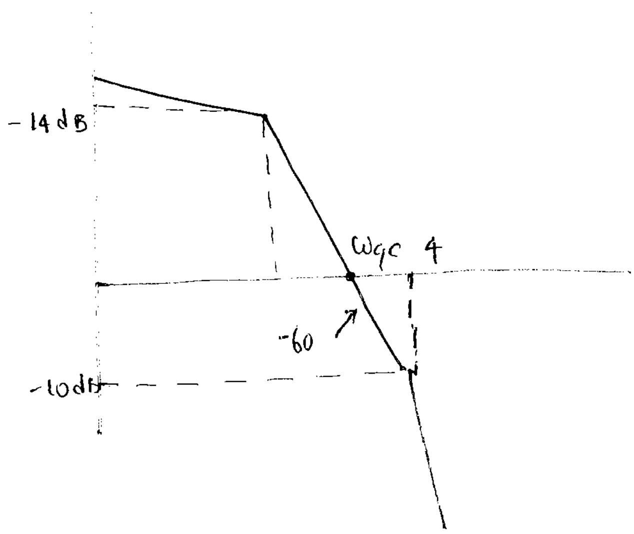
Q. The open loop T.F of a unity feed back control system have an initial value slope of -20 dB/dec and passing through 14 dB and also at simple corner frequencies of 1 & 4. Determine the T.F & ω_{gc}.



$$T.F = \frac{K}{S(1+S)(1+S/4)}$$

$$-40 = \frac{14-M}{\log 1 - \log 4} \qquad M = -10\,dB.$$

The plot should be.



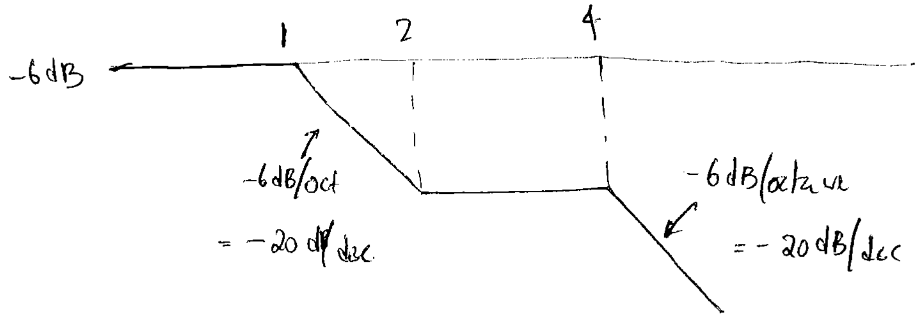
$$-40 = \frac{(14)}{\log 4 - \log \omega_{gc}}$$

$$40 = \frac{14}{\log 4 - \log \omega_{gc}}$$

$$24.08 - 40\log \omega_{gc} = 10$$

$$\omega_{gc} = 2.24 \text{ rad/s}$$

Q. what is a Bode plot. Explain the technique for drawing it. Find the Transfer function to the given assymptotic magnitude plots.



$$G(s)H(s) = \frac{K(1+s/2)}{(1+s)(1+s/4)}$$

$$-6 = 20\log K$$

$$\underline{K = 0.5}$$

$$G(s)H(s) = \frac{0.5(1+s/2)}{(1+s)(1+s/4)}$$

Q, Draw the Bode plot for $G(s)H(s) = \dfrac{10^3(s+20)}{(s^2+210s+2000)}$.

Determine the value of $G(j1000)$.

$$|G(j\omega)|_{\omega=1000} = \frac{10^3\sqrt{\omega^2+20^2}}{\sqrt{(\omega^2+10^2)}\sqrt{(\omega^2+200^2)}}$$

$$= \frac{10^3\sqrt{10^6+20^2}}{\sqrt{10^6+10^2}\sqrt{10^6+200^2}} = \underline{0.98} \approx 1$$

COMPLEX BODE PLOTS

I n- complex poles.

$$GH(s) = \left(\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}\right)^n$$

$s \to j\omega_n$

$$GH(j\omega) = \left(\frac{\omega_n^2}{-\omega^2+j2\xi\omega_n\omega+\omega_n^2}\right)^n$$

$\div\ \omega_n^2$

$$GH(j\omega) = \left[\frac{1}{\left(1-\left(\frac{\omega^2}{\omega_n^2}\right)\right)+j2\xi\left(\frac{\omega}{\omega_n}\right)}\right]^n$$

$$\boxed{M_{actual\ dB} = -20n\log\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\xi\left(\frac{\omega}{\omega_n}\right)\right)^2}}$$

$$\boxed{\phi_{actual} = -n\tan^{-1}\left(\frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right)}$$

ASSYMPTOTIC / APPROXIMATE ANALYSIS

Case (i) $\frac{\omega}{\omega_n} < 1$, neglect $\left(\frac{\omega}{\omega_n}\right)$ and $\left(2\xi\left(\frac{\omega}{\omega_n}\right)\right)$

$\therefore$ $M_{assymp} = 0$

slope = 0 dB/decade.

$\phi_{assymp} = 0^\circ$

Case (ii) $\frac{\omega}{\omega_n} > 1$, neglect 1 and also $\left(2\xi\left(\frac{\omega}{\omega_n}\right)\right)$

$\therefore$ $M_{assymp} = -20n\log\left(\frac{\omega}{\omega_n}\right)^2$

$M_{assymp} = -40n\log\omega + 40n\log\omega_n.$

$$\boxed{S = \frac{dM}{d\log W} = -40n \text{ dB/decade}}$$

$$\phi_{assymp} = -n\tan^{-1}\left[\frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{-\left(\frac{\omega}{\omega_n}\right)^2}\right]$$

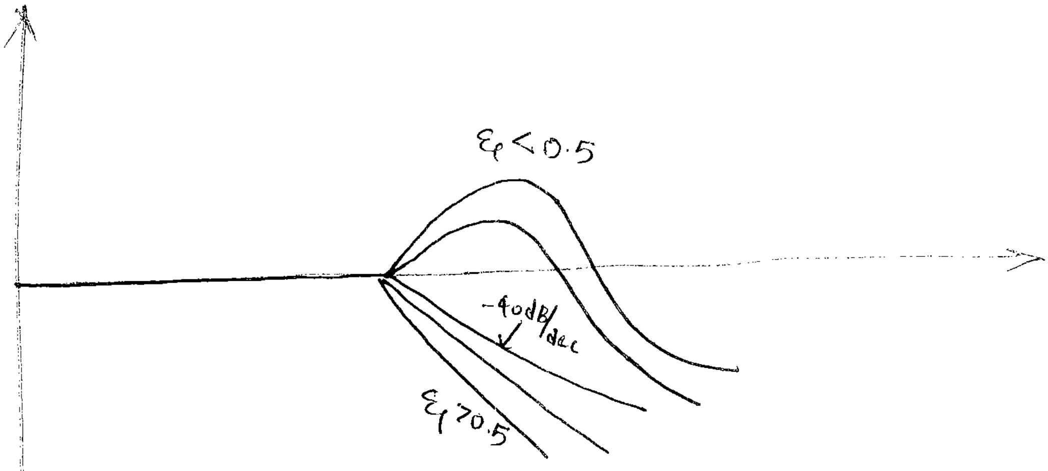
$$= -n \tan^{-1}\left(-\theta_{v.small}\right)$$

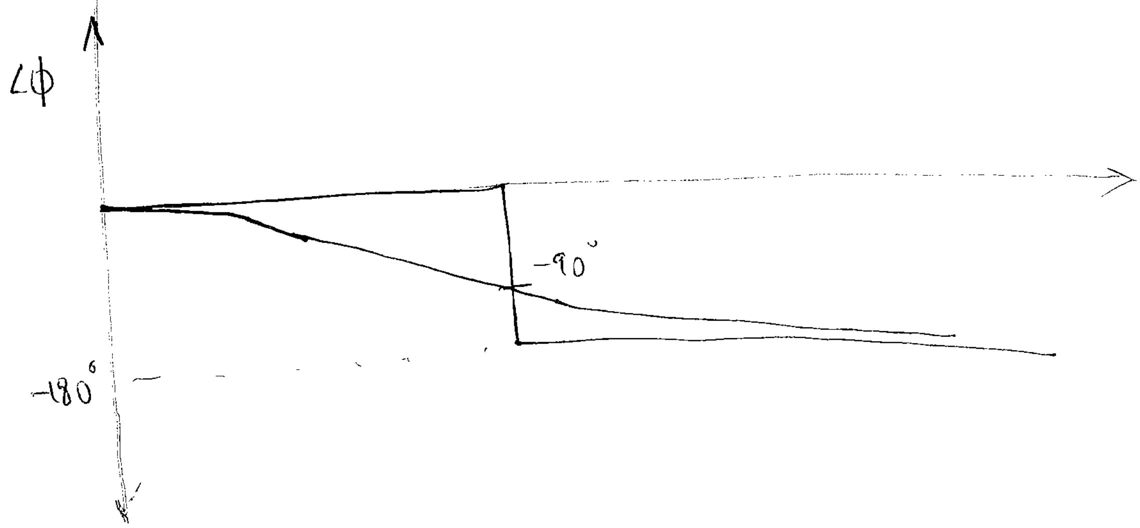
$$= -n\left(180 - \tan^{-1}\theta_{v small}\right)$$

$$\boxed{\phi_{assym} = -180^\circ n}$$

'wn is called corner frequency.

	S	$\angle\phi$
$<$CF	0dB/decade	0°
$>$CF	−40ndB/dec	$-180^\circ n$





CORRECTION AT CORNER FREQUENCY

$$\boxed{M_{correction} \text{ at CF } (\omega = \omega_n) = -20n\log(2\xi)}$$

$$\boxed{\phi_{correction} \text{ at CF} = -90^\circ n}$$

The correction at corner frequencies depends on ξ. Other than corner frequencies, it depends on ξ and ω_n, in the magnitude plot, where as in the phase plot, the correction at corner frequency is constant $(-90n)$ other than corner frequencies, it depends on ξ and ω_n.

n - Complex zeros

$$\boxed{M_{actual \text{ in dB}} = +20n\log\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi\left(\frac{\omega}{\omega_n}\right)\right)^2}}$$

Case (i) $\frac{\omega}{\omega_n} < 1$

$$M_{assymp} = +20n\log 1 = 0$$

$$\xi = 0 \qquad \phi_{assymp} = 0^\circ$$

Case (ii) $\frac{\omega}{\omega_n} > 1$

$$M_{assymp} = 20n\log\left(\frac{\omega}{\omega_n}\right)^2$$

$$= 40\log\omega - 40n\log\omega_n.$$

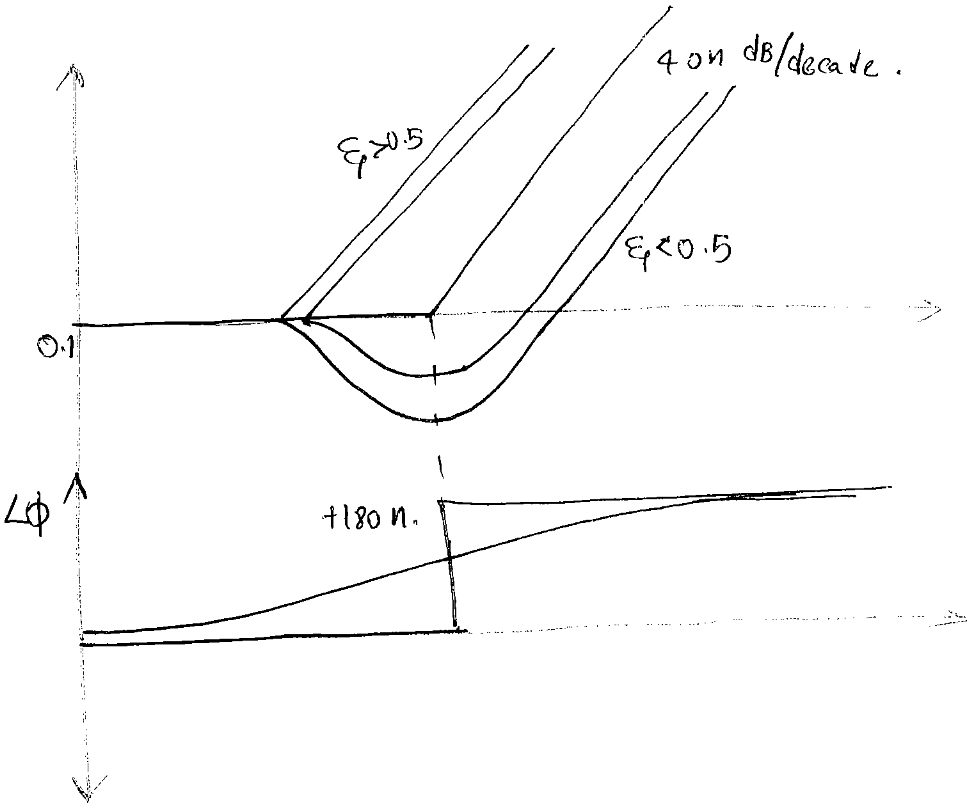
$S = 40n\,dB/dec.$

$\phi_{assympt} = +180^\circ n$

	S	$\angle\phi$
< CF	0 dB/decade	0°
> CF	+40n dB/decade	$+180^\circ n$

$M_{correction}$ at CF $= +20n\log(2\zeta)$

$\phi_{correction}$ at CF $= +90^\circ n$



Q, Draw the Bode plot to the given T.F.

$$GH(s) = \frac{s^2\left(1+\frac{s}{20}+\frac{s^2}{100}\right)^3}{\left(1+\frac{s}{3}+\frac{s^2}{9}\right)^2\left(1+\frac{s}{50}\right)^4}$$

$\omega_n^2 = 100$

$\omega_n = 10$

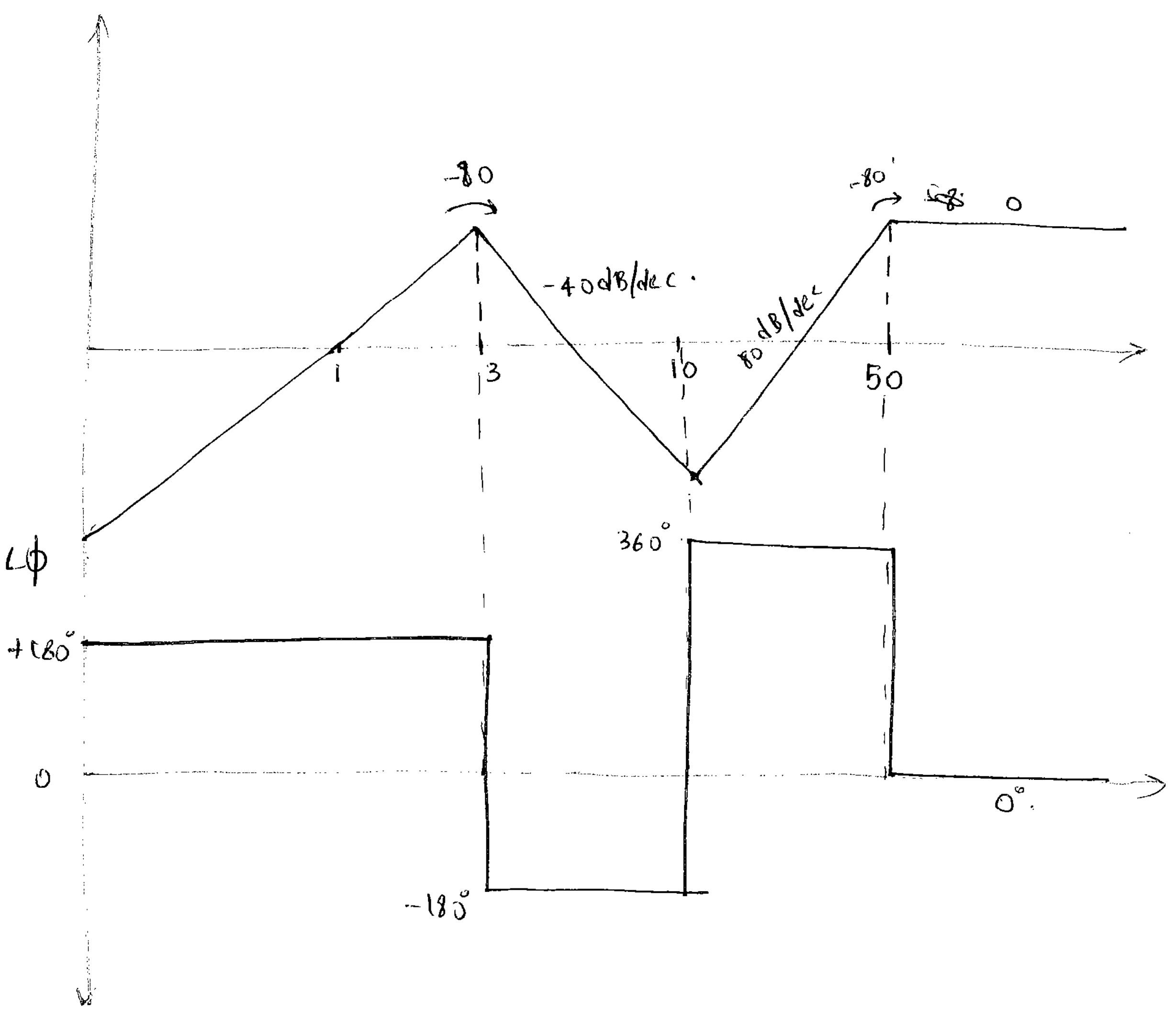
$\xi = 0.25$

$\frac{2\xi}{\omega_n} = \frac{1}{20}.$

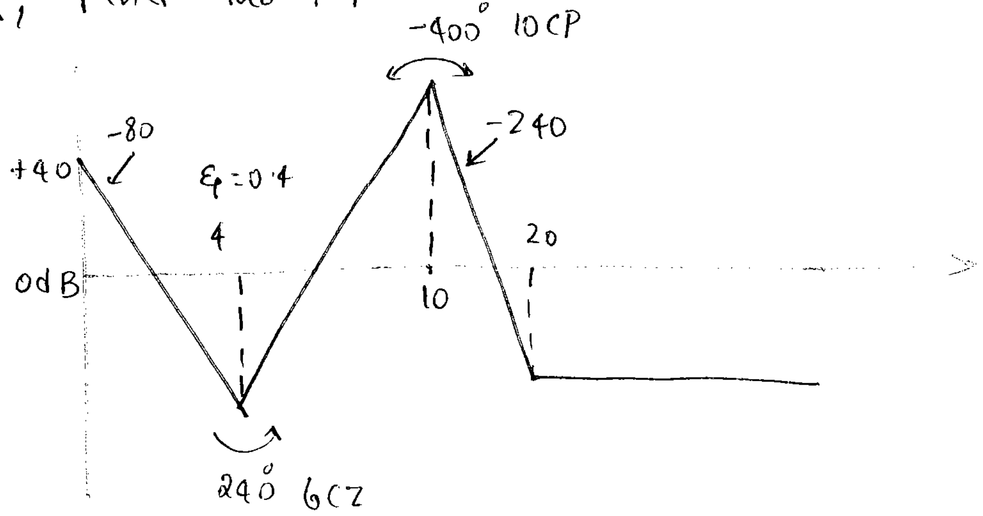
$\omega_n^2 = 9$

$\omega_n = 3$

$\xi = 0.5$



Q, Find the T.F



$$GH(s) = \frac{K}{s^4}\left(\frac{s^2+3.2s+16}{16}\right)^6\left(\frac{100}{s^2+14s+100}\right)^{10}\left(1+s/20\right)^{12}$$

$$40 = 20\log K - 80\log 0.1$$

$$\underline{K = 0.01}$$

POLAR PLOT

PURPOSE

- → To draw the frequency response of OLTF
- → To find closed loop system stability
- → To find GM, PM, ω_{gc}, and ω_{pc}
- → To find the relative stability.

$\boxed{\text{Polar plot is magnitude v/s phase plot.}}$

* Note:

The polar plots are used in the Nyquist plots, to find the closed loop stability. The frequency range for the polar plot is 0 to ∞, whereas Nyquist plot is $-\infty$ to ∞

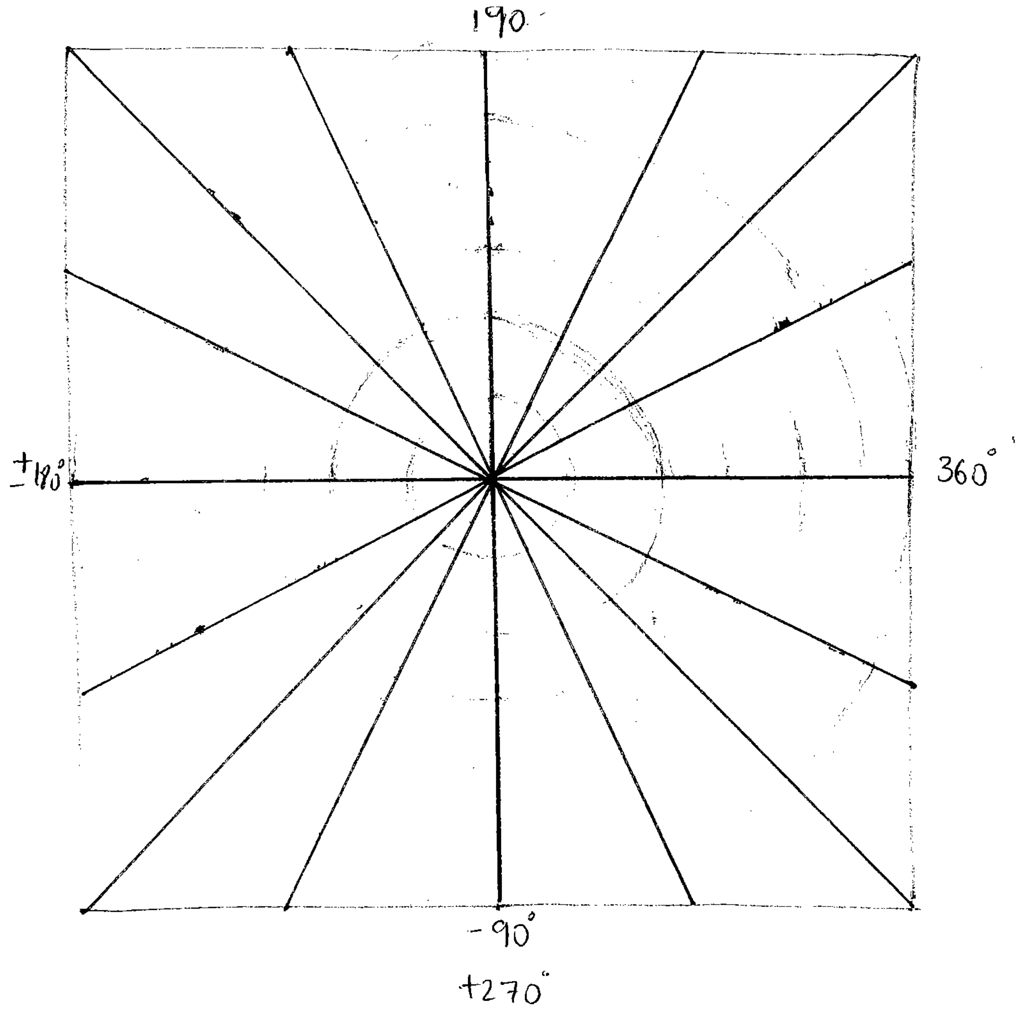
Q. Draw the polar plot for $G(s)\,H(s) = 1/s$

$s \to j\omega$.

$GH(j\omega) = \frac{1}{j\omega}$

$M = \frac{1}{\omega}$, $\angle\phi = -90^\circ$

ω	M	$\angle\phi$
0	∞	-90°
1	1	-90°
2	0.5	-90°
5	0.2	-90°
10	0.1	-90°
∞	0	-90°



Q, $GH(j\omega) = \frac{1}{s\tau + 1}$

$s \rightarrow j\omega$

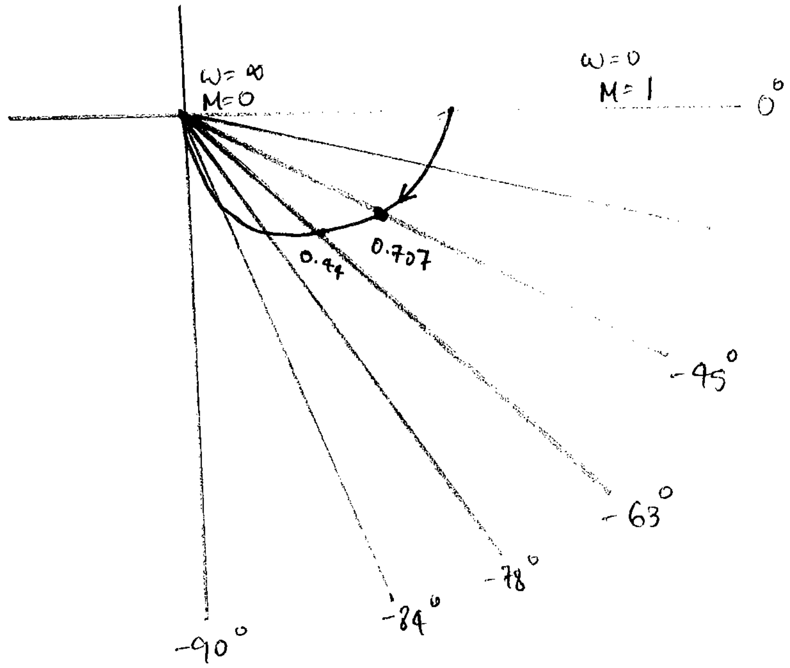
$GH(j\omega) = \frac{1}{j\omega\tau + 1}$

$M = \frac{1}{\sqrt{(\omega\tau)^2 + 1}}$

$\cancel{\angle\phi = \frac{1}{j}}$

$\angle\phi = -\tan^{-1}(\omega\tau)$

ω	M	ϕ
0	1	0
$\frac{1}{\tau}$	$1/\sqrt{2}$	-45°
$2/\tau$	0.44	-63°
$5/\tau$	0.19	-78°
$10/\tau$	0.1	-84°
⋮	⋮	⋮
∞	0	-90°



Q) $GH(s) = \left(\frac{s+1}{s+10}\right)$, $GH(s) = \left(\frac{s+10}{s+1}\right)$

$\Downarrow$ Lead Compensator HPF

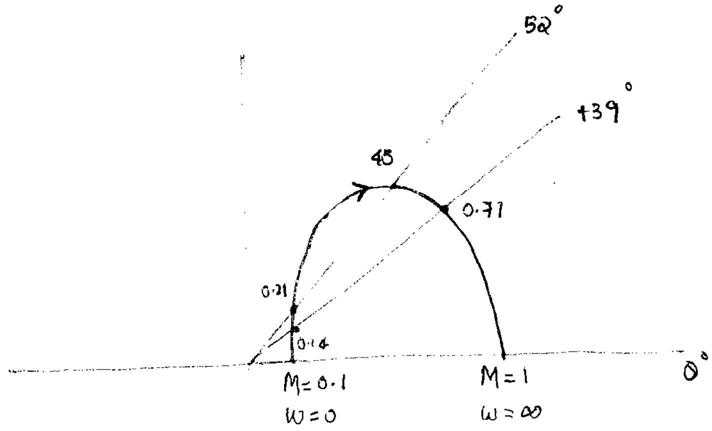
$\Downarrow$ Lag Compensator LPF

$s \to j\omega$

$$GH(j\omega) = \frac{j\omega+1}{j\omega+10}$$

$$M = \sqrt{\frac{\omega^2+1}{\omega^2+100}} \quad , \quad \phi = \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{10}\right)$$

ω	M	$\angle\phi$
0 →	0.1 →	0°
1 →	0.14 →	39°
2 →	0.21 →	52°
5 →	0.45 →	52°
10 →	0.71 →	39°
∞ →	1 →	0°

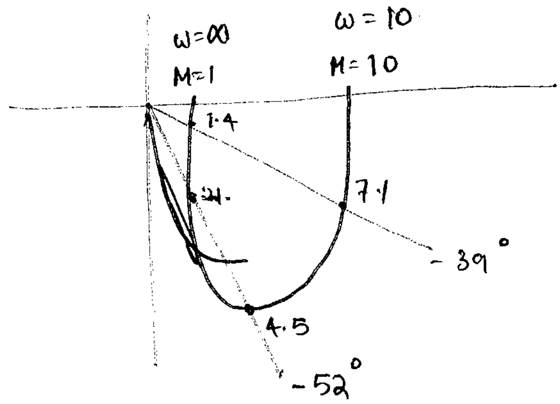


$$GH(s) = \frac{(s+10)}{(s+1)} \quad (\text{Lag/LPF})$$

$$M = \frac{\sqrt{\omega^2+100}}{\sqrt{\omega^2+1}}$$

$$\phi = \tan^{-1}\left(\frac{\omega}{10}\right) - \tan^{-1}(\omega)$$

ω	M	$\angle\phi$
0	10	0°
1	7.1	-39°
2	4.5	-52°
5	2.1	-52°
10	1.4	-39°
:		
∞	1	0



PROCEDURE TO DRAW THE POLAR PLOT

* This procedure is valid when the magnitude at $\omega=0$ greater than the magnitude at ∞. $\boxed{M|_{\omega=0} > M|_{\omega=\infty}}$ 99% are LPF.

If the magnitude at $\omega=0$ less than the magnitude at ∞ like high pass filter, then get the plot by using standard procedure. (Above method).

Step 1 : Find the magnitude and phase at $\omega=0$.

Step 2 : Find the magnitude and phase at $\omega=\infty$

Step 3   Ending Direction → $\phi_1 \; \phi_2$. +ve :⇒ Clockwise

-ve :⇒ Anti clockwise.

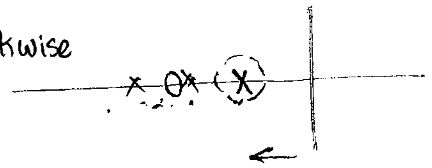
Step 4   Starting Direction

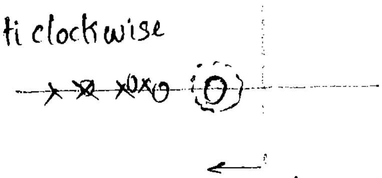
Considering the transfer functions, it should have positive sign only.

If the finite pole near to imaginary, starting direction is clockwise.

If finite zero near to imaginary, the starting direction is anticlockwise.

Clockwise   Anti clockwise





← Nearest one   ← Nearest one.

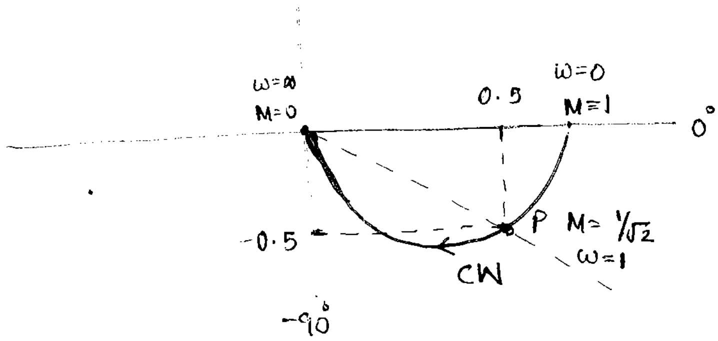
Q. Draw the polar plot.

$s \to j\omega$

$$GH(s) = \frac{1}{(s+1)}$$

$s \to j\omega$

$$GH(j\omega) = \frac{1}{(j\omega+1)}$$



$$\angle\phi = -\tan^{-1}\omega$$

At ω=0

$M = \frac{1}{\sqrt{\omega^2+1}}$

$M = 1$, $\angle\phi = 0°$

At ω=∞

$M = 0 \angle -90°$.

E.D → CW

S.D → pole. so CW

Q. If a point is given on curve, and its angle is given, then find cordinates.

At point P

$\angle GH = -45^\circ$

$-\tan^{-1}(\omega) = -45^\circ$

$\underline{\underline{\omega = 1}}$

$$M = \frac{1}{\sqrt{\omega^2+1}} = \frac{1}{\sqrt{2}} = 0.707$$

The Intersection gives the magnitude value.

Point $\Rightarrow \frac{1}{\sqrt{2}} \angle -45^\circ$. $\rightarrow$ polar.

In rect $= \underline{\underline{0.5 - j0.5}}$

So cordinates is $(0.5, -0.5)$

Q $$GH(j\omega) = \frac{1}{(s+1)(s+2)}$$

$s \rightarrow j\omega$

$$GH(j\omega) = \frac{1}{(j\omega+1)(j\omega+2)}$$

$$M = \frac{1}{\sqrt{\omega^2+1}\sqrt{\omega^2+4}} \qquad \angle\phi = -\tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right)$$

At $\omega = 0$

$$M = \frac{1}{2} = \frac{1}{2} = \underline{\underline{0.5}}$$

$\angle\phi = 0$

At $\omega = \pi/2$

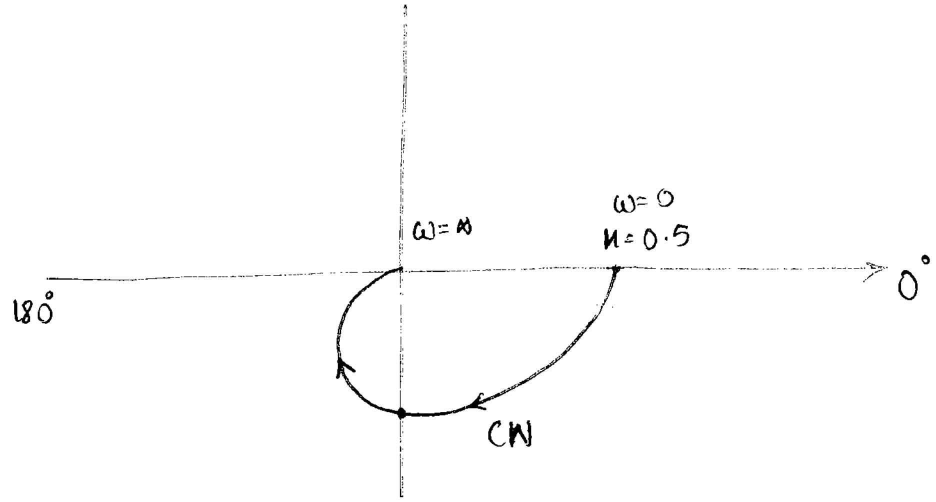
$M = 0$

$\angle\phi = -\pi/2 - \pi/2$

$= \underline{\underline{-\pi}} = \underline{\underline{-180^\circ}}$

E.D $\Rightarrow$ CW

S.Direction $\Rightarrow$ CW.



Q. $\angle GH = 90^\circ$

$$-90^\circ = -\tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$$

$$90^\circ = \tan^{-1}\left(\frac{\omega + \omega/2}{1-\omega^2/2}\right) \quad 1-\omega^2/2 = 0$$

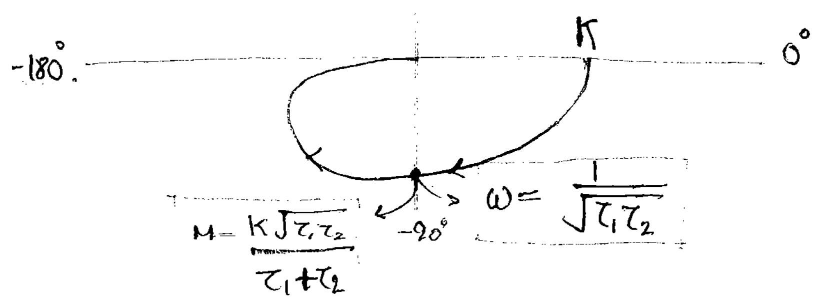
$$\omega = \sqrt{2} \text{ rad/s}.$$

$$M = \frac{1}{\sqrt{\frac{1}{2}+1}\,\sqrt{\frac{1}{2}+4}} = \frac{1}{\sqrt{18}}$$

Intersection point with $-90^\circ \longrightarrow \frac{1}{\sqrt{18}} \angle -90^\circ$

$$\left(0, -j\,1/\sqrt{18}\right)$$

Q $\quad GH(s) = \dfrac{K}{(s\tau_1+1)(s\tau_2+1)}$



Q. $GH(s) = \dfrac{1}{(s+1)(s+2)(s+3)}$

$GH(j\omega) = \dfrac{1}{(j\omega+1)(j\omega+2)(j\omega+3)}$

$M = \dfrac{1}{\sqrt{\omega^2+1}\sqrt{\omega^2+4}\sqrt{\omega^2+9}}$

$\angle\phi = -\tan^{-1}\omega - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{3}$

At $\omega = 0$

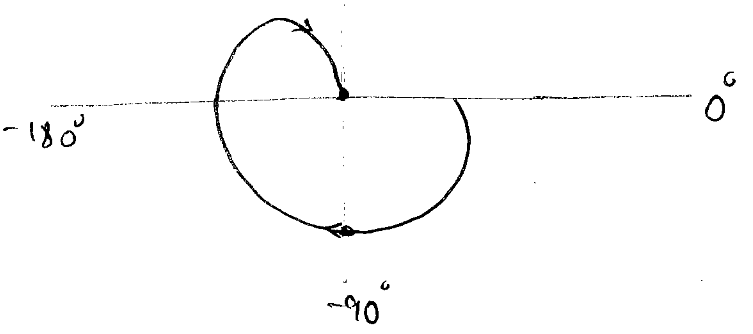
$M = \dfrac{1}{1\times 2\times 3} = \dfrac{1}{6} = 0.1667$

$\angle GH = 0$

At $\omega = \infty$

$M = 0$

$\angle GH = -270^\circ$



ED → CW

SD → CW

Intersection at -90°

$-90^\circ = -\tan^{-1}\omega - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{3}$

$+90^\circ = \tan^{-1}\left(\dfrac{\omega + \frac{\omega}{2}}{1-\frac{\omega^2}{2}}\right) + \tan^{-1}\left(\frac{\omega}{3}\right)$

$$= \tan^{-1} \frac{\dfrac{\omega + \frac{\omega}{2}}{1 - \frac{\omega^2}{2}} + \dfrac{\omega}{3}}{1 - \dfrac{\frac{\omega^2}{3} + \frac{\omega^2}{6}}{1 - \omega^3/6}}$$

$$= \tan^{-1}\left(\frac{\omega + \frac{\omega}{2} + \frac{\omega}{3}\left(1 - \frac{\omega^2}{2}\right)}{1 - \frac{\omega^3}{6} - \frac{\omega^2}{3} - \frac{\omega^2}{6}}\right) \Rightarrow 1 - \frac{5\omega^2}{6-\omega^2} = 0$$

$$6 - \omega^2 - 5\omega^2 .$$

$$\Rightarrow \omega = 1 \text{ rad/sec}$$

$$\cancel{1 - \frac{\omega^2}{6} - \frac{\omega^2}{3} - \frac{\omega^2}{6} = 0}$$

$$\cancel{1 = \frac{\omega^2}{6} + \frac{\omega^2}{3} + \frac{\omega^2}{6}}$$

$$\cancel{1 - \omega^2\left[\frac{2}{3}\right]}$$

$$= \tan^{-1}\left(\frac{\omega + \frac{5\omega}{6-\omega^2}}{1 - \frac{5\omega^2}{6-\omega^2}}\right)$$

$$M = \frac{1}{\sqrt{2 \times 5 \times 10}} = \underline{0.1}$$

Intersection with -180°.

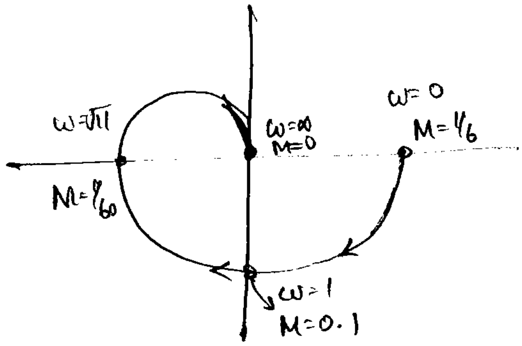
$$\cancel{\omega} + \frac{5\cancel{\omega}}{6-\omega^2} = 0$$

$$6 - \omega^2 + 5 = 0$$

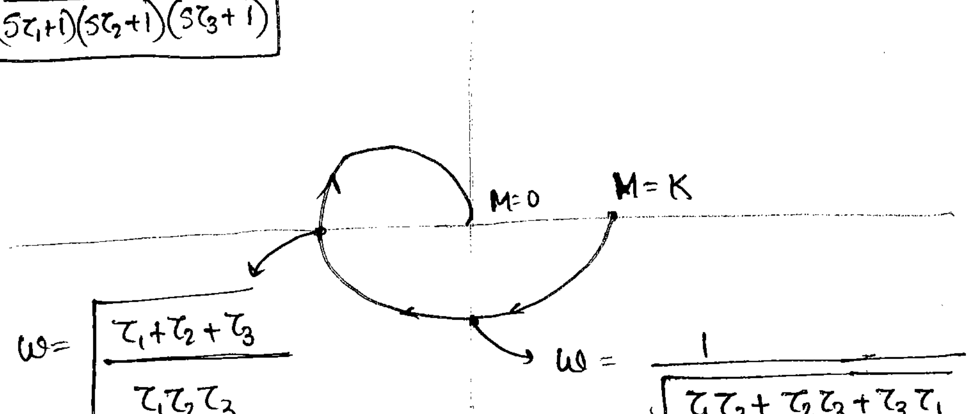
$$\omega = \sqrt{11} \text{ rad/s.}$$

$$M_{-180^\circ} = \frac{1}{\sqrt{12 \times 15 \times 20}}$$

$$= \frac{1}{60}$$



$$\boxed{GH(s) = \frac{K}{(s\tau_1+1)(s\tau_2+1)(s\tau_3+1)}}$$



$$\omega = \sqrt{\frac{\tau_1+\tau_2+\tau_3}{\tau_1\tau_2\tau_3}} \qquad \omega = \frac{1}{\sqrt{\tau_1\tau_2+\tau_2\tau_3+\tau_3\tau_1}}$$

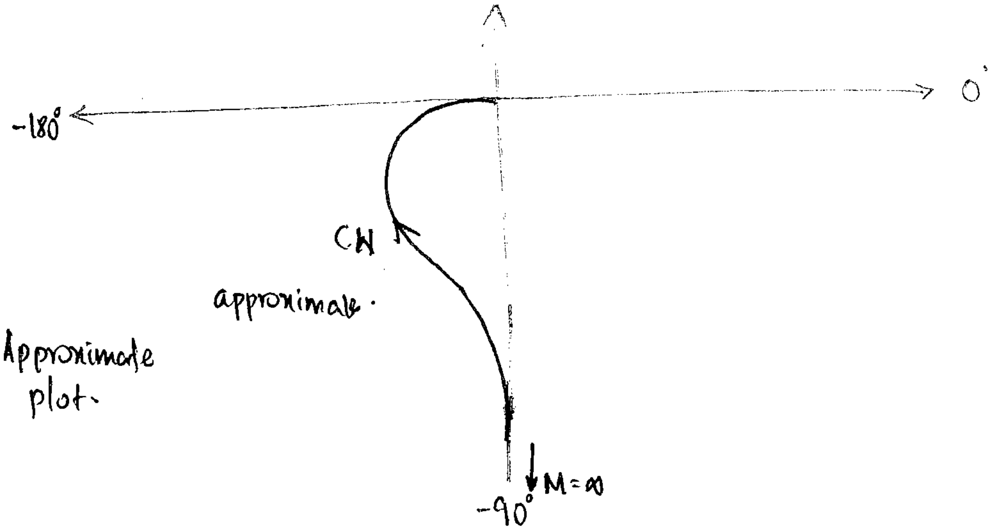
Note: The addition of each finite pole shift the ending angle by -90°, in the clockwise direction.

Q. $GH(s) = \dfrac{1}{s(s+1)}$

$s \to j\omega \qquad GH(j\omega) = \dfrac{1}{j\omega(j\omega+1)}$

$M = \dfrac{1}{\omega\sqrt{\omega^2+1}} \qquad \angle\phi = -90^\circ - \tan^{-1}\dfrac{\omega}{1}$

$\omega = 0$	$\omega = \infty$
$M = \infty$	$M = 0$
$\angle GH = -90^\circ$	$\angle GH = -180^\circ$



$$GH(j\omega) = \frac{1}{j\omega(j\omega+1)}$$

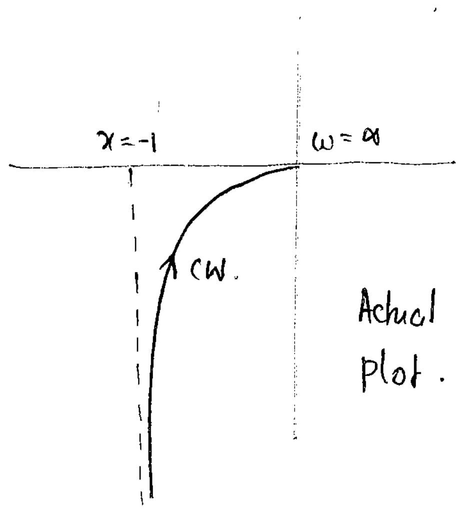
Rationalizing,

$$GH(j\omega) = \frac{(1-j\omega)}{j\omega(1+\omega^2)} = \frac{-j(1-j\omega)}{\omega(1+\omega^2)} = \frac{-\omega-j}{\omega(1+\omega^2)}$$

$$= \frac{-\cancel{\omega}}{\cancel{\omega}(1+\omega^2)} - \frac{j}{\omega(1+\omega^2)}$$

$$= \frac{-1}{1+\omega^2} - \frac{j}{\omega(1+\omega^2)}$$

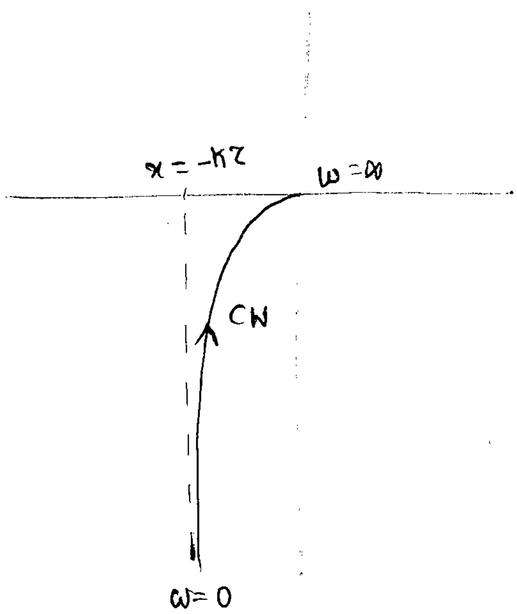
At $\omega = 0$, $(-1-j\infty)$



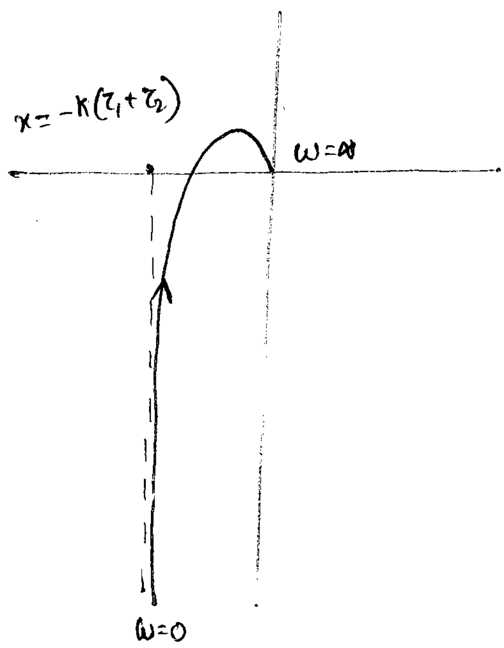
Note
In the above polar plot, $\omega = 0$, assymptotic to the straight line $x = -1$

In all such cases where the plot contains some assymptote, go for rationalization and find x value.

$GH(s) = \dfrac{K}{s(s\tau+1)}$



$GH(s) = \dfrac{K}{s(s\tau_1+1)(s\tau_2+1)}$



Q, $GH(s) = \dfrac{1}{s^2(s+1)}$

$s \to j\omega$

$GH(j\omega) = \dfrac{1}{(j\omega)^2(j\omega+1)} =$

$M = \dfrac{1}{\omega^2\sqrt{1+\omega^2}}$, $\angle\phi = -180^\circ - \tan^{-1}(\omega)$

<u>$\omega = 0$</u>

$M = \infty$

$\angle\phi = -180^\circ$

<u>$\omega = \infty$</u>

$M = 0$

$\angle\phi = -270$

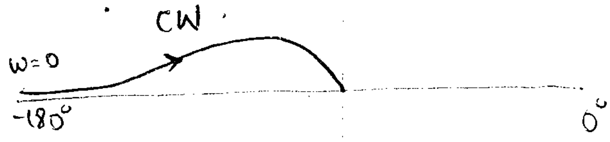
ED $\longrightarrow$ CW

SD $\longrightarrow$ CW

$$GH(j\omega) = \frac{-1}{\omega^2(j\omega+1)} \times \frac{1-j\omega}{1-j\omega}$$

$$= \frac{-(1-j\omega)}{\omega^2(1+\omega^2)}$$

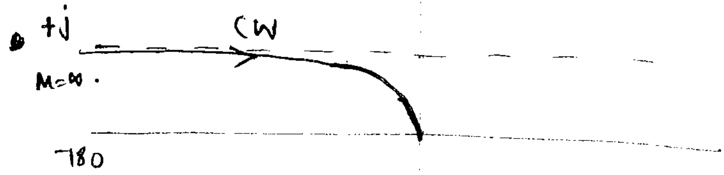
Approximate.



$$= \frac{-1}{\omega^2(1+\omega^2)} + \frac{j\cancel{\omega}}{\omega^{\cancel{2}}(1+\omega^2)}$$

$$= \frac{-1}{\omega^2(1+\omega^2)} + \frac{j}{\omega(1+\omega^2)}$$

Actual.



Q1 $GH(s) = \frac{1}{s^3(s+1)}$

$$M = \frac{1}{\omega^3\sqrt{\omega^2+1}}$$

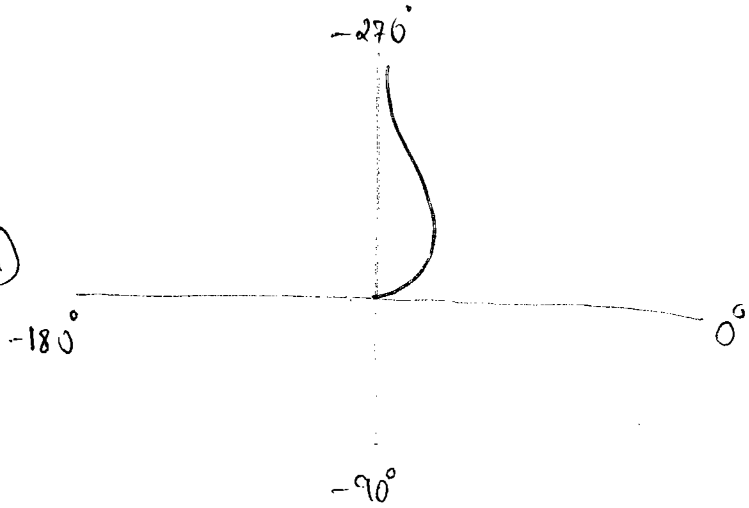
$$\phi = -270^\circ - \tan^{-1}(\omega)$$

$\omega = 0 \Rightarrow \infty\angle -270^\circ$

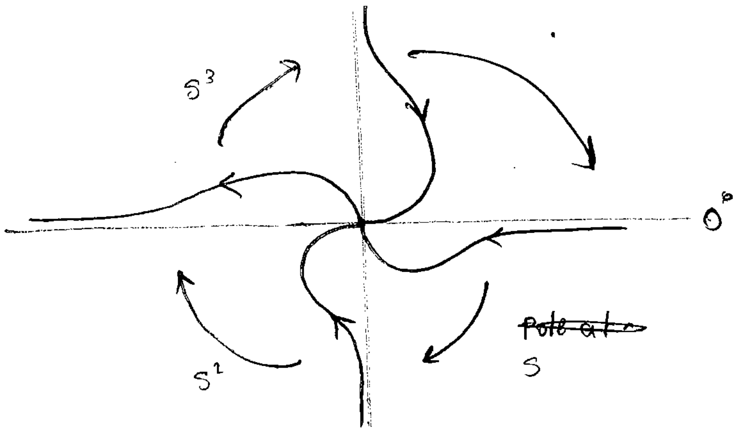
$\omega = \infty \Rightarrow 0\angle -360^\circ$

ED → CW

SD → CW



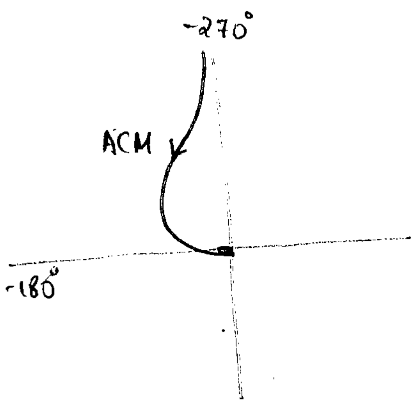
Note: The addition of each pole at the origin shift the total plot by -90° in the clockwise direction.



Q, $GH(s) = \dfrac{s+1}{s^3}$

$s \to j\omega$, $\quad GH(j\omega) = \dfrac{j\omega+1}{(j\omega)^3}$

$M = \dfrac{\sqrt{\omega^2+1}}{\omega^3} \qquad \angle\phi = +\tan^{-1}\omega - 270^\circ$



$\underline{\omega = 0}$

$M = \infty$

$\angle GH = -270^\circ$
ϕ_1

$\underline{\omega = \infty}$

$M = 0$

$\angle GH = -180^\circ$
ϕ_2

ED $\to$ $\phi_1 - \phi_2 = -ve \Rightarrow$ Anticlockwise.

SA $\to$ Anticlockwise (No finite pole)

pole at zero not considered as finite pole.

Q, $GH(s) = \dfrac{(s+1)(s+2)}{s^3}$

$$M = \frac{\sqrt{\omega^2+1}\,\sqrt{\omega^2+4}}{\omega^3}$$

$$\angle\phi = \tan^{-1}\omega + \tan^{-1}\frac{\omega}{2} - 270^\circ$$

$\underline{\omega = 0}$

$M = \infty$

$\angle GH = -270^\circ$

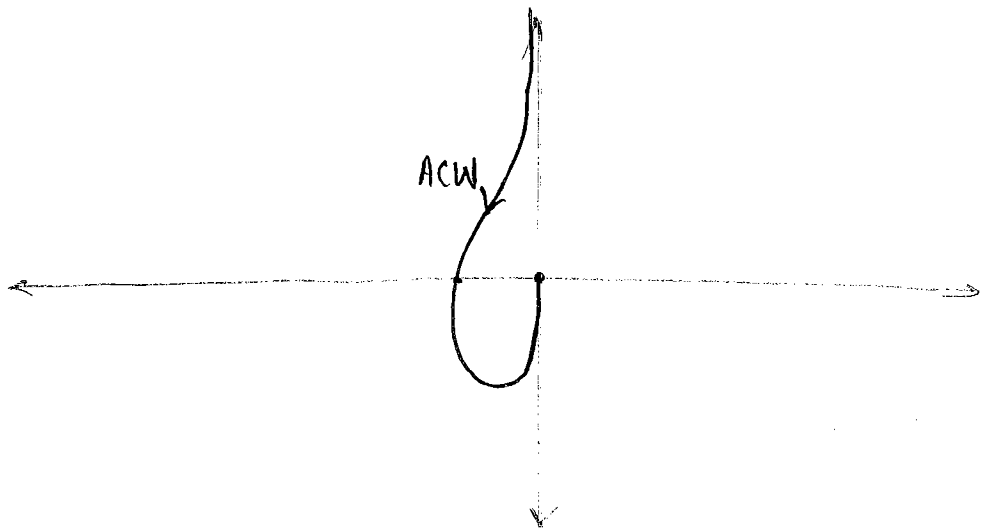
$\underline{\omega = \infty}$

$M = 0$

$\angle GH = 180 - 270^\circ$
$= -90^\circ$

ED $\Rightarrow$ $\phi_1 - \phi_2 = -180 \longrightarrow$ Anticlockwise.

SD $\Rightarrow$ $\underline{\text{ACW}}$



Q, $GH(s) = \dfrac{(s+1)(s+2)(s+3)}{s^3}$

$$M = \frac{\sqrt{\omega^2+1}\,\sqrt{\omega^2+4}\,\sqrt{\omega^2+9}}{\omega^3}$$

$$\angle\phi = \tan^{-1}\omega + \tan^{-1}\frac{\omega}{2} + \tan^{-1}\frac{\omega}{3} - 270^\circ$$

$\underline{\omega = 0}$

$M = \infty$

$\angle GH = -270^\circ$

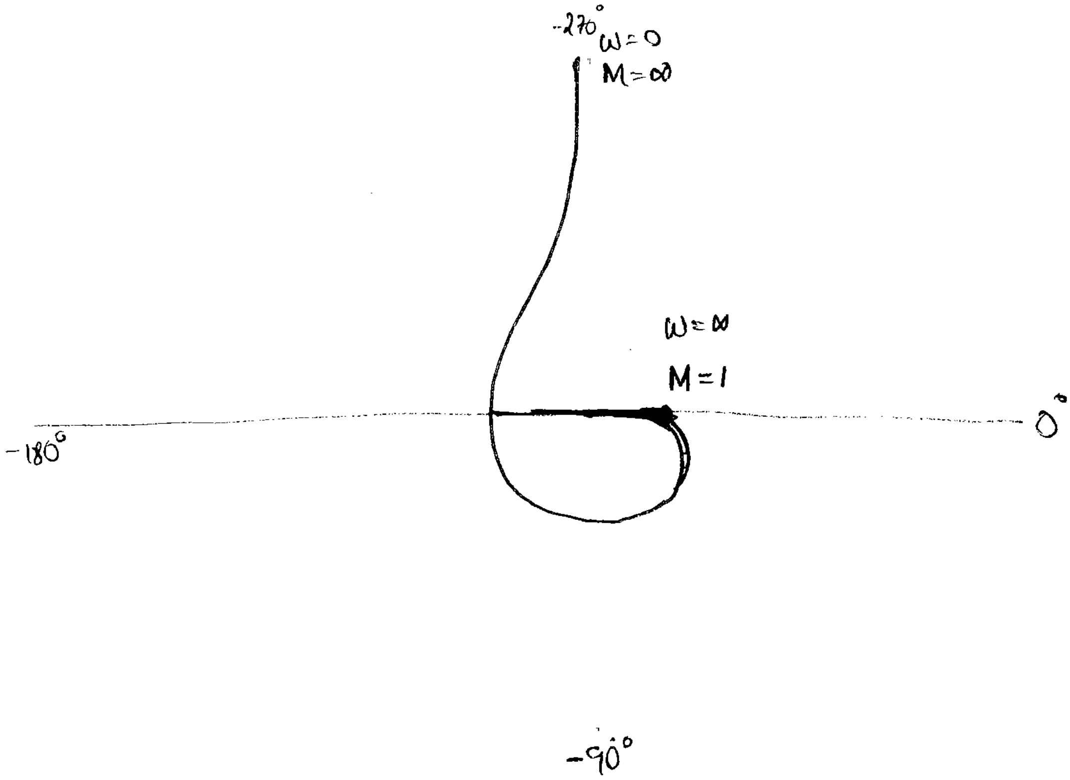
$\underline{\omega = \infty}$

$\cancel{M = 0}$ $M = \dfrac{\cancel{\omega^3}\sqrt{\left(1+\frac{1}{\omega^2}\right)\left(1+\frac{2}{\omega^2}\right)\left(1+\frac{3}{\omega^2}\right)}}{\cancel{\omega^3}}$

$\underline{\angle GH = 0^\circ}$ $\underline{M = 1}$

ED $\Rightarrow$ ACW

SD $\Rightarrow$ ACW.



Note: The addition of each finite zero shift the ending angle by +90°, in the anticlockwise direction.

Q, $GH = S$

$s \to j\omega$

$M = \omega$ $\angle\phi = 90^\circ$

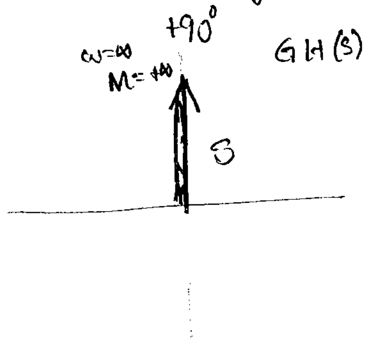
$\omega = 0$ $0\angle +90^\circ$

$\omega = 1$ $1\angle +90^\circ$

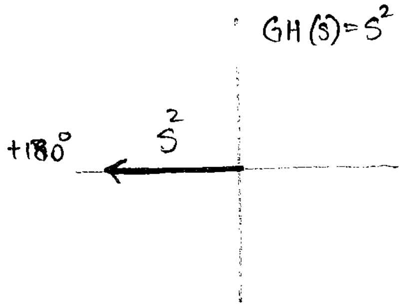
$\omega = \infty \longrightarrow \infty \angle +90^\circ$

Note: whenever the T.F consist only poles at the origin or only zeros at the origin, then the polar plot is nothing but angle line.

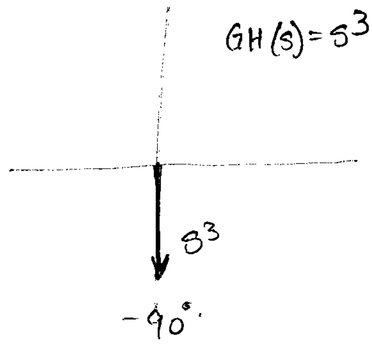
$GH(s) = s$



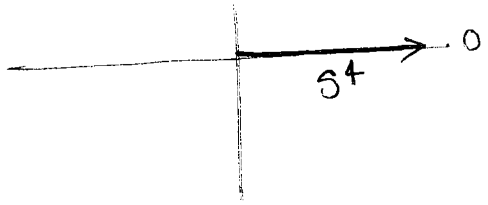
$GH(s) = s^2$



$GH(s) = s^3$



$GH(s) = s^4$



Note: The addition of each zero at origin shift the total plot. by $+90^\circ$ in the anticlockwise direction.

$GH(s) = 1/s$

$s \to j\omega$

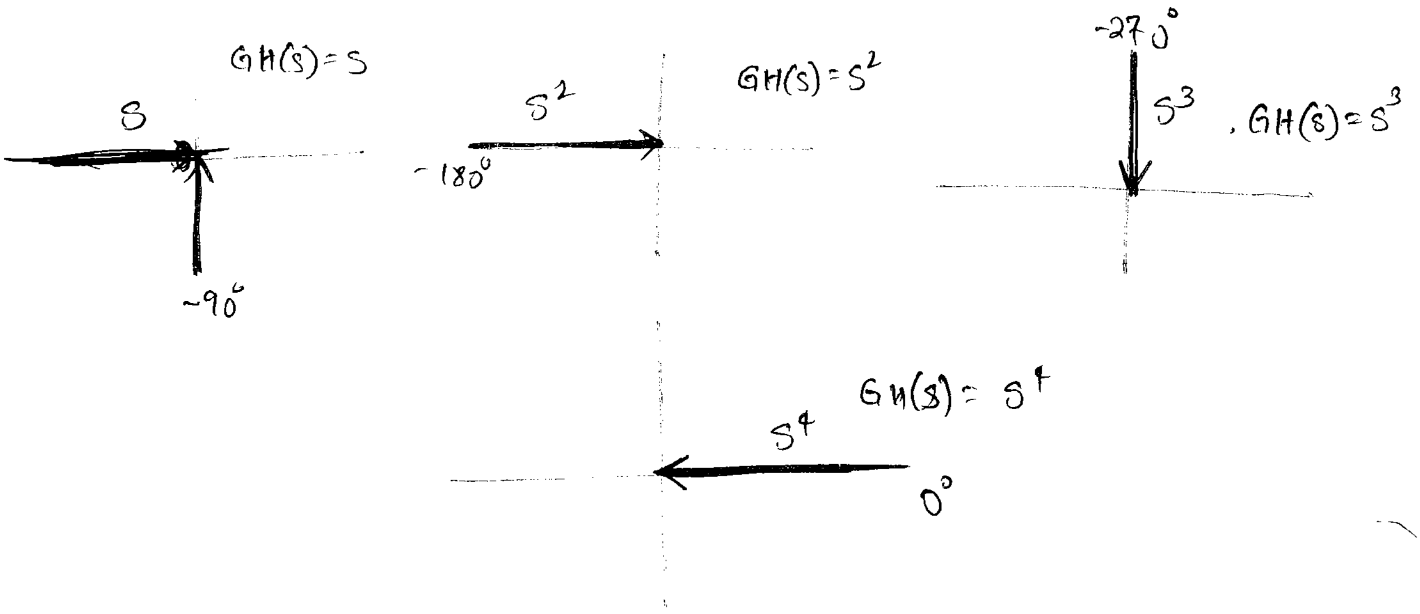
$GH(j\omega) = 1/j\omega$

$M = 1/\omega \quad \angle\phi = -90^\circ$

$\omega = 0 \Rightarrow \infty \angle -90^\circ$

$\omega = \infty \Rightarrow 0 \angle +90^\circ$

$\left.\begin{array}{l} ED \rightarrow \times \\ SD \rightarrow \times \end{array}\right\}$ straight line along -90°



Q $GH(s) = \dfrac{(s+1)}{s^3(s+2)}$

$M = \dfrac{\sqrt{\omega^2+1}}{\omega^3\sqrt{\omega^2+4}}$, $\angle\phi = -270^\circ + \tan^{-1}\omega - \tan^{-1}\left(\dfrac{\omega}{2}\right)$

$\underline{\omega = 0}$

$M = \infty$

$\angle GH = -270^\circ$

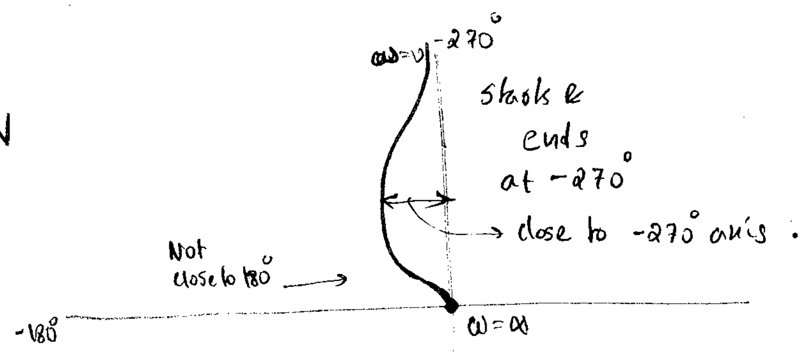
$\underline{\omega = \infty}$

$M = 0$

$\angle GH = -270^\circ$

ED = X

SD = ACW



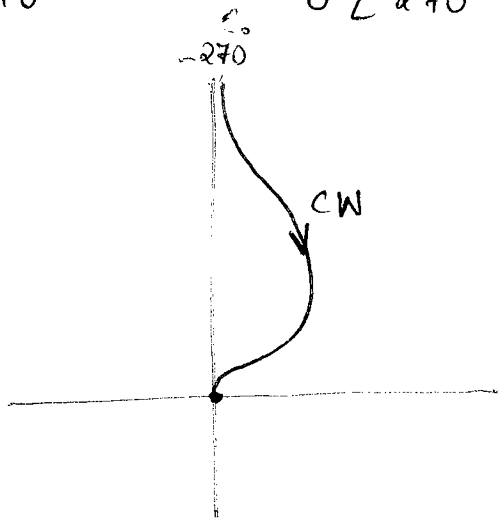
→ In phase equ, constant term determines initial angle, but finite ~~angle~~ terms depends on ω, so finite zeros & poles determines the direction, how much it is pushed left or right.

Q, $\angle GH(S) = \frac{[illegible]}{s^3(s+1)}$

$M = \frac{\sqrt{\omega^2+4}}{\omega^3\sqrt{\omega^2+1}}$ $\qquad \angle\phi = -270 - \tan^{-1}\omega + \tan^{-1}\omega/2$.

$\underline{\omega = 0}$ $\qquad$ $\underline{\omega = \infty}$

$\infty \angle -270^\circ$ $\qquad$ $0 \angle -270^\circ$



Q, $GH(s) = \frac{(s+1)}{s^2(s+2)(s+3)}$

$M = \frac{\sqrt{\omega^2+1}}{\omega^2\sqrt{(\omega^2+4)(\omega^2+9)}}$

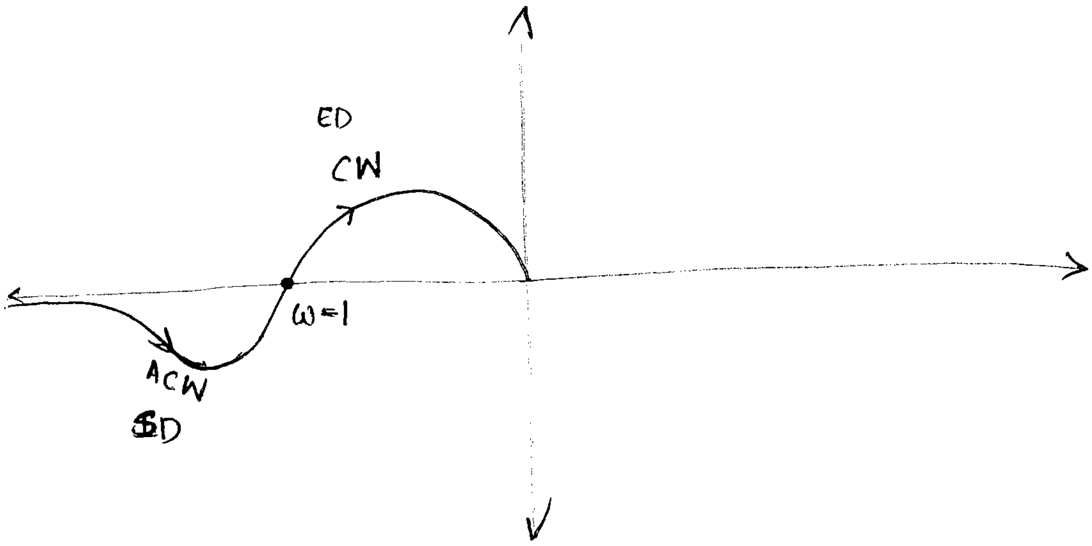
$\angle\phi = -180^\circ + \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{3}$

$\underline{\omega = 0}$ $\qquad$ $\underline{\omega = \infty}$

$M = \infty$ $\qquad$ $M = 0$

$\angle GH = -180^\circ$ $\qquad$ $\angle GH = -270^\circ$

$\left.\begin{array}{l} ED \rightarrow CW \\ SD \rightarrow ACW \end{array}\right\}$ starting, ending direction different.



Intersection point at -180°.

$$-180^\circ = -180^\circ + \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{3}$$

$$\tan^{-1}\omega = \tan^{-1}\frac{\frac{\omega}{2}+\frac{\omega}{3}}{1-\frac{\omega^2}{6}}$$

ie, At intersection point cw effect will cancel ACW effect.

ie, +ve ~~tan~~ $\tan^{-1}$ terms must equal to -ve $\tan^{-1}$ terms.

$$\cancel{\omega} = \frac{\frac{\cancel{\omega}}{2}+\frac{\cancel{\omega}}{3}}{1-\frac{\omega^2}{6}}$$

$$1 = \frac{\frac{5}{\cancel{6}}}{\frac{6-\omega^2}{\cancel{6}}}$$

$$6-\omega^2 = 5$$

$$\omega^2 = 1$$

$$\underline{\underline{\omega = 1}}$$

For $0 < \omega < 1 \implies \phi < -180^\circ$ (ie, less negative)

For $1 < \omega < \infty \implies \phi > -180^\circ$ (ie, more negative)

Q4 $GH(s) = \dfrac{\quad}{s^2(s+1)(s+2)}$

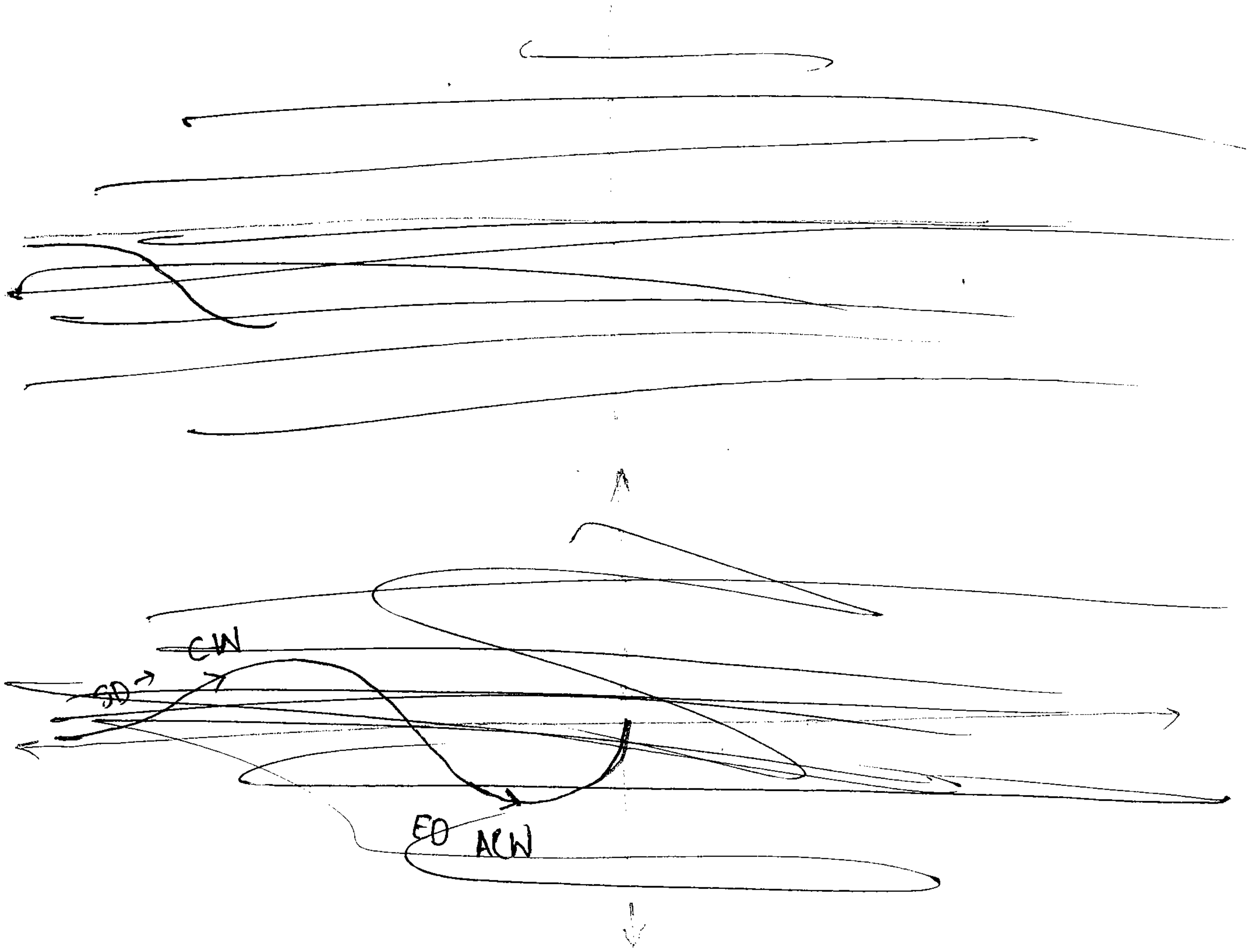
$$M = \frac{\sqrt{\omega^2+9}}{\omega^2\sqrt{(\omega^2+1)(\omega^2+4)}}$$

$$\angle\phi = -180^\circ + \tan^{-1}\frac{\omega}{3} - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}.$$

$\omega = 0$	$\omega = \infty$
$M = \infty$	$M = 0$
$\angle GH = -180^\circ$	$\angle GH = -270^\circ$

ED $\Rightarrow$ CW

SD $\Rightarrow$ ~~ACW~~ CW



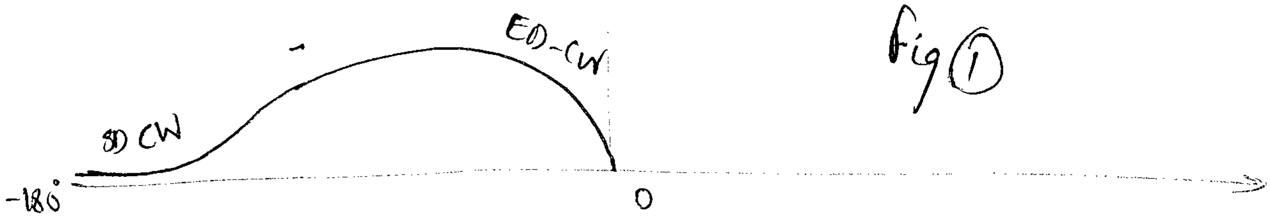
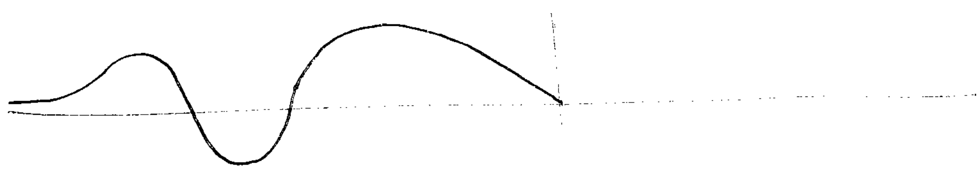


fig ①

OR

fig ②



Verification

Intersection point with -180°

$$\cancel{-180} = \cancel{-180} - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2} + \tan^{-1}\left(\frac{\omega}{3}\right)$$

$$\frac{\omega}{3} = \frac{\cancel{\omega + \frac{\omega}{2}}}{1 - \frac{\omega^2}{2}}$$

$$1 - \frac{\omega^2}{2} = 9/2$$

$$2 - \omega^2 = 9$$

$$\omega = \pm j\sqrt{7} \quad \text{Invalid.}$$

$\therefore$ No intersection

So fig ① is correct.

Q_1 $\quad GH(s) = \dfrac{(s+1)(s+2)}{s^2(s+3)}$

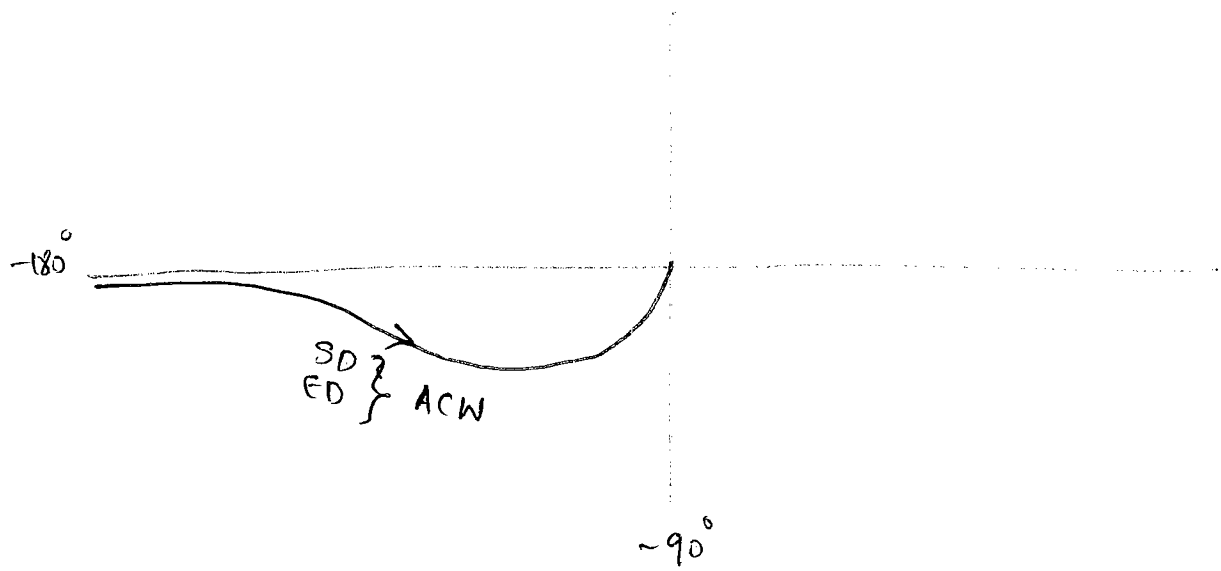
$$M = \frac{\sqrt{(\omega^2+1)(\omega^2+4)}}{\omega^2\sqrt{\omega^2+9}}$$

$$\angle\phi = -180^\circ + \tan^{-1}\omega + \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{3}$$

$\underline{\omega = 0.}$	$\underline{\omega = \infty}$
$M = \infty$	$M = 0$
$\angle GH = -180^\circ$	$\angle GH = \cancel{-270^\circ}\ -90^\circ$

$$ED \Rightarrow ACW$$

$$SD \Rightarrow ACW$$



Q $GH(s) = \dfrac{(s+2)(s+3)}{s^2(s+1)}$

$$M = \frac{\sqrt{\omega^2+4}\,\sqrt{\omega^2+9}}{\omega^2\sqrt{\omega^2+1}}$$

$$\angle\phi = -180^\circ + \tan^{-1}\frac{\omega}{2} + \tan^{-1}\frac{\omega}{3} - \tan^{-1}\omega$$

$\underline{\omega = 0}$

$M = \infty$

$\angle GH = -180^\circ$

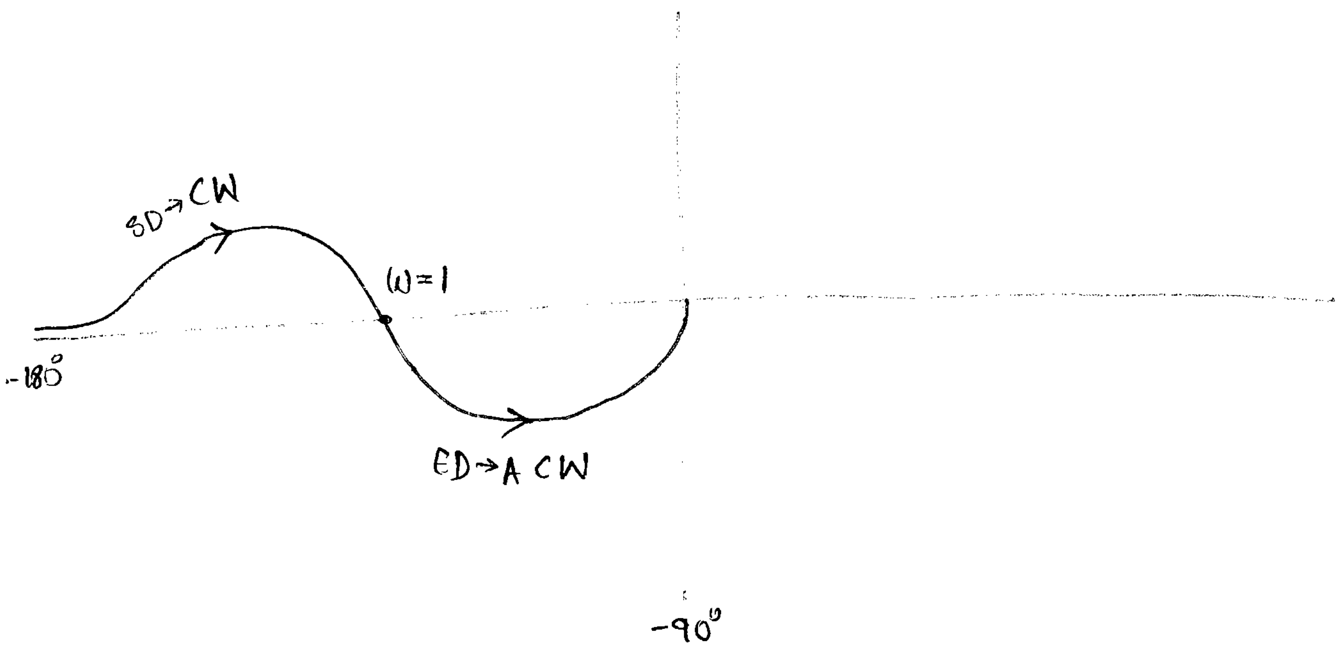
$\underline{\omega = \infty}$

$M = 0$

$\angle GH = -90^\circ$

ED = ACW

SD = CW



Intersection with -180°

$$\cancel{-180} = \cancel{-180^\circ} + \tan^{-1}\frac{\omega}{2} + \tan^{-1}\frac{\omega}{3} - \tan^{-1}\omega,$$

$\omega = \frac{\omega}{2} + \frac{\omega}{3}$ $\qquad\qquad \omega = 1$

Q) $GH(s) = \frac{1}{s(s+1)}$

$$M = \frac{1}{\omega\sqrt{\omega^2+1}}$$

$$\angle\phi = -90^\circ - \tan^{-1}\omega$$

$GH(s) = \frac{1}{s(s-1)}$

$$GH(j\omega) = \frac{1}{j\omega(j\omega-1)}$$

$$\cancel{\angle\phi = -90^\circ + \tan^{-1}\omega}$$

$$\phi = -90^\circ - (180^\circ - \tan^{-1}(\omega))$$

$$\boxed{\phi = -270 + \tan^{-1}(\omega)}$$

$GH(s) = \frac{1}{s(-s-1)}$

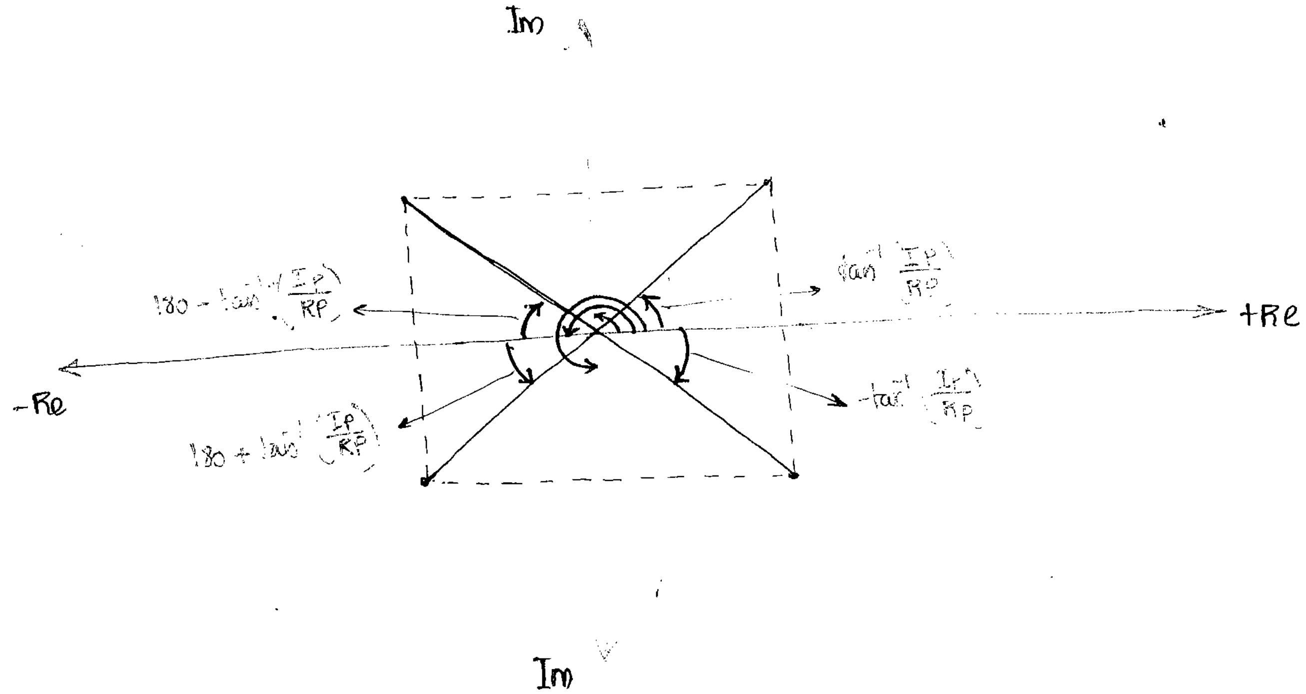
$$GH(j\omega) = \frac{1}{j\omega(-j\omega-1)}$$

$$\cancel{\angle\phi = -90^\circ - \tan^{-1}\omega}$$

$GH(s) = \frac{1}{s(1-s)}$

$$\cancel{\phi}\ GH(j\omega) = \frac{1}{j\omega(1-j\omega)}$$

$$\angle\phi = -90^\circ + \tan^{-1}\omega$$



$GH(s) = \frac{1}{s(s+1)}$

$M =$

$\phi = -90^\circ - \tan^{-1}\omega$

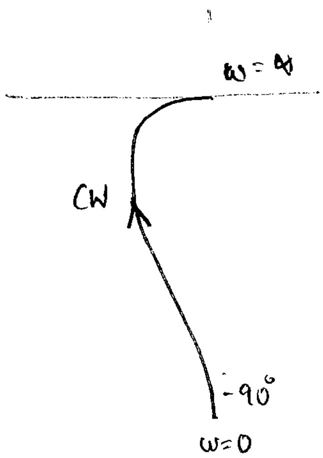
$\boxed{\phi = -90^\circ - \tan^{-1}\omega}$

$\omega = 0 \rightarrow \infty \angle -90^\circ$

$\omega = \infty \Rightarrow 0 \angle -180^\circ$

ED → CW

SD → CW



$GH(s) = \frac{1}{s(s-1)}$

$\phi = -90^\circ - (180^\circ - \tan^{-1}\omega)$

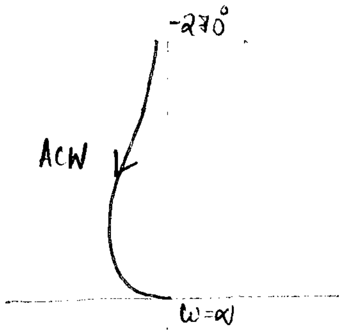
$\boxed{\phi = -270^\circ + \tan^{-1}\omega}$

$\omega = 0 \quad \infty \angle -270^\circ$

$\omega = \infty \quad 0 \angle -180^\circ$

ED → ACW

SD → X



$GH(s) = \frac{1}{s(-s-1)}$

$\phi = -90^\circ - (180^\circ + \tan^{-1}\omega)$

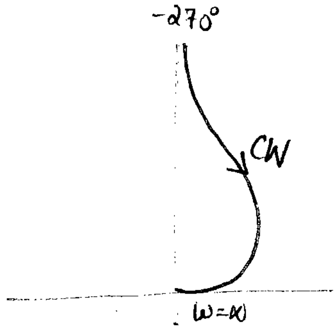
$\boxed{\phi = -270^\circ - \tan^{-1}\omega}$

$\omega = 0 \Rightarrow \infty \angle -270^\circ$

$\omega = \infty \Rightarrow 0 \angle -360^\circ$

ED → CW

SD → X



$GH(s) = \frac{1}{s(1-s)}$

$\phi = -90^\circ + \tan^{-1}\omega$

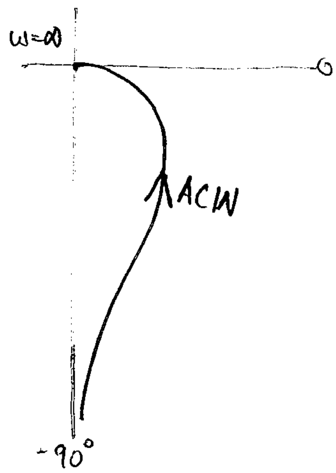
$\boxed{\phi = -90^\circ + \tan^{-1}\omega}$

$\omega = 0 \Rightarrow \infty \angle -90^\circ$

$\omega = \infty \Rightarrow 0 \angle 0^\circ$

ED → ACW

SD → X



$$GH(s) = \frac{(s+2)}{(s+1)(s-1)}$$

$$M = \frac{\sqrt{\omega^2+2}}{\sqrt{(\omega^2+1)(\omega^2+1)}} = \frac{\sqrt{\omega^2+2}}{\omega^2+1}$$

$$\phi = \tan^{-1}\frac{\omega}{2} - (180 - \tan^{-1}(\omega)) - \tan^{-1}\omega$$

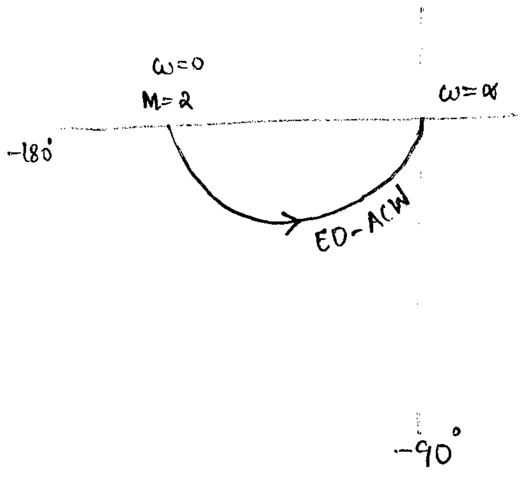
$$= \phi - 180 + \tan^{-1}\frac{\omega}{2}$$

$\omega = 0 \Rightarrow \sqrt{2} \angle -180^\circ$

$\omega = \infty \Rightarrow 0 \angle -90^\circ$

ED $\Rightarrow$ ACW

SD $\Rightarrow$ X



$$GH(s) = \frac{(s-3)}{s(s+1)}$$

$$M = \frac{\sqrt{\omega^2+9}}{\omega\sqrt{\omega^2+1}}$$

$$\phi = -90 - \tan^{-1}\omega + \left(180 - \tan^{-1}\frac{\omega}{3}\right)$$

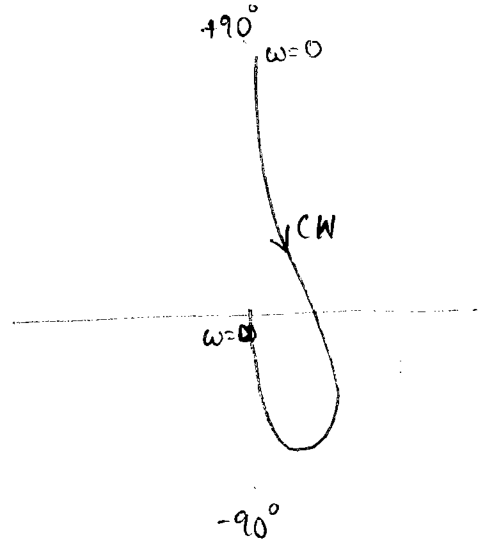
$$= 90^\circ - \tan^{-1}\omega - \tan^{-1}\left(\frac{\omega}{3}\right)$$

$\omega = 0 \Rightarrow \infty \angle +90^\circ$

$\omega = \infty \Rightarrow 0 \angle -90^\circ$

ED $\Rightarrow$ CW

SD $\Rightarrow$ X



$$GH(s) = \left(\frac{s+2}{s-2}\right)$$

$$M = 1$$

$$\Phi = \tan^{-1}\frac{\omega}{2} - \left(180 - \tan^{-1}\frac{\omega}{2}\right)$$

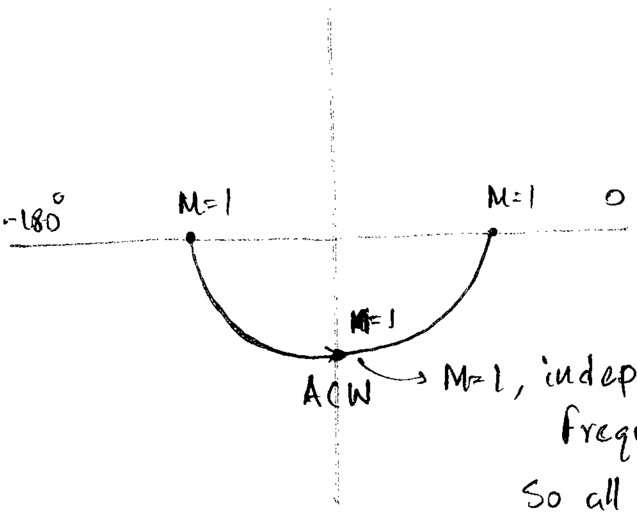
$$= -180 + 2\tan^{-1}\frac{\omega}{2}$$

$\omega = 0 \Rightarrow 1\angle -180^\circ$

$\omega = \infty \Rightarrow 1\angle 0^\circ$

ED $\Rightarrow$ ACW

SD $\Rightarrow$ X



M=1, independant of frequency.
So all point.

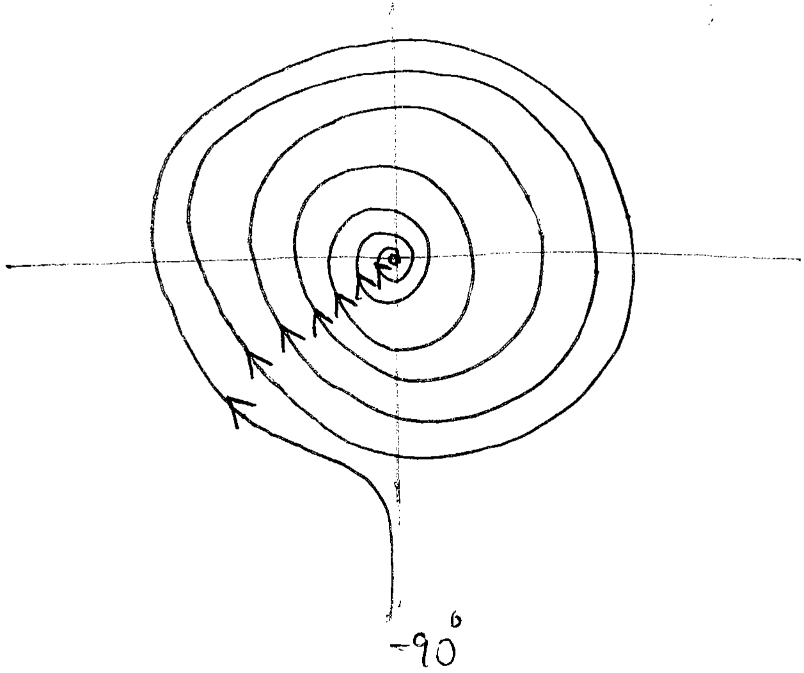
Q, $GH(s) = \frac{e^{-s}}{s(s+1)}$

$$GH(j\omega) = \frac{e^{-j\omega}}{\omega\sqrt{\omega^2+1}}$$

$$M = \frac{1}{\omega\sqrt{\omega^2+1}}$$

$$\phi = -90^\circ - \tan^{-1}\omega - \omega \times \frac{180^\circ}{\pi}$$

ω	M	$\angle\phi$
0 →	∞ →	90°
1 →	0.707 →	-192°
2 →	0.22 →	-267°
5 →	0.04 →	-453°
10 →	0.01 →	-744°
⋮		⋮
∞ →	0 →	$-\infty$

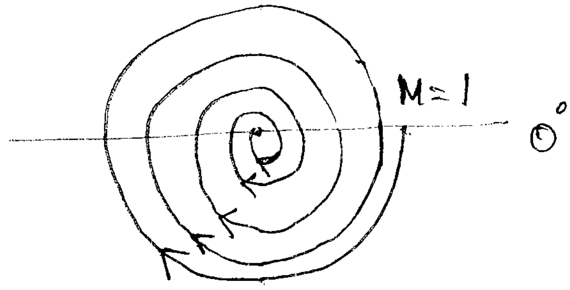


Q, $GH(s) = \frac{e^{-s}}{(s+1)}$

$$M = \frac{1}{\sqrt{\omega^2+1}}$$

$$\angle\phi = -\tan^{-1}\omega - \omega \times \frac{180^\circ}{\pi}$$

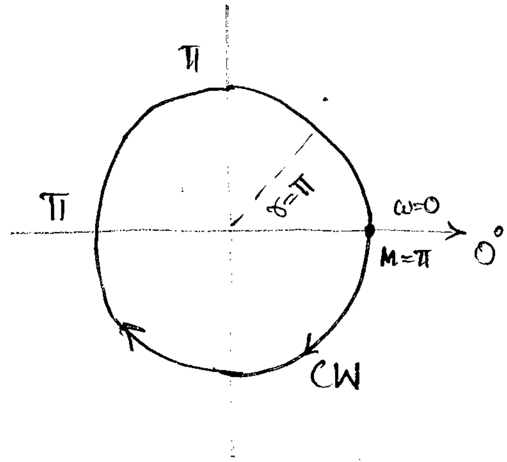
ω	M	$\angle\phi$
$\omega = 0$	$M = 1$	0°
⋮	⋮	
$\omega = \infty$	0 →	$-\infty$



$$GH = \pi e^{-2s}$$

$$M = \pi, \quad \angle\phi = -2\omega \times \frac{180}{\pi}$$

ω	M	$\angle\phi$
0 →	π →	0°
$\pi/4$ →	π →	-90°
$\pi/2$ →	π →	-180°
$3\pi/4$ →	π →	-270°
2π →	π →	-360°
⋮	⋮	⋮



NYQUIST PLOT

PURPUSE

→ To draw the complete frequency response of the open loot T.F

→ To find the closed loop system stability

→ To find the number of closed loop poles, in the right hand side.

→ To find the ~~weight~~ range of K value for system stability.

→ To find the Gain margin, phase margin, Gain cross over frequency and phase cross ~~offo~~ over frequency.

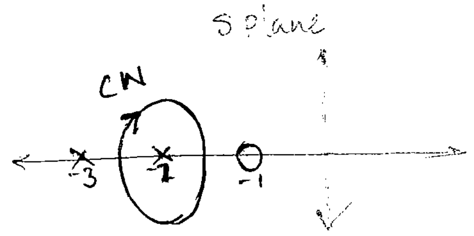
→ To find the relative stability using Gain margin and phase margin.

* The Nyquist stability criteria is developed by using the mathamatical principle known as PRINCIPLE OF ARGUMENTS

PRINCIPLE OF ARGUMENTS

It states that if there are P poles and Z zeros are enclosed by the random selected closed path in the S plane, then the corresponding G(s)H(s) plane encircles the origin with (P−Z) times.

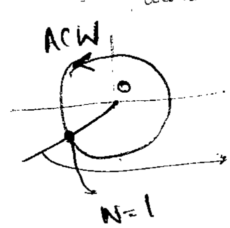
ie, $\boxed{N = P - Z}$ where N : number of encirclements about origin.



$N_{(origin)} = P - Z$ $N = 1 - 0 = 1$

eg: $GH(s) = \frac{(s+1)}{(s+2)(s+3)}$

G(s)H(s) plane

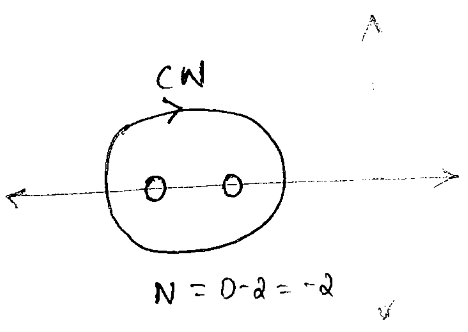


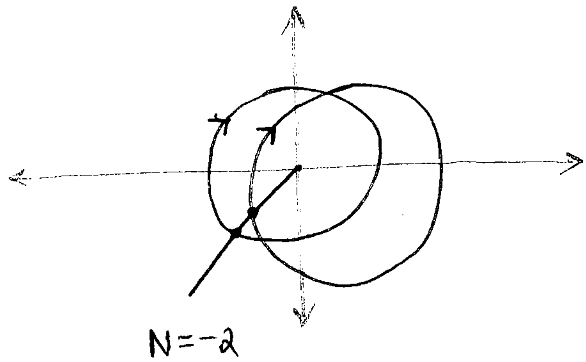
A line drawn from origin will cut only one circle.

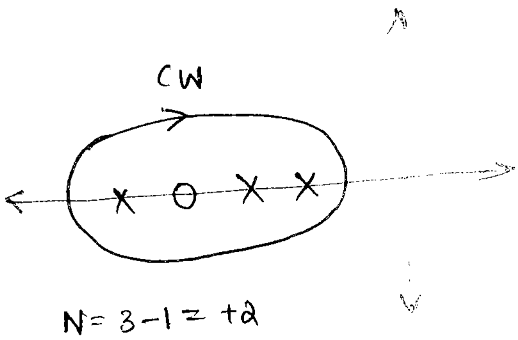
If enclosed POLE $:\Longrightarrow$ change in direction.

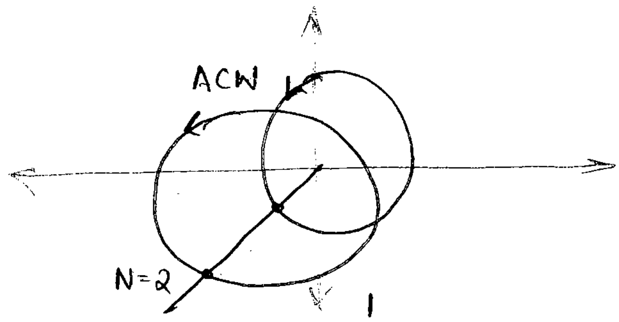
If enclosed ZERO $:\Longrightarrow$ No change in direction.

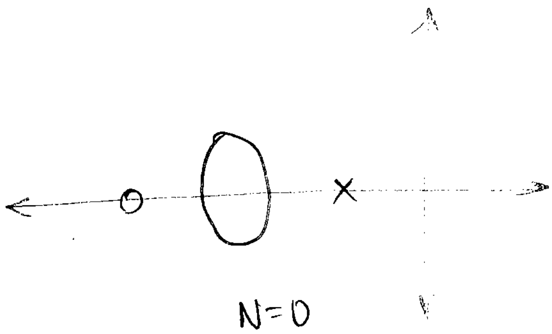
SPLANE	GH(S) PLANE

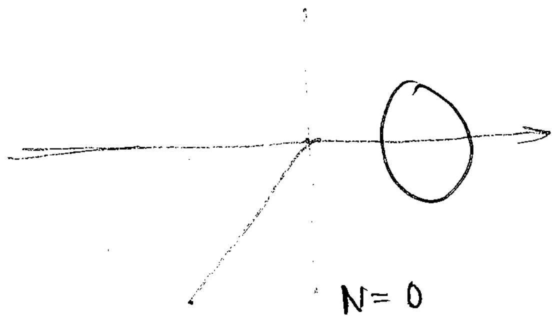












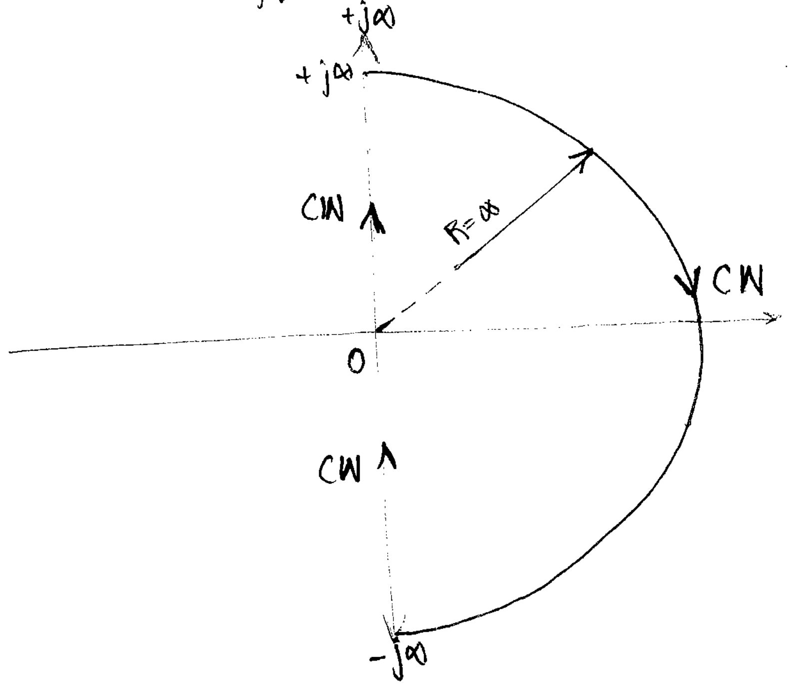
N → +ve : change in direction.

N → −ve : No change in direction.

* The random selected path should not pass through 'pole or zero.

NYQUIST PLOT

The principle of Argument concept is applied to the total right half of the s plane by selecting as a closed path. The selected total right half of s plane with a radius of ∞ is called Nyquist contour.



→ The Nyquist stability criteria is a right of s-plane analysis.

pole zero Configuration.

The open loop T.F

$$\boxed{G(s)H(s) = \frac{K N(s)}{D(s)}} \longrightarrow (1)$$

The closed loop T.F

$$CLTF \Rightarrow \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{G(s)}{1+K\frac{N(s)}{D(s)}}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)D(s)}{D(s)+KN(s)}} \rightarrow (2)$$

The closed loop ~~poles~~ stability are given by characteristic equation,

$$q(s) = 1 + G(s)H(s)$$

Substituting $G(s)H(s)$

$$q(s) = 1 + \frac{KN(s)}{D(s)}$$

$$\boxed{q(s) = \frac{D(s)+KN(s)}{D(s)}} \rightarrow (3)$$

Compare (1) & (3)

※ $\boxed{\text{poles of CE} \equiv \text{OLTF poles}}$

Compare (2) & (3)

※ $\boxed{\text{Zeros of CE} \equiv \text{CLTF poles}}$

$$\boxed{N = P - Z}$$

N :→ No:of Encirclements about Critical point. $(-1+j0)$. (origin shifted).

Z:→ Zeros of CE in the RH-s plane.

:→ CLTF poles in the RH s plane

P:→ poles of CE in the RH-s-plane

P:→ OLTF poles ~~of~~ in the RH-

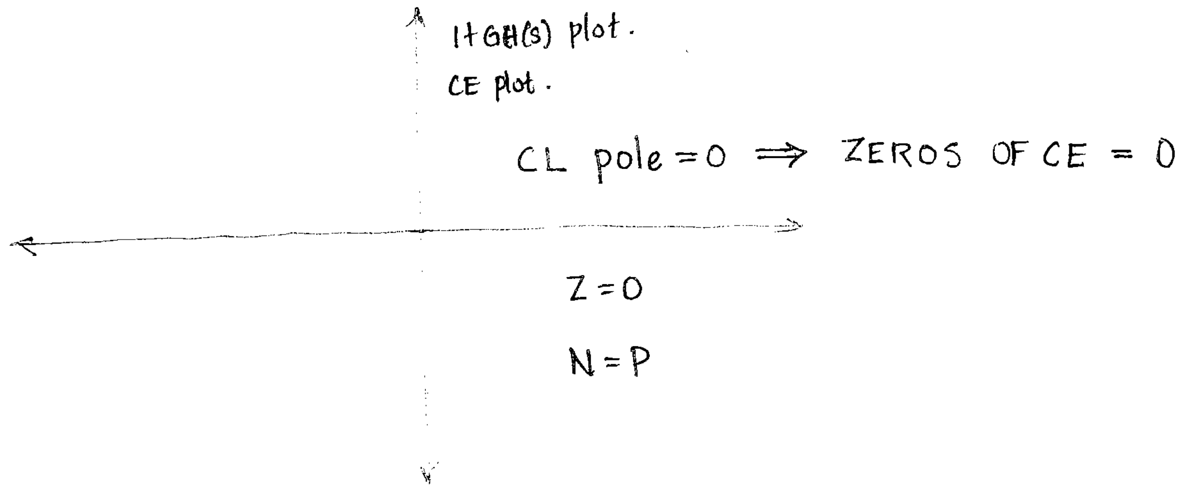
" If the plot is GH(s), then origin is 0.

If the plot is 1+G.H(s), then encirclements will be around "-1". ie, origin shifted.

→ Because, GH(s) corresponds to OLTF, But 1+GH(s) is CE which give closed loop stability.

Nyquist stability Criteria.

To become the closed loop system stable, there should not be any closed loop pole in the right of s plane. The closed loop pole is zeros of characteristic equ. which must be zero in the right of s plane, That Means $\boxed{Z=0 \;,\; \therefore N=P}$ (Then stable.



Nyquist statement.

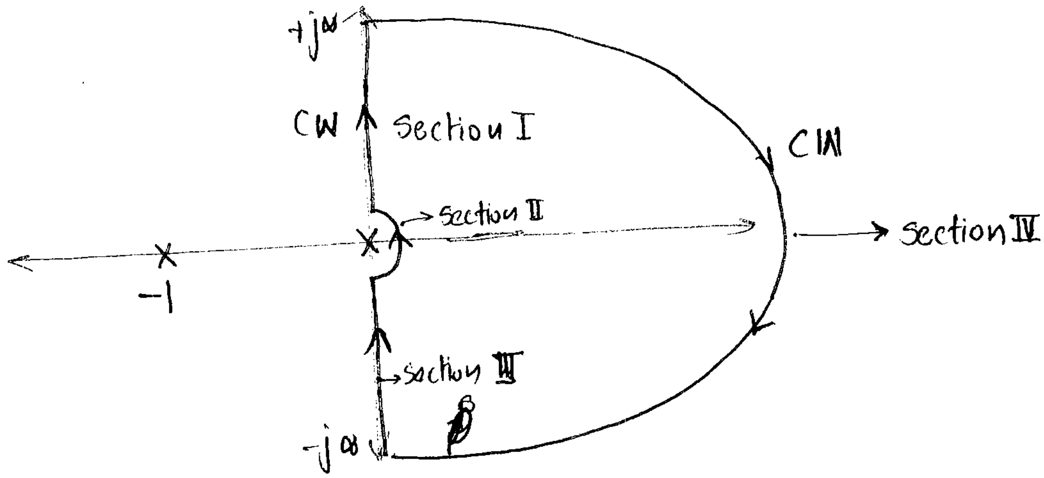
"It states that number of encirclements about the critical point must be equal to poles of characteristic equation which are equal to open loop transfer function poles in the right of s plane." $\boxed{N=P}$

Q. For $G(s)H(s) = \dfrac{1}{s(s+1)}$ Draw the Nyquist plot.

$$1 + G(s)H(s) = 1 + \frac{1}{s(s+1)}$$

$$= \frac{s(s+1)+1}{s^2+s}.$$

S1 Locate the poles and zeros in s-plane.



S2 Draw ~~polar plot for~~ polar plot for each section.

Section I

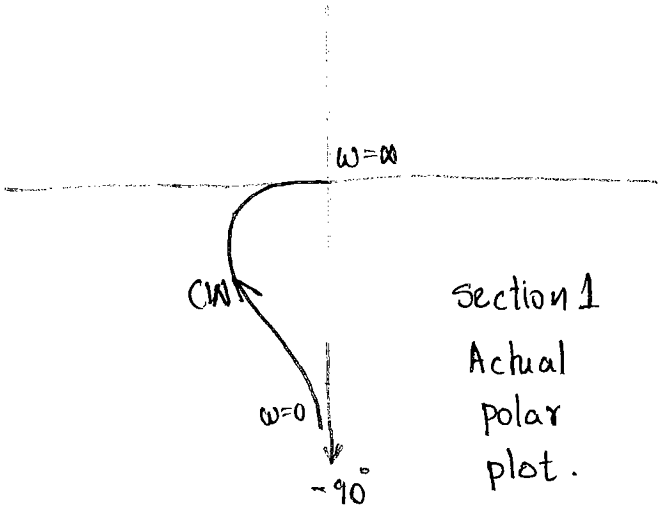
$\omega = 0^+ \quad \omega = \infty^+$

$$M = \frac{1}{\omega\sqrt{\omega^2+1}}$$

$$\angle\phi = -90^\circ - \tan^{-1}(\omega)$$

$\omega = 0^+ \longrightarrow \infty \angle -90^\circ$

$\omega = \infty^+ \longrightarrow 0 \angle -180^\circ.$



Section 1 Actual polar plot.

ED $\Rightarrow$ CW

SD $\Rightarrow$ CW

Section-II

$\omega = 0^-$ $\qquad$ 0 CW

$\omega = 0^+$

$\omega = 0^- \longrightarrow \infty \angle$

$\omega = 0^+ \longrightarrow \infty \angle$

Here analysis is required.

90°

0^+ $+j0$

$$\left.\frac{1}{s}\right|_{s=+j0} = \frac{1}{\angle +j0} = \frac{1}{+90^\circ} = -90^\circ$$

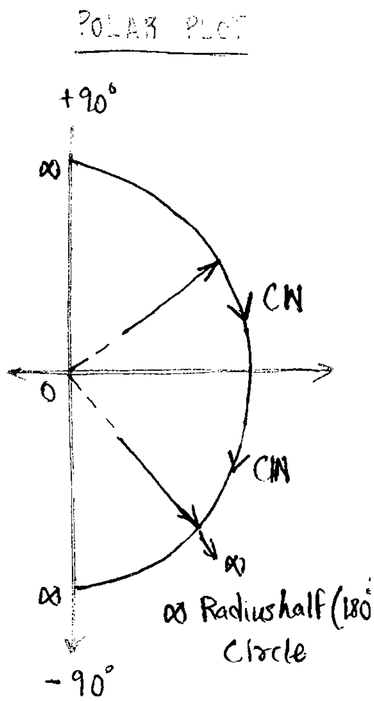
0^- $-j0$

$$\left.\frac{1}{s}\right|_{s=-j0} = \frac{1}{\angle -j0} = \frac{1}{-90^\circ} = +90^\circ$$

$$\overline{180^\circ}$$

Direction (diff) $\Rightarrow$

-90°



$+j0$

$$\angle\left.\frac{1}{s^3}\right|_{s=+j0} = \frac{1}{3\times\angle +j0} = \frac{1}{3\times 90} = -270^\circ$$

$-j0$

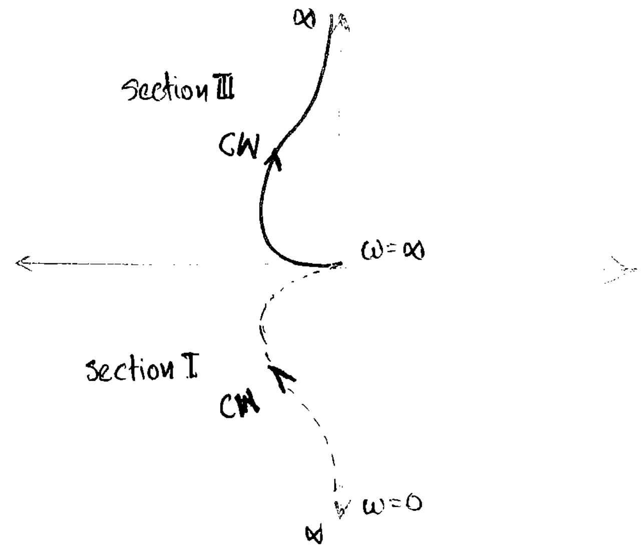
$$\left.\frac{1}{s^3}\right|_{s=-j0} \quad \frac{1}{3\times\angle -j0} = \frac{1}{3\times -90} = 270^\circ$$

Direction (diff)

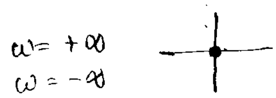
$\boxed{\text{Number of "}\infty\text{" Radius half circle} = \text{N of poles at origin}}$

Section-III

Mirror image of section I w.r.t Real axis. But the direction is direction is continuous.

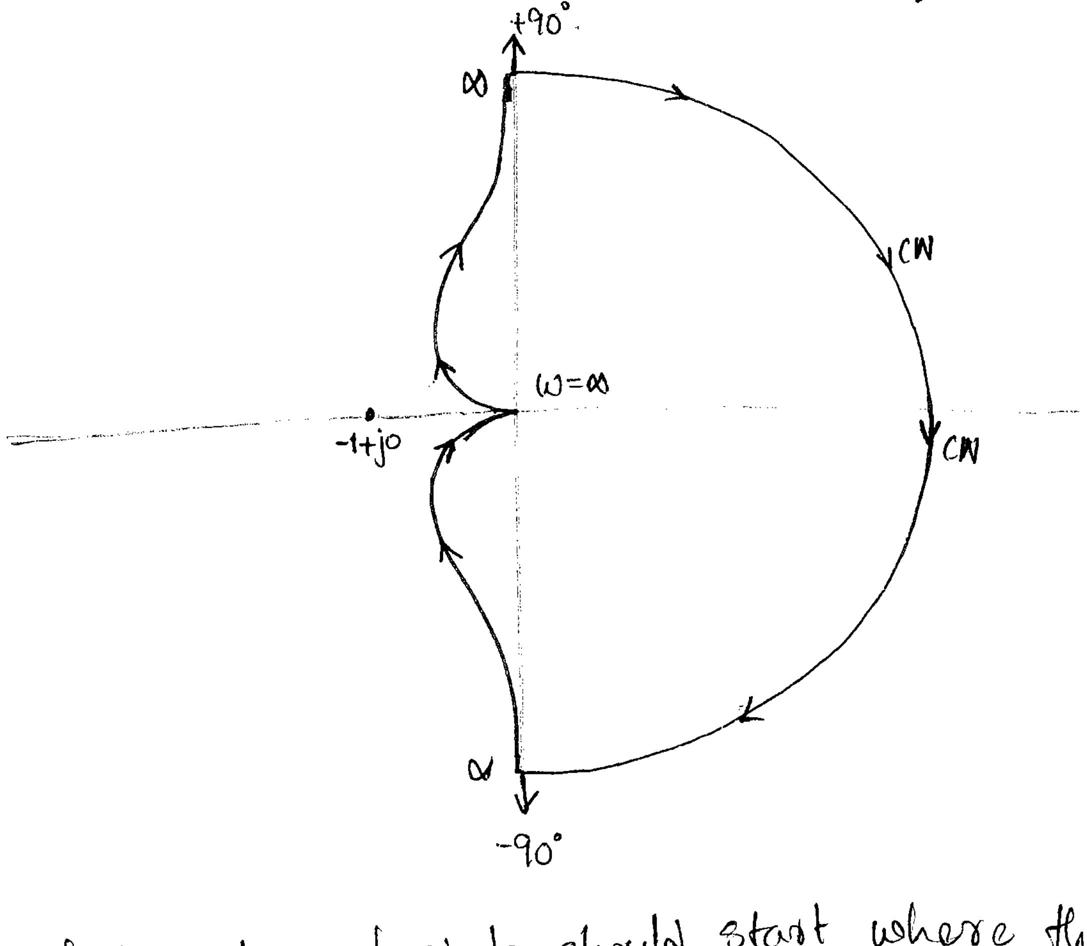


<u>Section IV</u>



The section 4 gives the Magnitude of Zero. Hence neglect the Section 4. Because it is a point at origin.

<u>S3</u> Join all polar plot then we will get Nyquist plot.



* The infinite radius of circle should start where the mirror image stop and infinite radius of circle should end where

the actual polar plot started.

* The Infinite radius half circle always clockwise. Because it depends on the Nyquist contour direction.

Nyquist stability

$N = P, \; Z = 0$

$P \rightarrow$ OL pole in the RH-s-plane

$GH(s) = \frac{1}{s(s+1)}$ Stable.

$P = 0$

$N = 0$

Here critical point $-1+j0$

Here not encirclement around $-1+j0$.

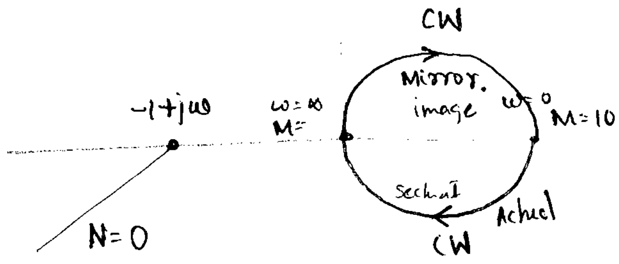
Q $GH(s) = \frac{10}{(s+1)}$

$M = \frac{10}{\sqrt{(\omega^2+1)}}$ $\angle\phi = -\tan^{-1}\omega$

Section I

$\omega = 0, \Rightarrow 10 \angle 0^\circ$

$\omega = \infty \Rightarrow 0 \angle -90^\circ$



$ED \Rightarrow CW$

$SD \Rightarrow CW$

$N = 0 = P$ Hence stable.

Q, $GH(s) = \frac{10}{(s+1)(s+2)}$ $\longrightarrow P=0$

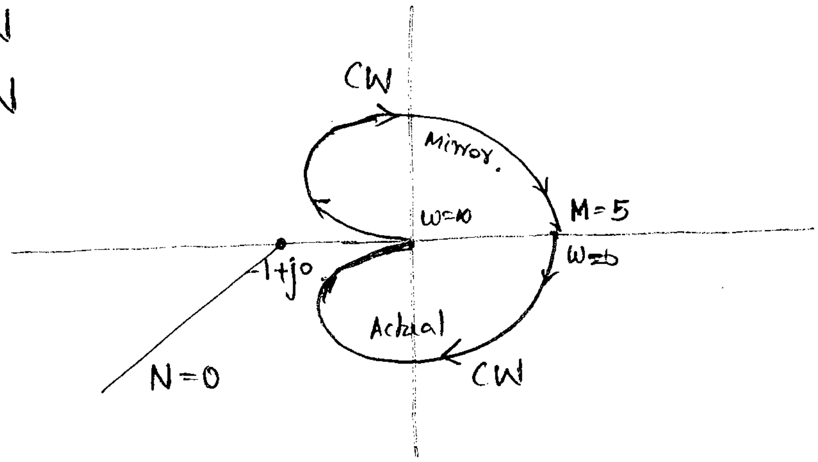
$M = \frac{10}{\sqrt{(\omega^2+1)(\omega^2+4)}}$ $\quad \angle\phi = -\tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$

$\omega = 0 \Rightarrow 5\angle 0^\circ$

$\omega = \infty \Rightarrow 0\angle -180^\circ$

ED $\Rightarrow$ CW

SD $\Rightarrow$ CW



$N = 0 = P \Rightarrow$ CL system stable.

Q, $GH(s) = \frac{10}{s^2(s+2)(s+4)}$ $\quad P=0$

$M = \frac{10}{\omega^2\sqrt{\omega^2+4}\sqrt{\omega^2+16}}$

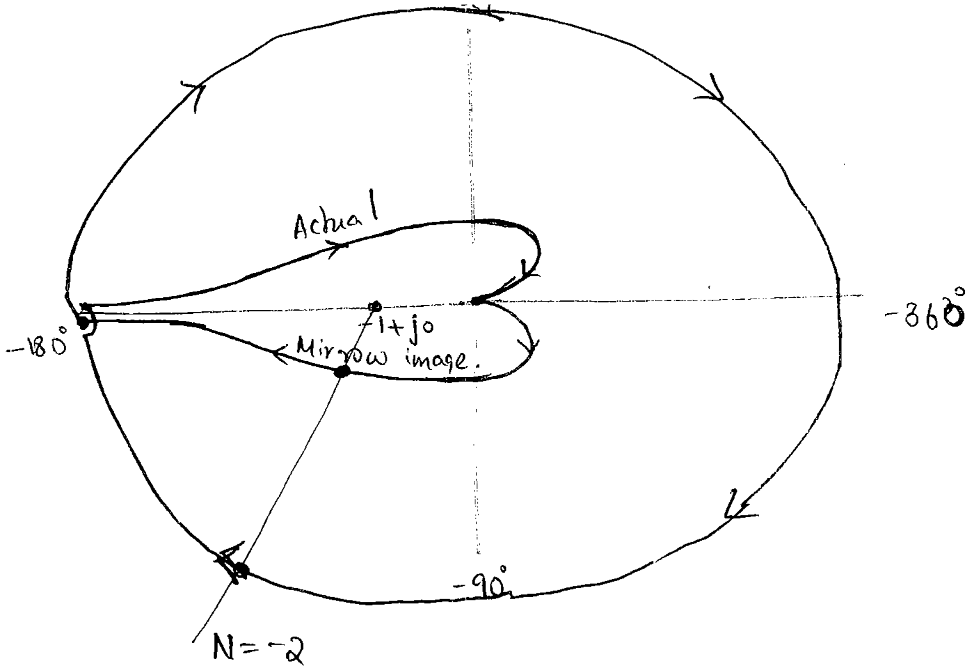
$\angle\phi = -180 - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{4}$

$\omega = 0 \Rightarrow \infty\angle -180^\circ$

$\omega = \infty \Rightarrow 0\angle -360^\circ$

ED $\Rightarrow$ CW

SD $\Rightarrow$ CW



start where mirror stop, start and at start of actual.

N ≠ P.

$P = 0$
$N = -2$

The number of closed loop poles in the RH side is given by

$N = P - Z$

$N = P - Z$
$-2 = 0 - Z$
$Z = 2$ CL poles in the right of s plane.

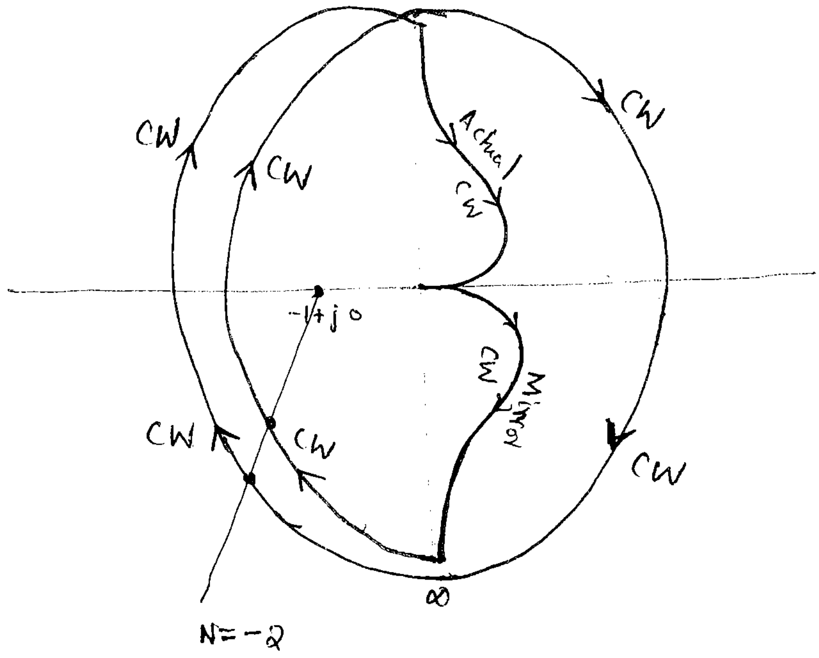
Q, $GH = \dfrac{10}{s^3(s+10)}$ $P = 0$

$M = \dfrac{10}{\omega^3\sqrt{\omega^2+100}}$ $\angle\phi = -270^\circ - \tan^{-1}\dfrac{\omega}{10}$

$\omega = 0 \Rightarrow \infty \angle -270^\circ$

$\omega = \infty \Rightarrow 0 \angle -360^\circ$

ED = CW
SD = CW



$N \neq P$

Q $GH(s) = \dfrac{1}{s(1-s)}$

$M = \dfrac{1}{\omega\sqrt{1+\omega^2}}$

$\angle\phi = -90 \cancel{\tan} - (-\tan^{-1}\omega)$

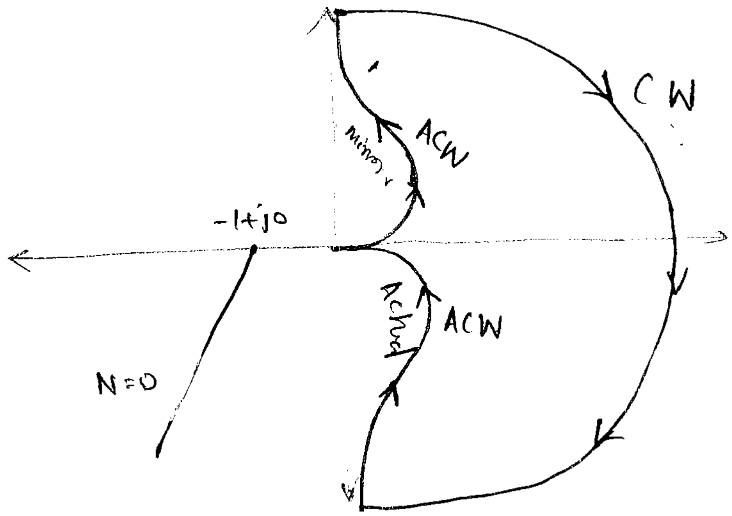
$\phi = -90 + \tan^{-1}\omega$.

$\omega = 0 \Rightarrow \infty \angle -90^\circ$

$\omega = \infty \Rightarrow 0 \angle 0^\circ$

ED = ACW

SD = X



~~Always~~

Always infinite radius half circle will be CW always

~~N=0=P~~

N=0, P=1, Not stable.

Q, $GH(s) = \frac{1}{s(s-1)} \rightarrow P=1$

$M = \frac{1}{\omega\sqrt{\omega^2+1}}$

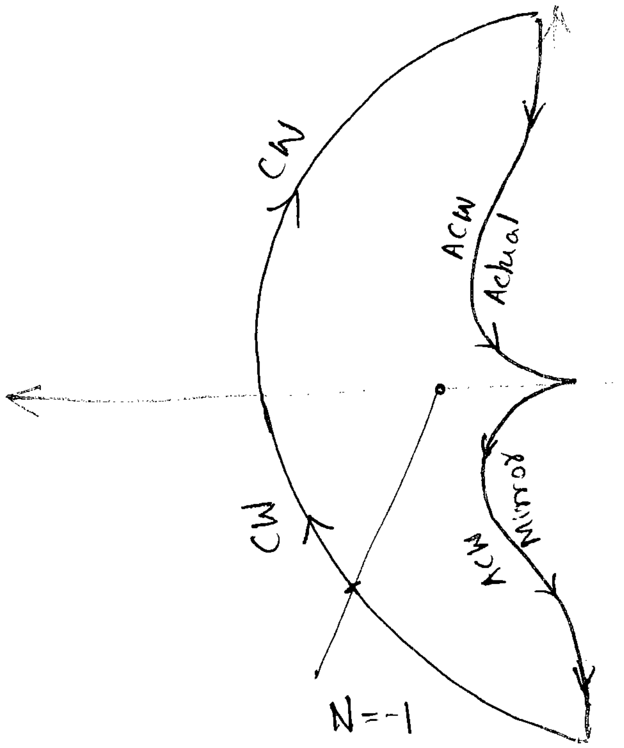
$\angle\phi = -90^\circ - (180 - \tan^{-1}\omega)$

$\angle\phi = -270 + \tan^{-1}\omega$.

$\omega=0 \Rightarrow \infty \angle -270^\circ$

$\omega=\infty \Rightarrow 0 \angle -180$

END ⇒ ACW.



$N \neq P$

Hence Unstable.

Q, Find the range of K value for closed ~~sp~~ loop stability by Nyquist stability analysis.

$$GH(s) = \frac{K}{(s+1)(s+2)(s+3)}$$

$$M = \frac{K}{\sqrt{(\omega^2+1)(\omega^2+4)(\omega^2+9)}}$$

$$\phi = -\tan^{-1}\omega - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{3}$$

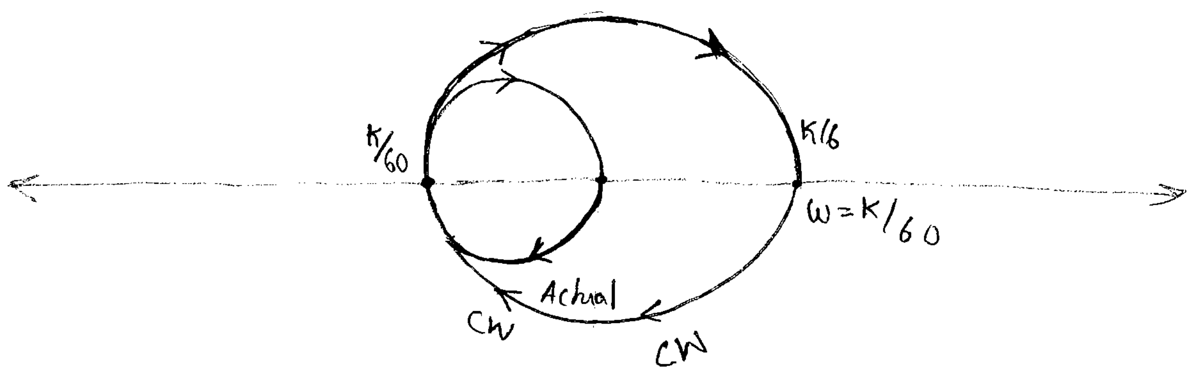
$$\omega = 0 \Rightarrow \frac{K}{2\times 3} = \frac{K}{6}\angle 0^\circ$$

$$\omega = \infty \Rightarrow 0\angle -270^\circ$$

ED $\Rightarrow$ CW

SD $\Rightarrow$ CW

Refer the polar plot for intersection point ~~at~~ with -180°.



IP with $-180 = (-1+j0)$

$$\left|\frac{-K}{60}\right| = |-1+j0|$$

for $N = P$ $\quad P = 0$

so for $N = 1$

the intersection point must be the ~~origin, of the~~ critical point.

procedure to find range of K value.

S1 : Assume that the intersection point with $-180°$ equal to the critical point. That means the magnitude of the intersection point equal to the magnitude of critical point.

ie, $M = 1$ (in above problem).

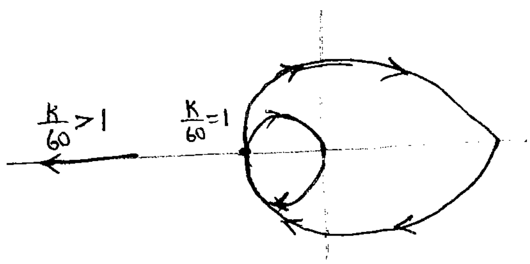
$$\left|\frac{-K}{60}\right| = 1$$

$$\frac{K}{60} = 1$$

S2 : Shift the intersection point towards $-\infty$ by considering magnitude > 1

In this case, the critical point is inside the loop.

For this Get the no: of encircle ment and condition for system stability



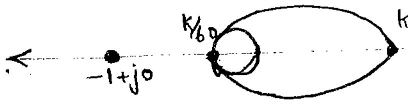
$\frac{K}{60} > 1 \Rightarrow K > 60$ unstable

$N = P - Z$

$-2 = 0 - Z \Rightarrow$

$Z = 2$ CL poles RH-S-plane $(K > 60)$.

S3 : Shift the Intersection towards origin. by considering magnitude < 1. In this case the critical point is outside the loop. For this get the no: of encirclement and condition for stability.



N=0
P=0

~~Zeros~~ ∴ CL poles = Zeros of CL = 0 Hence stable,

$$\frac{K}{60} < 1$$

$$\boxed{K < 60 \Rightarrow \text{System stable.}}$$

There is also a lower limit for K.

~~ie, if~~ It is given by intersection point K/6.

For eg: If K = −12, then point K/6 ~~plot~~ shifts left ~~with~~ to −2 which again makes System Unstable.

so poin K/6 must also greater than −1

$$\therefore \frac{K}{6} > -1$$

$\boxed{K > -6}$ system stable.

In net $\boxed{-6 < K < 60}$

* whenever one of the stability condition is less than certain value, the lower limit is decided by intersection point with 0°. The Intersection point with 0° must be greater than −1. Then closed loop system stable $\boxed{-6 < K < 60}$

Q) Find the range of K value for system stability.

$$GH(s) = \frac{K(s+3)}{s(s-1)}$$

~~$\angle\phi = \tan^{-1}\frac{\omega}{3}$~~

$$M = \frac{K\sqrt{\omega^2+9}}{\omega\sqrt{\omega^2+1}}, \quad \angle\phi = -90^\circ + \tan^{-1}\frac{\omega}{3} - (180 - \tan^{-1}\omega)$$

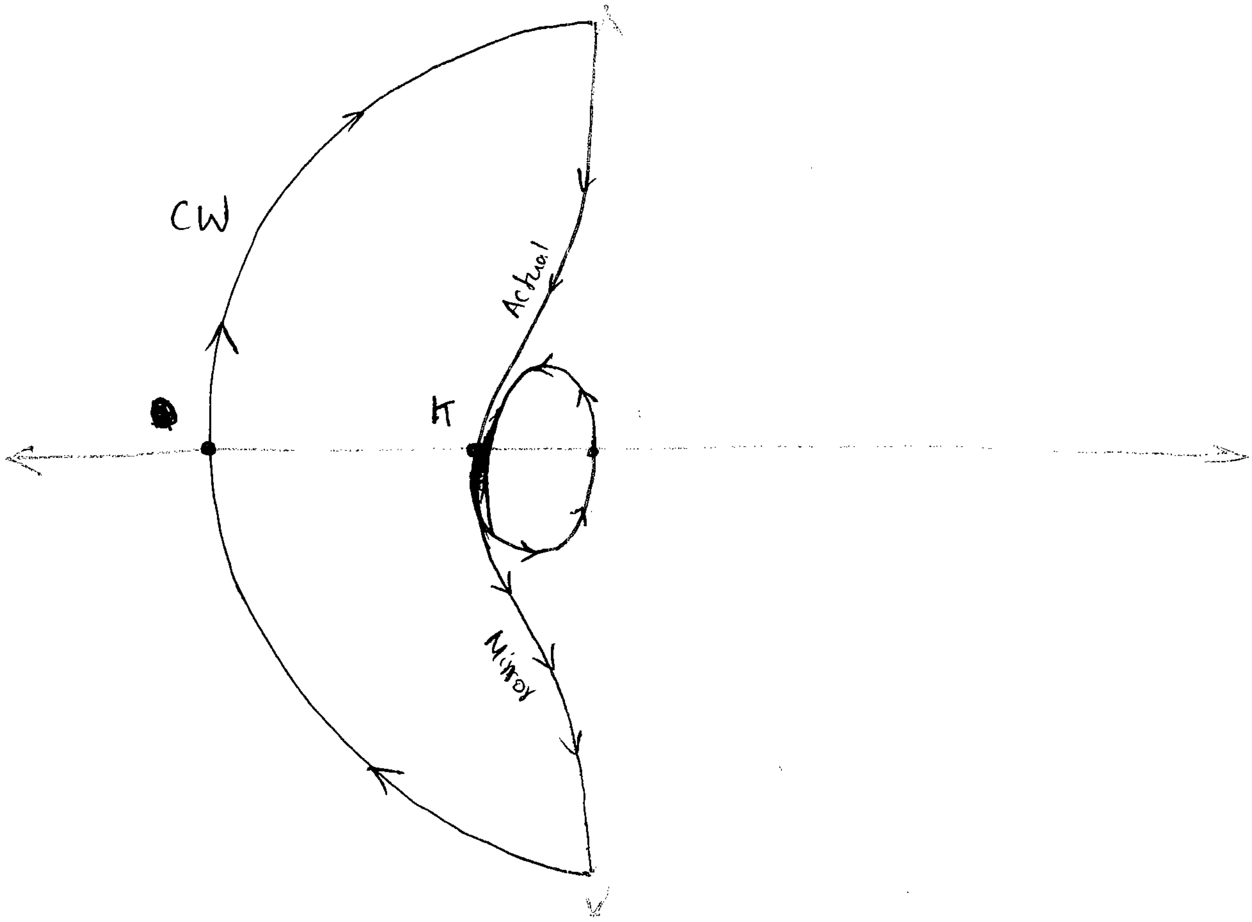
$$= -270^\circ + \tan^{-1}\frac{\omega}{3} + \tan^{-1}\omega$$

$\omega = 0 \Rightarrow \infty\angle -270^\circ$.

$\omega = \infty \Rightarrow 0\angle -90^\circ$

ED $\Rightarrow$ ACW

SD $\Rightarrow$ X



Intersection point with -180°

$$-180^\circ = -270^\circ + \tan^{-1}\frac{\omega}{3} + \tan^{-1}\omega .$$

$$+90^\circ = \tan^{-1}\left(\frac{\omega\left(\frac{4}{3}\right)}{1-\frac{\omega^2}{3}}\right)$$

$$1-\frac{\omega^2}{3} = 0$$

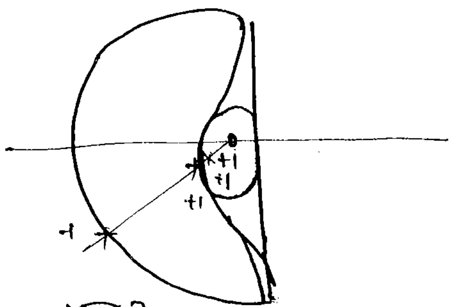
$$\frac{\omega^2}{3} = 1$$

$$\omega = \sqrt{3} \text{ rad/s} .$$

$$M\Big|_{\omega=\sqrt{3}} = \frac{K\sqrt{12}}{\sqrt{3}\sqrt{4}} = \underline{\underline{K}}$$

Shift left.

$K > 1$



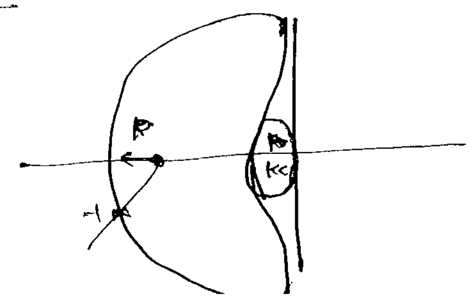
$N = +1+1-1$

$= \underline{\underline{+1}}$

Unstable .

Shift right

$K < 1$



$N = -1$

$P = 1$

$N \neq P$

Unstable for $K < 1$

4) $GH(s) = \dfrac{K(s+2)}{(s+1)(s-1)} \rightarrow P = 1$

$$M = \frac{K\sqrt{\omega^2+4}}{\sqrt{(\omega^2+1)(\omega^2+1)}}$$

$$\angle\phi = \tan^{-1}\frac{\omega}{2} \cancel{- \tan^{-1}\omega} - 180 \cancel{+ \tan^{-1}\omega}$$

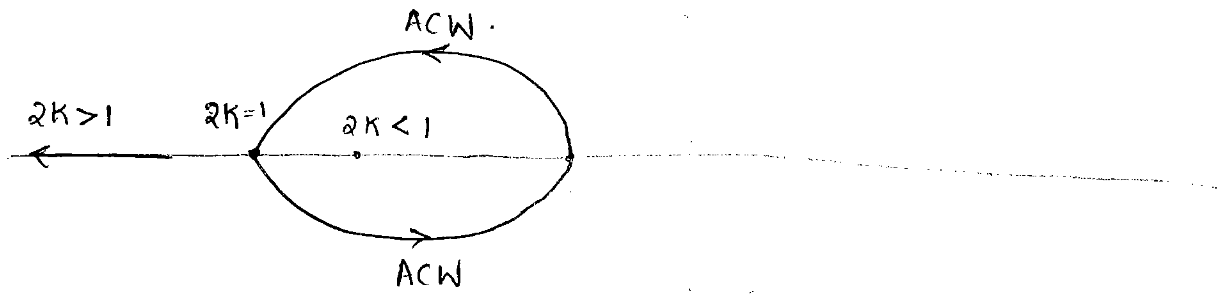
$$= -180 + \tan^{-1}\frac{\omega}{2}$$

$\omega = 0 \Rightarrow 2K \angle -180^\circ$

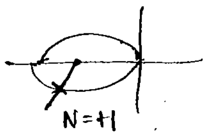
$\omega = \infty \Rightarrow 0 \angle -90^\circ$

ED $\rightarrow$ ACW

SD $\rightarrow$ X



$\rightarrow$ shift, $2K > 1$



$N = P$

$2K > 1 \Rightarrow \underline{\underline{K > 1/2}}$ stable.

Q $GH(s) = \frac{K(s-2)}{(s+2)}$ Find the range of K.

$M = \frac{K\sqrt{\omega^2+4}}{\sqrt{\omega^2+4}}$ $\angle\phi = 180 - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{2}$

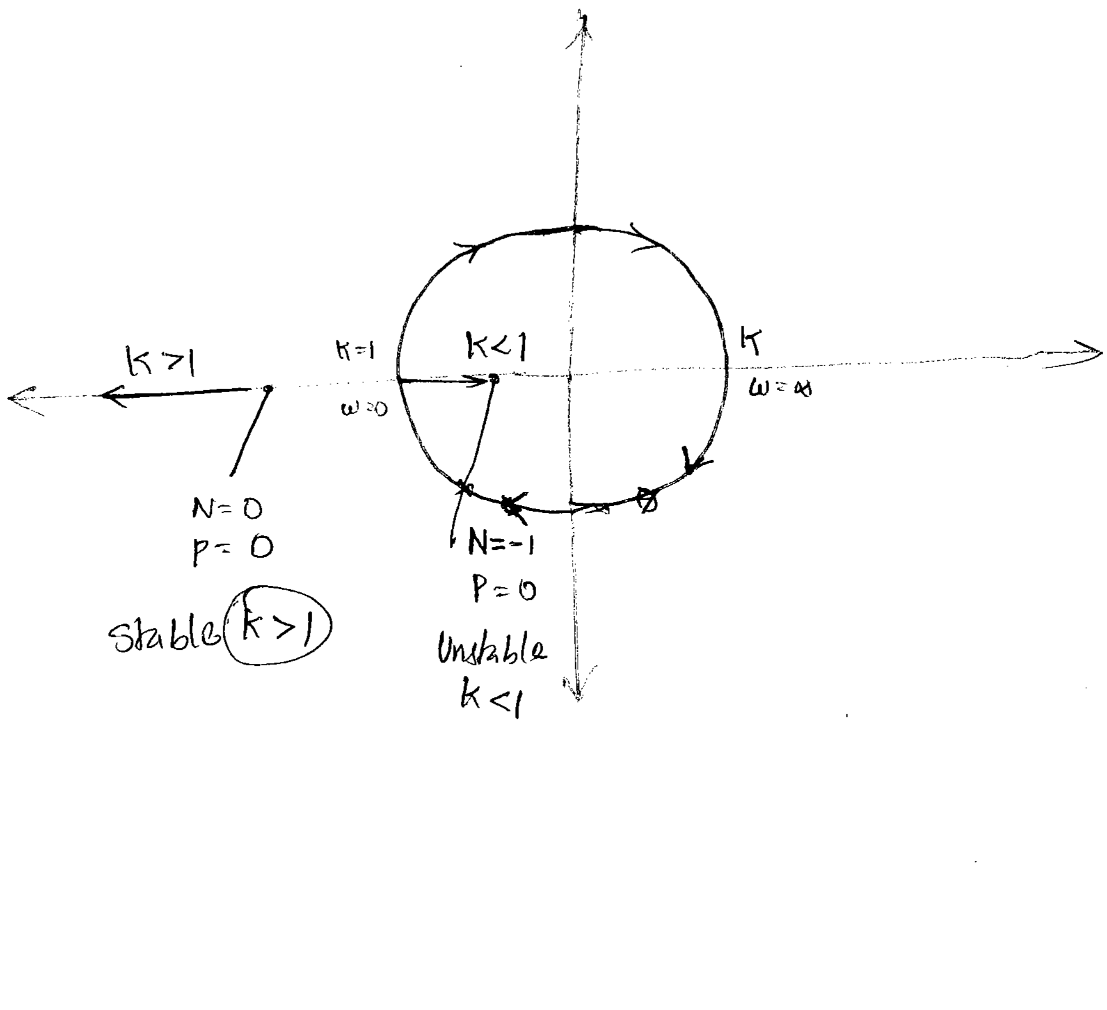
$= 180 - 2\tan^{-1}\left(\frac{\omega}{2}\right)$

$\omega = 0 \Rightarrow K\angle 180^\circ$

$\omega = \infty \Rightarrow K\angle 0^\circ$

ED $\Rightarrow$ CW

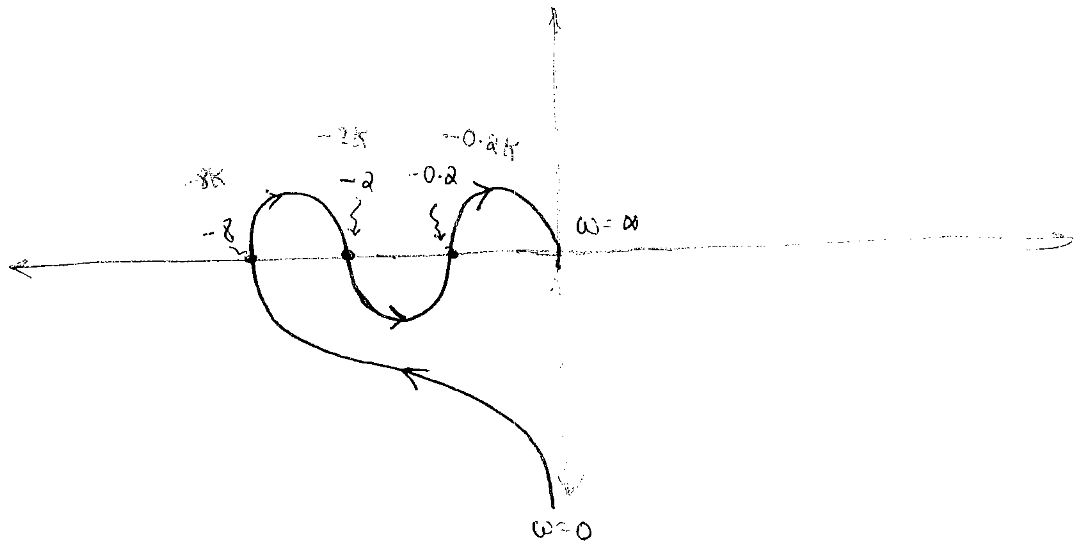
SD $\Rightarrow$ X



Q. To the given Nyquist plot, there is no open loop pole in the right of s plane. The range of K value for closed loop system stability is

Given. P = 0,

Actually it is Polar plot only

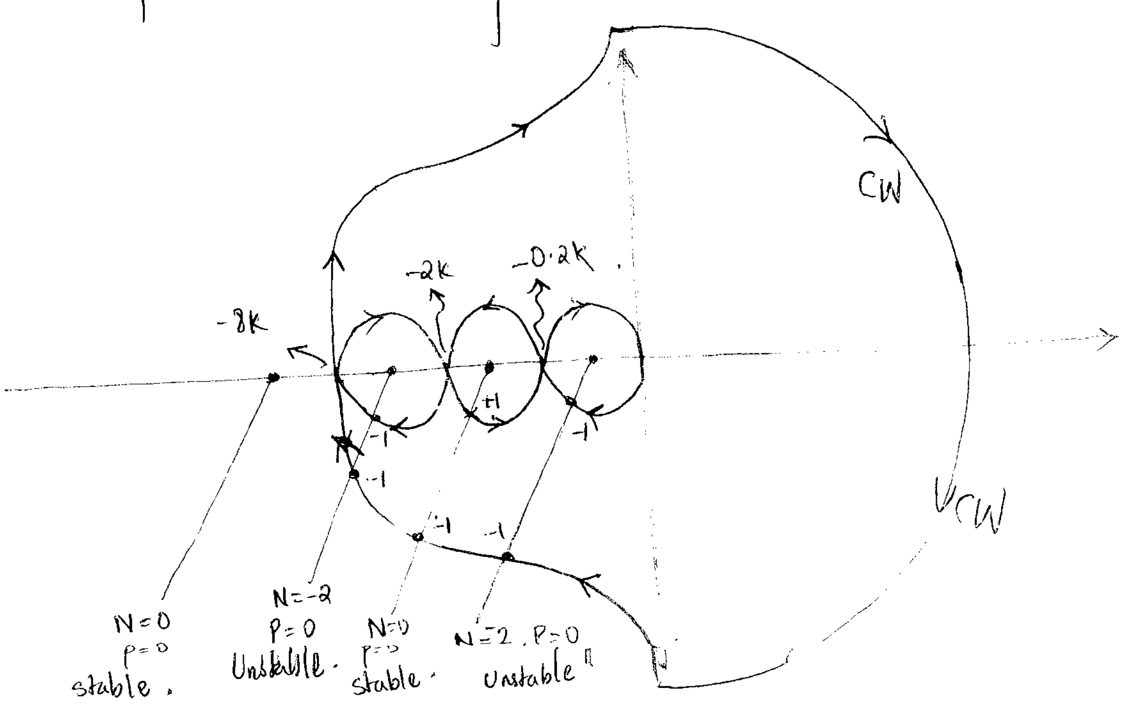


whenever such qus came, No K given anywhere

Note: To get the range of K value for stability, multiply the K with the intersection point and write the magnitude.

-8 → -8K, -2 → -2K, -0.2K.

Considering the mirror image and the infinite radius halfcircle.



we require $-1+j0$ in the stable region.

Region 1

$0.2K < 1$ & $2K > 1$

$K < 5$ & $K > 0.5$

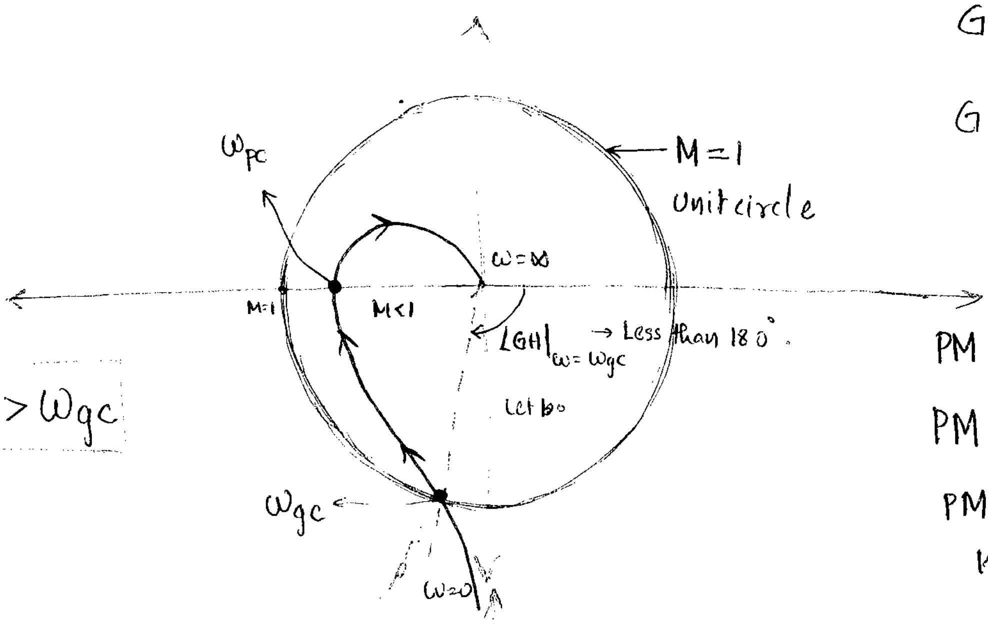
$\therefore$ $\boxed{0.5 < K < 5}$

Region 2

$8K < 1$

$\boxed{K < 1/8}$ & $\boxed{0.5 < K < 5}$ system stable.

Q, Identify the stability to the given polar plots.



$\boxed{\omega_{pc} > \omega_{gc}}$

$GM = \frac{1}{M|_{\omega pc}}$

$GM = \frac{1}{<1} = >1$

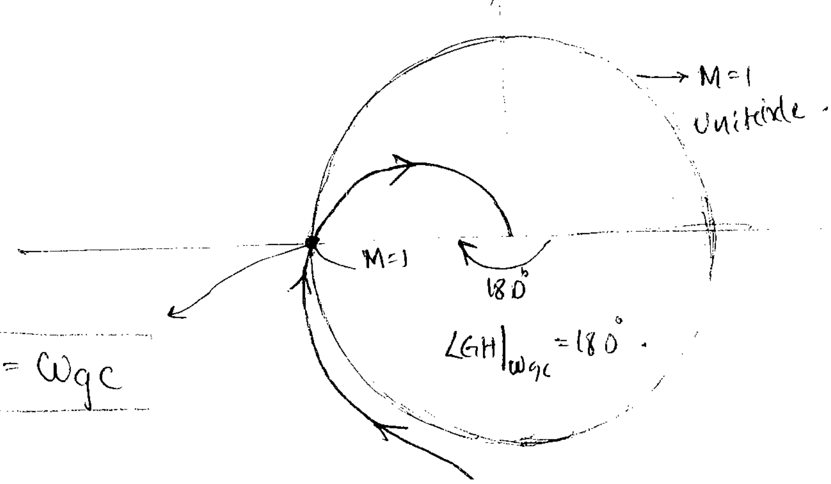
Linear.

$PM = 180° + \angle GH|_{\omega gc}$

$PM = 180° - 100$

$PM = 80°$

Hence stable.



$\boxed{\omega_{pc} = \omega_{gc}}$

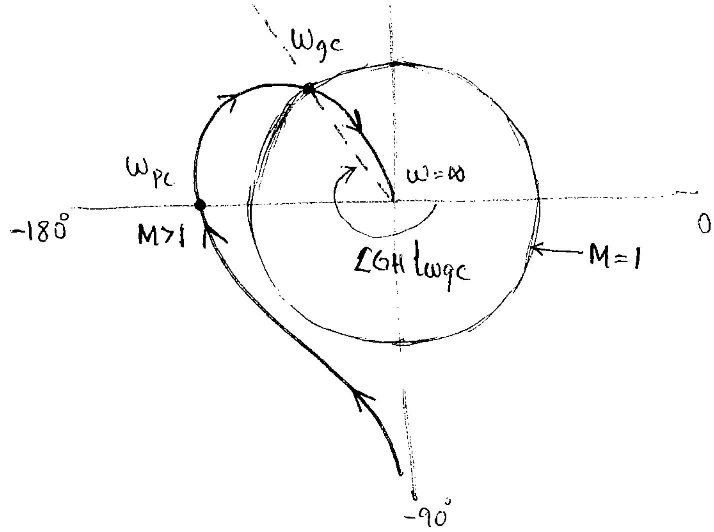
$GM = \frac{1}{M|_{\omega pc}}$

$GM = 1$

$PM = 180° + \angle GH|_{\omega gc}$

$= 180 - 180°$

$PM = \underline{\underline{0}}$



$GM = \frac{1}{M|_{pc}} = \frac{1}{>1} \doteq <1$

$PM = 180^\circ + \angle GH$

$PM = 180^\circ - 250^\circ$

$= -70^\circ$

system unstable.

$\omega_{gc} \Rightarrow M=1$

$\omega_{pc} \Rightarrow \angle -180^\circ$

$GM \Rightarrow \frac{1}{|M|_{\omega=\omega_{pc}}}$

$PM \Rightarrow 180^\circ + \angle GH|_{\omega=\omega_{gc}}$.

---

* whenever the plot intersects -180° line with a magnitude less than one, the system is stable. Because here $\omega_{pc} > \omega_{gc}$.
* whenever the plot intersects -180° line with a magnitude $= 1$ then the system is marginal stable. Because here $\omega_{pc} = \omega_{gc}$
* whenever the plot intersects -180° line with a magnitude greater than one, the system is Unstable. Because here $\omega_{pc} < \omega_{gc}$.

Calculations of Gain Margin and phase Margin.

Q. calculate the Gain margin $G(s)H(s) = \frac{1}{s(s+1)(s+2)}$

$$\cancel{M = \frac{1}{\omega\sqrt{(\omega^2+1)(\omega^2+4)}}} \qquad M = \frac{1}{\omega\sqrt{(\omega^2+1)(\omega^2+4)}}$$

$$\cancel{\angle\phi = -90^\circ - \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right)} \qquad \angle\phi = -90^\circ - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$$

$$GM = \frac{1}{|GH(j\omega)|_{\omega=\omega_{pc}}}$$

$$= -20\log|GH(j\omega)|_{\omega_{pc}}$$

Step 1 : Find $\omega_{pc} \Rightarrow \angle GH = -180^\circ$

$$-180^\circ = -90^\circ - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$$

$$90^\circ = \tan^{-1}\frac{\frac{3\omega}{2}}{1-\frac{\omega^2}{2}}$$

$$1 = \frac{\omega^2}{2}$$

$$\omega = 2 \qquad \underline{\omega = \sqrt{2}}$$

Step 2 :

$$|GH(j\omega)|_{\omega=\omega_{pc}} = \frac{1}{\sqrt{2}\sqrt{(2+1)(2+4)}}$$

$$= \frac{1}{\sqrt{2}\sqrt{3\times 6}} = \frac{1}{\sqrt{36}} = \underline{\frac{1}{6}}$$

Step 3 $\quad GM = \dfrac{1}{|GH(j\omega)|_{\omega=\omega_{pc}}} = \underline{6}$

In dB, $\quad GM_{dB} = -20 \log \left| |GH(j\omega)| \right|_{\omega=\omega_p}$

$= \underline{15.56 \text{ dB}}.$

Q calculate the phase margin for $GH(s) = \dfrac{1}{s(s+1)}$

$$PM = 180 + \angle GH(j\omega) \Big|_{\omega = gc}$$

~~M = 1~~

Step 1 : Find ω_{gc}

$M = \dfrac{1}{\omega\sqrt{\omega^2+1}}$, $M = 1$

$1 = \dfrac{1}{\omega\sqrt{\omega^2+1}}$

$\omega^2(\omega^2+1) = 1$

$\cancel{\omega^2 = -1}$

$\cancel{\omega = \pm j, 0}$

$\omega^4 + \omega^2 - 1 = 0$

$x = \omega^2$

$\omega_{gc} = 0.786$ r/sec.

Step 2 : PM

$PM = 180^\circ + (-90^\circ - \tan^{-1}\omega)$

$\underline{PM = 52^\circ}$

Q, Find the K value to get the PM = 30°.

$$GH(s) = \frac{K}{s(s+1)}$$

$$M = \frac{K}{\omega\sqrt{\omega^2+1}}$$

$$PM = 180^\circ + (-90^\circ - \tan^{-1}\omega)$$

$$= 90 - \tan^{-1}\omega_{gc}$$

$$30^\circ = 90 - \tan^{-1}\omega_{gc}$$

$$\tan^{-1}\omega_{gc} = 60^\circ$$

$$\omega_{gc} = \sqrt{3}$$

$$1 = \frac{K}{\sqrt{3}\sqrt{3+1}}$$

$$\sqrt{3}(2) = K$$

$$K = 2\sqrt{3}$$

Q, (i) Find the K value to the get the PM = 60°.

(ii) Find the K value to the get the GM = 20 dB

~~PM = 180~~

$$GH(s) = \frac{K}{s(s+2)(s+4)}$$

(i) $PM = 180^\circ + \left(-90^\circ - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{4}\right)$

$60^\circ - 90^\circ - \tan^{-1}\omega \ \ - \tan^{-1}\omega$

$$30 = \tan^{-1}\left(\frac{\frac{3}{4}\omega}{\left(1 - \frac{\omega^2}{8}\right)}\right)$$

$$\frac{1}{\sqrt{3}} = \frac{\frac{3}{\cancel{4}}\omega}{\frac{8-\omega^2}{\cancel{8}2}}$$

$$\frac{1}{\sqrt{3}} = \frac{6\omega}{8-\omega^2}$$

$$8-\omega^2 = 6\sqrt{3}\,\omega$$

$$\omega^2 + 6\sqrt{3}\,\omega - 8 = 0$$

$$\omega_{gc} = 0.7199$$

$$M = \frac{K}{\omega\sqrt{(\omega^2+4)(\omega^2+16)}}$$

$$M = 1$$

$$\underline{\underline{K = 6.2193}}$$

(ii) GM = 20dB,

$$GM = -20\log|GH(j\omega)|_{\omega_{pc}}$$

$$\cancel{20} = -\cancel{20}\log|GH(j\omega)|_{\omega_{pc}}$$

$$|GH(j\omega)| = \frac{1}{10}$$

$\angle GH = -180^\circ$

$\angle GH = -90^\circ - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{4}$

$-180^\circ = -90^\circ - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{4}$

$$90 = \tan^{-1}\frac{\frac{3}{4}\omega}{1-\frac{\omega^2}{8}}$$

$$1 - \frac{\omega^2}{8} = 0$$

$$\omega^2 = 8$$

$$\omega_{pc} = 2\sqrt{2} \text{ rad/s.}$$

$$M = \frac{K}{\omega\sqrt{(\omega^2+4)(\omega^2+16)}}$$

$\cancel{20 = 20\log|GH(j\omega)|_{\omega_{pc}}}$

$$\frac{1}{10} = \frac{K}{2\sqrt{2}\sqrt{(8+4)(8+16)}}$$

$$= \frac{K}{2\sqrt{2}\sqrt{(12)(24)}}$$

$$K = \frac{48}{10}$$

$$K = 4.8$$

Q, The open loop T.F of a unity feedback system is

$$G(s) = \frac{as+1}{s^2}$$

The value of a to get the PM = 45° is.

$$PM = 180^\circ + \left(-180^\circ + \tan^{-1}(a\omega)\right)\Big|_{\omega}$$

$$45^\circ = \tan^{-1}(a\omega)$$

$$a\omega = 1$$

$$\omega_{gc} = 1/a.$$

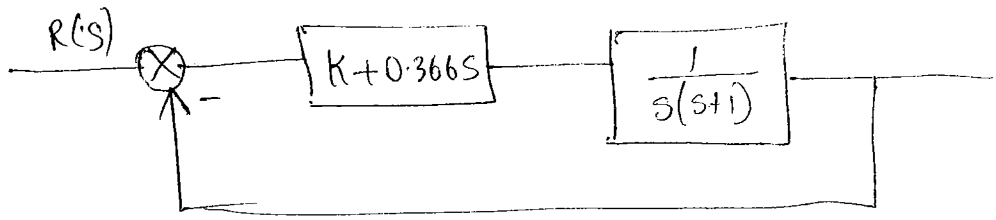
$$M = \frac{\cancel{\sqrt{a^2+1}}\ \sqrt{a^2\omega_{gc}^2+1}}{\omega^2}$$

$$M = \frac{\sqrt{2}}{1/a^2}$$

$$1/a^2 = \sqrt{2}$$

$$a^2 = \frac{1}{\sqrt{2}} = \underline{\underline{0.84089}}$$

Q, If the compensator system shown in figure has a phase margin of 60° and a cross of frequency of 1 rad/s. then the value of system Gain K is.



$$GH(s) = \frac{(K + 0.366s)}{s(s+1)}$$

$$PM = 180 + -90 - \tan^{-1}\omega + \tan^{-1}\left(\frac{0.366\omega}{K}\right)$$

$$60^\circ = 90 - \tan^{-1}\omega + \tan^{-1}\left(\frac{0.366\omega}{K}\right)$$

$\cancel{30^\circ = \tan^{-1}}$

$\omega_{gc} = 45^\circ$

$$60^\circ = 45^\circ + \tan^{-1}\left(\frac{0.366 \times 1}{K}\right)$$

$$15^\circ = \tan^{-1}\left(\frac{0.366}{K}\right)$$

$$2 - \sqrt{3} = \frac{0.366}{K}$$

$\cancel{K = 3.732}$

$$\underline{K = 1.3659}$$

Q. The loop gain of a closed loop system

$$GH(s) = \frac{\pi e^{-0.25s}}{s}$$ passes through the negative real axis, at the point is

$$\angle GH(s) = -90^\circ - \frac{0.25\omega 180}{\pi} \times \cancel{\frac{\pi}{180}}$$

passing through -ve real axis, means intersection point

$$-180 = -90^\circ - \frac{0.25\,\omega \times 180^\circ}{\pi}$$

$$-90^\circ = -\frac{0.25 \times 180^\circ \omega}{\pi}$$

$$\omega_{pc} = 2\pi \text{ rad/sec}$$

$$M\Big|_{\omega_{pc}} = \frac{\pi}{\omega} = \frac{\cancel{\pi}}{2\cancel{\pi}} = \frac{1}{2}$$

Intersection point with -180° = $(-0.5, j0)$ (−ve real axis)

Q. The gain ~~#~~ margin and phase to the above problem is,

$$GM = -20\log|GH(j\omega)|_{\omega_{pc}}$$

$$= -20\log 0.5$$

$$= 6.021 \text{ dB}$$

$$\omega_{gc} \Rightarrow M = 1$$

$$\frac{\pi}{\omega} = 1$$

$$\omega_{gc} = \pi$$

$$PM = 180 + \cancel{\phi} - 90^\circ - \frac{0.25}{\pi}\,\omega_{gc} \times 180^\circ$$

$$= 90 - \frac{45^\circ \times \cancel{\pi}}{\cancel{\pi}}$$

$$= 45^\circ$$

Q calculate the Gain margin and phase Margin

$$GH(s) = \frac{e^{-s}}{s(s+1)}$$

~~GM~~, PHASE MARGIN

$$M = \frac{1}{\omega\sqrt{\omega^2+1}}$$

$M = 1$

$$\omega_{gc}\sqrt{\omega_{gc}^2+1} = 1$$

$$\omega^2(\omega^2+1) = 1$$

$$\omega^4+\omega^2-1 = 0$$

$$\omega_{gc} = 0.786 \text{ rad/s}$$

~~GM =~~

$$PM = 180 + \left(-90^\circ - \tan^{-1}\omega - \omega \times \frac{180}{\pi}\right)$$

$$- ~~45~~$$

$$PM = \underline{6.833}$$

GAIN MARGIN

$$\angle GH = -90 - \tan^{-1}\omega - \omega \times \frac{180}{\pi}$$

$$-180^\circ = -90 - \tan^{-1}\omega - \omega \times \frac{180}{\pi}$$

$$+90^\circ = \tan^{-1}\omega + \omega \times \frac{180}{\pi}$$

$$\omega_{pc} = \underline{0.8603}$$

$$GM = -\cancel{20} - 20\log\left|\frac{1}{0.86\sqrt{0.86^2+1}}\right|$$

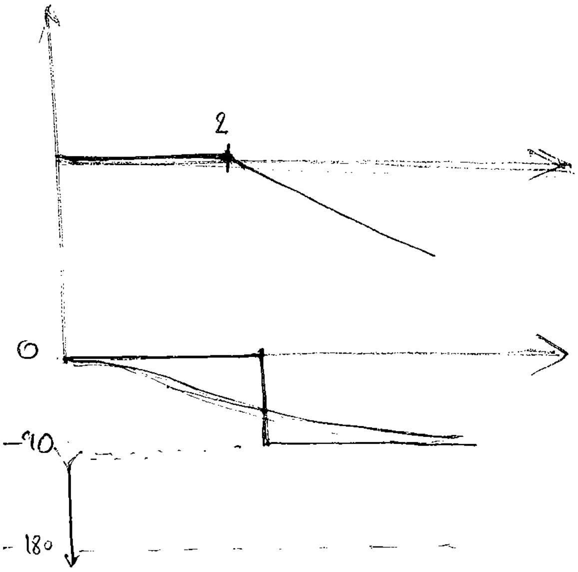
$$GM = \frac{1}{M_{pc}} = \underline{1.13} \quad \cancel{= 20\log}$$

$$GM_{dB} = \cancel{1.13\ 1.0}$$

$$GM = \frac{1}{M_{pc}} = \frac{1}{0.88} = \underline{1.13}$$

Q,

$$GH(s) = \frac{1}{(s+2)}$$



GM

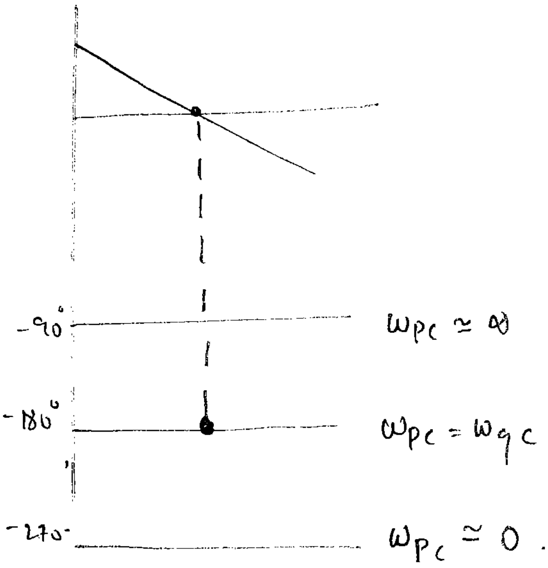
$\omega_{pc} \rightarrow \angle GH = -180^\circ$

$$-180^\circ = -\tan^{-1}(\omega/2)$$

$\omega = 0 \Rightarrow 0^\circ$

$\omega = \infty \Rightarrow -90^\circ$

ie, -180°, doesnotexist, ie, angle from 0 to -90°
so it never goes to -180° as given in bode plot above.



$\omega_{pc} \approx \infty$

~~$\omega_{pc} = \omega_{gc}$.~~

$$M|_{\omega_{pc}} = \frac{1}{\sqrt{\omega^2+4}} = 0$$

$$GM = \frac{1}{M|_{\omega_{pc}}} = \frac{1}{0} = \infty$$

If ∞ not there
then 2nd priority is, ω_{gc} or $\omega_{pc} = 0^\circ$.

Phase Margin — In some other qn.

if

$\omega_{gc} \rightarrow M = 1$

$$\frac{1}{\sqrt{\omega^2+4}} \neq 1$$

& if $\omega = 0$ $M = 0.5$

$\omega = 0$ $M = 0$

Then the $\omega_{gc} = \infty$ Does not exist

Note: whenever the plot or transfer function, less magnitude
and less ~~magnitude than~~ negative phase angle than -180°,
at all the frequency range then gain margin is
equal to phase margin which is equal to ∞.

$GH = 1/s$

GM $\xrightarrow{} \omega_{pc}$

$\angle GH = -180°$

$-90° \neq -180°$

Always $\angle -180°$

Does not exist.

$\omega_{pc} = \infty$

$M = 0$

$GM = \infty$

PM $\xrightarrow{} \omega_{gc}$

$M = 1$

$\frac{1}{\omega} = 1$

$\omega_{gc} = 1$

$PM = 180 - 90°$

$PM = 90°$

stable.

$GH(s) = 1/s^2$

Gain margin.

$\omega_{pc} \rightarrow \angle GH = -180°$

$-180 = -180°$.

$\omega_{pc} = \omega_{gc}$

ω_{gc}. $\xrightarrow{} M = 1$

$\frac{1}{\omega^2} = 1$

$\omega = 1$

$\omega_{gc} = 1 = \omega_{pc}$

$GM = \frac{1}{M|_{\omega_{pc}}} = 1$

$PM = 180 - 180° = 0°$

Marginally stable.

$GH(s) = 1/s^3$

~~$GM = \frac{1}{M|_{\omega_{pc}}} = 1$ (linear)~~

GM

ω_{pc} $\angle GH = -180°$

$-270 \neq -180°$

$> -180°$

$\omega_{pc} = 0$

$M = \infty$

$GM = 0$ <1

PM

$M = 1$

$\frac{1}{\omega^3} = 1$

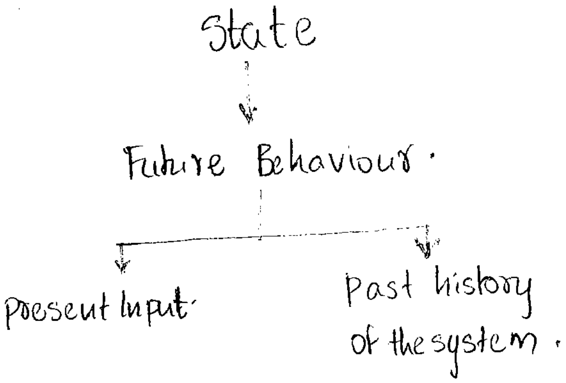
$\omega_{gc} = 1$ rad/s.

$PM = 180° - 270°$

$= -90°$

Unstable.

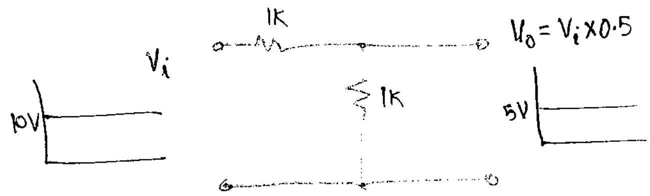
STATE SPACE ANALYSIS



State: The state gives the future behaviour of the system based on present input and past history of the system.

→ The past history of the system described by the state variables.

consider the system.



→ The resistive network is not having any state variables because no energy is stored in the system.

No storage elements.

No past history

No state variables.

The resistive n/w is called <u>memory less System.</u>

NUMBER OF STATE VARIABLES

~~The number of~~

* If the electrical network is given, the number of state variables = number of inductors and capacitors.

* If the differential equation, the number of state variables = Order of the differential equation.

Standard Form of State Model.

$$\dot{X} = AX + BU \longrightarrow \text{state equation / Dynamic equation.}$$

$$Y = CX + DU \Longrightarrow \text{output equation.}$$

$\dot{X}$: Differential state vector.

X : State vector.

U : Input vector.

A : State Matrix.

B : Input Matrix.

C : output Matrix

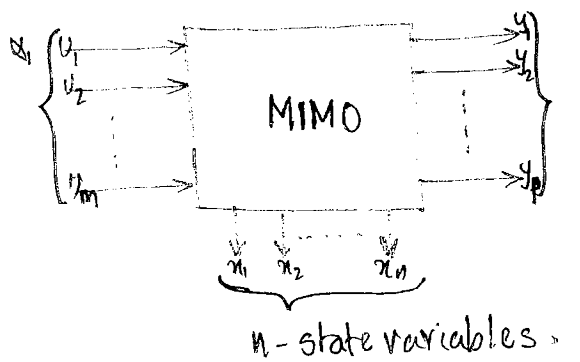
D : Transmission matrix.

The transmission is always zero, If the circuit not presents the any active element.

Order of matrix

Consider MIMO system.

m inputs



p outputs.

Input vector $U = \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_M \end{bmatrix}_{m\times 1}$ output vector, $= \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_p \end{bmatrix}_{p\times 1}$ State vector $= \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1}$

* The order of the differential state variable = Order of State Variable.

$$\underset{n\times 1}{\dot{X}} = \overset{n\times n}{A}\, \underset{n\times 1}{X} + \overset{n\times m}{B}\, \underset{m\times 1}{U}$$

$$\underset{p\times 1}{Y} = \overset{p\times n}{C}\, \underset{n\times 1}{X} + \overset{p\times m}{D}\, \underset{m\times 1}{U}$$

STATE MODEL TO DIFFERENTIAL EQUATIONS

Write the state model to the given system.

$$\dddot{y} + 2\ddot{y} + 5\dot{y} + 7y = 9U$$

3^{rd} order, $n=3$

Let $y = x_1 \rightarrow ①$, $\dot{y} = \dot{x}_1 = x_2 \rightarrow ②$, $\ddot{y} = \dot{x}_2 = x_3 \rightarrow ③$

$\dddot{y} = \dot{x}_3 \rightarrow ④$ (can use only 3 state variables)

* To get the $\dot{x}_3$ in terms of the state variable, substitute all the above equations in the given system.

$\dot{x}_3 + 2x_3 + 5x_2 + 7x_1 = 9U$

$\dot{x}_3 = 9U - 7x_1 - 5x_2 - 2x_3 \longrightarrow (5)$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -7 & -5 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 9 \end{bmatrix} [U]$$

$$[Y] = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Controllable Canonical Form. (CCF)

To write directly. Controllable canonical form.

$$\rightarrow A = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & & & & \ddots & \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ \text{Last row} & \multicolumn{5}{l}{\text{with opposite sign of coeff. from right to left}} \end{bmatrix}$$

eg: $\dddot{y} + 2\ddot{y} + 5\dot{y} + 7y = 0$

opposite sign of coef.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -7 & -5 & -2 \end{bmatrix}$$

$C = [1 \ 0 \ 0 \cdots]$

$y = x_1$

To write observable canonical form: (OCF)

$$\boxed{A_{OCF} = A_{CCF}^T}$$

eg: In above, $= \begin{bmatrix} 0 & 0 & -7 \\ 1 & 0 & -5 \\ 0 & 1 & -2 \end{bmatrix}$

$B_{CCF} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix}$ End ↑ Start

$B_{OCF} = \begin{bmatrix} b_3 \\ b_2 \\ b_1 \\ b_0 \end{bmatrix}$ Start ↓ End.

$B_{OCF} = \begin{bmatrix} 9 \\ 0 \\ 0 \end{bmatrix}$

$C_{CCF} = \begin{bmatrix} c_0 & c_1 & c_2 & c_3 \end{bmatrix}$ End ← Start

$C_{OCF} = \begin{bmatrix} c_3 & c_2 & c_1 & c_0 \end{bmatrix}$ Start → End.

$C_{OCF} = [0 \; 0 \; 1$

---

eg: $\ddddot{y} + 2\dddot{y} - 4\ddot{y} + 6\dot{y} + 8y = 10U$

$$A_{CCF} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -8 & -6 & 4 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

STATE MODEL TO TRANSFER FUNCTION

consider the T.F

$$\frac{Y(s)}{U(s)} = \frac{2s+3}{s^2+5s+6}$$

$$\boxed{s^n = \dot{x}_n}$$

$$s^2 \longrightarrow \dot{x}_2$$

$$s^1 \longrightarrow \dot{x}_1 = x_2$$

$$s^0 \longrightarrow x_1$$

seperating numerator & denominator,

$$U = \dot{x}_2 + 5x_2 + 6x_1$$

$$Y = 3x_1 + 2x_2$$

- Now writing matrix.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [U]$$

$$[y] = \begin{bmatrix} 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Short cut.

End Same sign Start

$$\frac{y(s)}{U(s)} = \frac{K(2s+3)}{s^2+5s+6}$$

C matrix

A matrix

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$ → common factor in the numerator.

K

Q $\dfrac{Y(s)}{U(s)} = \dfrac{2s^3+4s^2+6}{s^5+3s^4+5s^2+7s+9} = \dfrac{2(s^3+2s^2+3)}{s^5+3s^4+5s^2+7s+9}$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -9 & -7 & 0 & -5 & -3 \end{bmatrix}$$

$\cancel{C = \begin{bmatrix} 6 & 0 & 4 & 2 \end{bmatrix}}$

$$C = \begin{bmatrix} 3 & 0 & 2 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2 \end{bmatrix}$$

Q $\dfrac{Y(s)}{U(s)} = \dfrac{1}{(s+1)(s+2)(s+3)}$ Find Diagonalization form.

$$\frac{Y(s)}{U(s)} = \frac{1/2}{s+1} + \frac{-1}{s+2} + \frac{1/2}{s+3}$$

$$y = \frac{\frac{1}{2}U}{s+1} + \frac{-1U}{s+2} + \frac{\frac{1}{2}U}{s+3}$$

$$y = x_1 + x_2 + x_3$$

$x_1 = \dfrac{\frac{1}{2}U}{s+1}$ $\qquad sx_1 + x_1 = \frac{1}{2}U$

$$\frac{d}{dt}x_1 + x_1 + \frac{1}{2}U$$

$$\boxed{\dot{x} = \tfrac{1}{2}U - x_1} \longrightarrow (1)$$

$$x_2 = \frac{-1U}{s+2}$$

$$sx_2 + 2x_2 = -1U$$

$$\boxed{\dot{x}_2 = -1U - 2x_2} \rightarrow (2)$$

$$x_3 = \frac{\frac{1}{2}U}{s+3}$$

$$sx_3 + 3x_3 = \tfrac{1}{2}U$$

$$\boxed{\dot{x}_3 = \tfrac{1}{2}U - 3x_3} \rightarrow (3)$$

Now write Matrix form

$$\left.\begin{aligned}\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} &= \begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 0\\ 0 & 0 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}\frac{1}{2}\\ -1\\ \frac{1}{2}\end{bmatrix}[U]\\ [Y] &= \begin{bmatrix}1 & 1 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}\end{aligned}\right\}\text{DIAGONALIZATION FORM}$$

Instead of taking $y = x_1 + x_2 + x_3$

taking, $y = \frac{1}{2}x_1 - x_2 + \frac{1}{2}x_3$ such that $x_1 = \frac{1}{s+1}$, $x_2 = \frac{1}{s+2}$, $x_3 = \frac{1}{s+3}$

we get $B = \begin{bmatrix}1\\1\\1\end{bmatrix}$ & $C = \begin{bmatrix}\frac{1}{2} & -1 & \frac{1}{2}\end{bmatrix}$

In diagonilization form, B and C matrices are interchangable.

Q, Find JORDAN CANONICAL FORM

$$\frac{y(s)}{U(s)} = \frac{1}{(s+2)^2(s+3)}$$

$$\frac{y(s)}{U(s)} = \frac{1}{(s+2)^2} - \frac{1}{(s+2)} + \frac{1}{s+3}$$

$$y = \frac{1U}{(s+2)^2} - \frac{1U}{(s+2)} + \frac{1U}{s+3}$$

$$\boxed{y = 1x_1 - 1x_2 + 1x_3}$$

Here compulsarly partial fraction coeff must include in C matrix.

In x_1, x_2 x_3, numerator must be 1U only.

$$x_1 = \frac{U}{(s+2)^2} \qquad x_2 = \frac{U}{s+2} \qquad x_3 = \frac{U}{s+3}$$

$$x_1 = \left(\frac{U}{s+2}\right)\left(\frac{1}{s+2}\right) \qquad sx_2 + 2x_2 = U \qquad sx_3 + 3x_3 = U$$

$$\boxed{\dot{x}_2 = U - 2x_2} \qquad \boxed{\dot{x}_3 = U - 3x_3}$$

$$x_1 = \frac{x_2}{s+2}$$

$$sx_1 + 2x_1 = x_2$$

$$\boxed{\dot{x} = x_2 - 2x_1}$$

JORDAN BLOCK (Repeated poles)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \left[\begin{array}{cc|c} -2 & 1 & 0 \\ 0 & -2 & 0 \\ \hline 0 & 0 & -3 \end{array}\right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \left[\begin{array}{c} 0 \\ 1 \\ \hline 1 \end{array}\right] [U]$$

$$[y] = \begin{bmatrix} 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

partial fraction.

short cut directly.

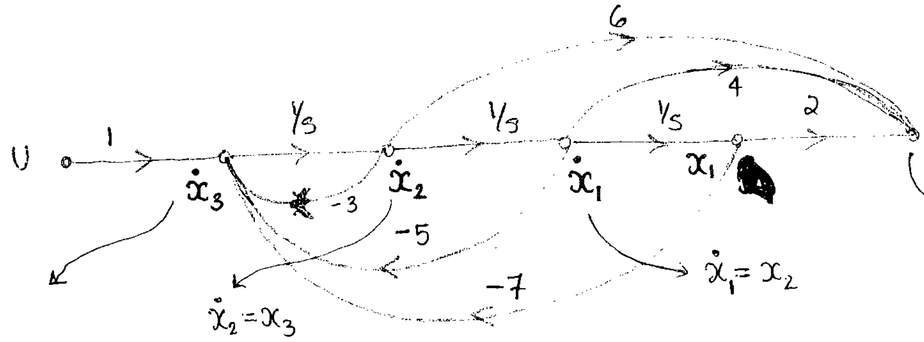
Q $$\frac{y(s)}{U(s)} = \frac{10}{(s+5)^3(s+10)}$$

Jordan block

(In Jordan block, along with diagonal element, above each diagonal element there will be one (indicate repetition.

$$A = \left[\begin{array}{ccc|c} -5 & 1 & 0 & 0 \\ 0 & -5 & 1 & 0 \\ 0 & 0 & -5 & 0 \\ \hline 0 & 0 & 0 & -3 \end{array}\right] \qquad B = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \qquad C = [P \;\; F]$$

Q. obtain the state model to the given signal flow graph



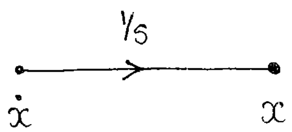
$y = 2x_1 + 4\dot{x}_1 + 6\dot{x}_2$

$y = 2x_1 + 4x_2 + 6x_3$

$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

Note: To select the node as a state variable, the incoming branch to that particular ~~branch~~ node is an integrator



$$\dot{x}_3 = 1U - 3\dot{x}_2 - 5\dot{x}_1 - 7x_1$$

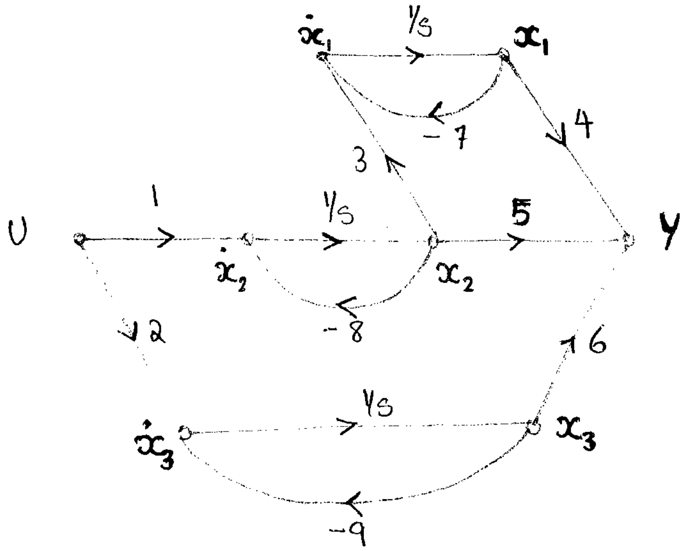
$$\boxed{\dot{x}_3 = 1U - 3x_3 - 5x_2 - 7x_1}$$

Now writing the matrix,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -7 & -5 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [U]$$

$$[y] = \begin{bmatrix} 2 & 4 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Q, write the state model.



$$\dot{x}_1 = 3x_2 - 7x_1$$

$$\dot{x}_2 = 1U - 8x_2$$

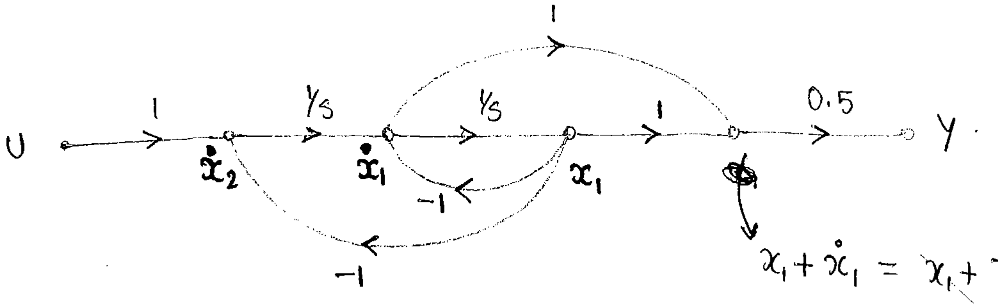
$$\dot{x}_3 = 2U - 9x_3$$

$$y = 4x_1 + 5x_2 + 6x_3$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -7 & 3 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} [U]$$

$$[y] = \begin{bmatrix} 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Q. Write the state model.



$x_1 + \dot{x}_1 = x_1 + x_2 - \cancel{x_1}$

$= x_2$

$\dot{x}_1 = x_2 - x_1$

$\dot{x}_2 = 1U - x_1$

$y = 0.5x_2 .$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [U]$$

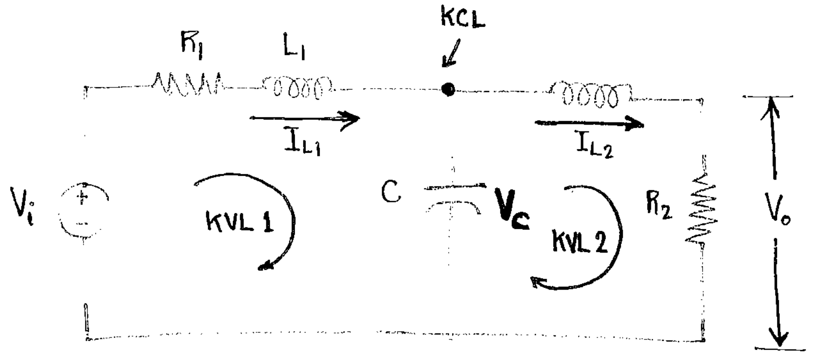
$$[y] = \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

STATE MODEL TO ELECTRICAL NETWORK

Procedure

- Select the state variables as voltage across capacitor, current through the inductor.
- The number of state variables equal to sum of inductors and Capacitors.
- write the independent KCL and KVL
  At capacitor junction, apply KCL and KVL through Inductor.
- The resultant equations should consist the state variables, differential state variables, Input and output variables.

Q. write the state model to the given electrical network.



$V_o = I_{L2} R_2 \longrightarrow$ (4)

state variables $= \begin{bmatrix} V_c \\ I_{L1} \\ I_{L2} \end{bmatrix}$

KCL

$I_C = I_{L1} - I_{L2}$

$C\dfrac{dV_c}{dt} = I_{L1} - I_{L2}$

$\boxed{\dot{V}_c = \dfrac{I_{L1}}{C} - \dfrac{I_{L2}}{C}} \longrightarrow$ (1)

KVL 1

$V_i = I_{L1}R_1 + L_1\dfrac{dI_{L1}}{dt} + V_c$

$\boxed{\dot{I}_{L1} = \dfrac{V_i}{L_1} - \dfrac{R_1}{L_1} I_{L1} - \dfrac{V_c}{L_1}} \longrightarrow$ (2)

KVL 2

$$V_C = L_2 \frac{dI_{L2}}{dt} + I_{L2} R_2$$

$$\boxed{\dot{I}_{L2} = \frac{V_C}{L_2} - \frac{R_2}{L_2} I_{L2}} \longrightarrow 3$$

$$\begin{bmatrix} \dot{V}_C \\ \dot{I}_{L1} \\ \dot{I}_{L2} \end{bmatrix} = \begin{bmatrix} 0 & 1/C & -1/C \\ -1/L_1 & -R_1/L_1 & 0 \\ 1/L_2 & 0 & -\frac{R_2}{L_2} \end{bmatrix} \begin{bmatrix} V_C \\ I_{L1} \\ I_{L2} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L_1 \\ 0 \end{bmatrix} [V_i]$$

$$[V_0] = \begin{bmatrix} 0 & 0 & R_2 \end{bmatrix} \begin{bmatrix} V_C \\ I_{L1} \\ I_{L2} \end{bmatrix}$$

TRANSFER FUNCTION FOR THE STATE MODEL

Transfer Function $\boxed{\mathbf{T \cdot F} = C [SI - A]^{-1} B + D}$

$$\boxed{T.F = C \frac{\text{Adj}[SI-A]}{|SI-A|} B + D}$$

$CE \Rightarrow |SI - A| = 0$ Gives Poles $\Longleftrightarrow$ Eigen values.

Q write the transfer function to the given state model.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \end{bmatrix} [u]$$

$$[y] = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\cancel{SI - A} = \cancel{\begin{bmatrix} s & 0 & 0 \\ 0 & s & \end{bmatrix}}$$

$$SI - A = \begin{bmatrix} s+2 & 3 \\ -4 & s-2 \end{bmatrix}$$

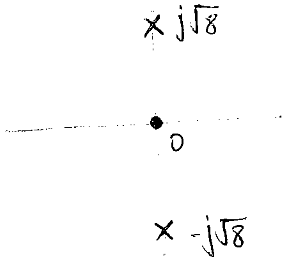
* To get SI−A, add 's' diagonally, and change sign of coefficients.

$$T.F = \frac{\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} s-2 & -3 \\ 4 & s+2 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix}}{s^2 - 4 + 12} + 0$$

$$= \frac{\begin{bmatrix} s-2+4 & -3+s+2 \end{bmatrix}}{s^2+8} \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$= \frac{\begin{bmatrix} s+2 & s-1 \end{bmatrix}}{s^2+8} \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \frac{8s+1}{s^2+8}$$

$$T.F = \frac{8s+1}{s^2+8}$$



Marginal system

Undamped system.

$$CE \Rightarrow s^2+8=0 \Rightarrow s = \pm j\sqrt{8}$$

Q

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$[y] = \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$sI - A = \begin{bmatrix} s & -3 \\ 2 & s+5 \end{bmatrix}$$

$$|sI-A| = s(s+5)+6 = \underline{s^2+5s+6}$$

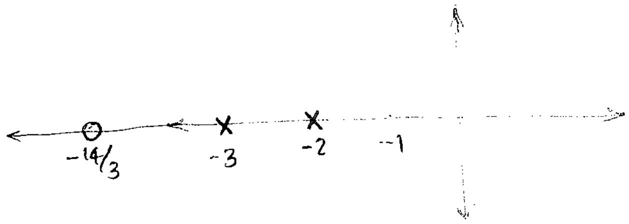
$$T.F = \frac{\begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} s+5 & 3 \\ -2 & s \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}}{s^2+5s+6} + 0$$

$$= \frac{\begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} s+8 \\ s-2 \end{bmatrix}}{s^2+5s+6} = \frac{2s+16+s-2}{s^2+5s+6}$$

$$T.F = \frac{3s+14}{s^2+5s+6}$$

CE $\Rightarrow$ $s^2+5s+6=0$

$(s+2)(s+3)=0 \quad s=-2,\ s=-3$



stable system. Under damped system.

SOLUTION TO THE STATE EQUATION

$\boxed{\dot{X} = AX + BU}$ $\longrightarrow$ Non-Homogeneous State Equation.

Method 1. : Laplace Transform Method :

$$sX(s) - X(0) = AX(s) + BU(s)$$

$$sX(s) - AX(s) = X(0) + BU(s)$$

$$X(s)\left[sI - A\right] = X(0) + BU(s)$$

$$X(s) = \left[sI-A\right]^{-1}X(0) + \left[\left[sI-A\right]^{-1}BU(s)\right]$$

Taking Inverse LT

$$\boxed{x(t) = \underbrace{\mathcal{L}^{-1}\left[\left[sI-A\right]^{-1}X(0)\right]}_{ZIR} + \underbrace{\mathcal{L}^{-1}\left[\left[sI-A\right]^{-1}BU(s)\right]}_{ZSR}} \quad (1)$$

* In the Response, ZIR is because of Initial conditions and ZSR is due to input.

Method II : CLASSICAL METHOD

$$\boxed{x(t) = \underbrace{\boxed{e^{At}}}_{\phi(t)} X(0) + \int_0^t \underbrace{\boxed{e^{A(t-\tau)}}}_{\phi(t-\tau)} B U(\tau)\, d\tau} \longrightarrow \text{(II)}$$

$$\underbrace{\phantom{e^{At}X(0)}}_{\text{ZIR}} \qquad \underbrace{\phantom{\int_0^t e^{A(t-\tau)} B U(\tau)\, d\tau}}_{\text{ZSR}}$$

⇒ Compare ZIR

STM ⇒ $\boxed{\phi(t) = e^{At} = \mathcal{L}^{-1}\left[[SI-A]^{-1}\right]}$ STATE TRANSITION MATRIX

$$\therefore \quad \boxed{[SI-A]^{-1} = L\left[\phi(t)\right] = \Phi(S)}$$

⇒ Compare ZSR

$$\boxed{\int_0^t \phi(t-\tau)\, B\, u(\tau)\, d\tau = L^{-1}\left[\phi(S) B U(S)\right]}$$

$$\boxed{x(t) = e^{At} X(0) + L^{-1}\left[\phi(S)\, B\, U(S)\right]} \longrightarrow \text{(III)}$$

I, II, III $\dot{X}$

(ii) Homogeneous state Equation. $(U=0)$

$$\boxed{\dot{X} = AX}$$

$$\boxed{x(t) = e^{At} X(0) = \text{ZIR} = L^{-1}[SI-A]^{-1} X(0)}$$

PROPERTIES OF STM $\phi(t)$ State Transition Matrix.

1. $\phi(0) = I$ (Identity Matrix)

2. $\phi^{K}(t) = \left(e^{At}\right)^{K} = e^{A(Kt)} = \phi(Kt)$

$$\phi^{-1}(t) = \phi(-t)$$

3. $\phi(t_1+t_2) = \phi(t_1)\,\phi(t_2)$

4. $\phi(t_2-t_1)\cdot\phi(t_1-t_0) = \phi(t_2-t_0)$

Q Find the Complete time response to the given state Model.

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} X, \quad X[0] = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad y = \begin{bmatrix} 1 & -1 \end{bmatrix} X$$

Soln ~~Its a~~ ~~OD: U=0~~

Complete time response means find y(t)

The given state model is homogenous U=0.

The solution is

$$x(t) = ZIR$$

$$= e^{At}X(0) = \phi(t)\,X(0)$$

$$STM \Rightarrow \phi(t) = e^{At} = L^{-1}\left[[SI-A]^{-1}\right] = L^{-1}\left[\frac{Adj\,[SI-A]}{|SI-A|}\right]$$

$$sI - A = \begin{bmatrix} s & -1 \\ +2 & s \end{bmatrix}$$

$$\frac{\text{Adj}\,[sI-A]}{|sI-A|} = \frac{\begin{bmatrix} s & 1 \\ -2 & s \end{bmatrix}}{s^2+2}$$

$$L^{-1}\left[[sI-A]^{-1}\right] = L^{-1}\begin{bmatrix} \frac{s}{s^2+2} & \frac{1}{s^2+2} \\ \frac{-2}{s^2+2} & \frac{s}{s^2+2} \end{bmatrix}$$

$$\phi(t) = \begin{bmatrix} \cos\sqrt{2}t & \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t & \cos\sqrt{2}t \end{bmatrix}$$

checking verification

Apply properties

$$\phi(0) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

satisfied.

$$x(t) = \phi(t)\,X(0)$$

$$= \begin{bmatrix} \cos\sqrt{2}t + \frac{1}{\sqrt{2}}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t + \cos\sqrt{2}t \end{bmatrix}$$

$y = [1\ -1]\,X$

$$y = [1 \quad -1]\begin{bmatrix} \cos\sqrt{2}t + 1/\sqrt{2}\sin\sqrt{2}t \\ -\sqrt{2}\sin\sqrt{2}t + \cos\sqrt{2}t \end{bmatrix} = \cancel{\cos\sqrt{2}t} + 1/\sqrt{2}\sin\sqrt{2}t + \sqrt{2}\sin\sqrt{2}t - \cancel{\cos\sqrt{2}t}$$

$$= 3/\sqrt{2}\ \sin\sqrt{2}t$$

Q. Obtain the time response for unit step input of the system given by

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} X + \begin{bmatrix} 0 \\ 5 \end{bmatrix} U, \quad X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad y = \begin{bmatrix} 0 & 1 \end{bmatrix} X$$

$$x(t) = e^{At} X(0) + L^{-1}\left[\cancel{X(0)}\ \Phi(s)\ B\,U(s) \right]$$

ZIR + ZSR

STM = $\phi(t)$

$$\{SI - A\} = \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}$$

$s(s+3) + 2 = s^2 + 3s + 2$

$(s+1)(s+2)$

$$[SI - A]^{-1} = \frac{\begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}}{s^2 + 3s + 2}$$

$$L^{-1}\left[[SI - A]^{-1}\right] = L^{-1}\begin{bmatrix} \frac{s+3}{s^2+3s+2} & \frac{1}{s^2+3s+2} \\ \frac{-2}{s^2+3s+2} & \frac{s}{s^2+3s+2} \end{bmatrix}$$

$$= L^{-1}\begin{bmatrix} \frac{s+3}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{bmatrix}$$

$A(s+2) + B(s+1) = s+3$

$A + B = 0$

$A = -B$

$2A + B = 3$

$-2B + B = 3$

$-B = 3$

$B = 3$

$A = 3$

$$= L^{-1}\begin{bmatrix} \frac{2}{s+1} - \frac{1}{s+2} & \frac{1}{s+1} - \frac{1}{s+2} \\ \frac{-2}{} + \frac{2}{} & \frac{-1}{} + \frac{2}{} \end{bmatrix}$$

$$\phi(t) = \underbrace{\begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -1e^{-t} + 2e^{-2t} \end{bmatrix}}_{\phi(t)}$$

$$ZIR = \phi(t)\,X(0)$$

$$= \begin{bmatrix} 2e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} \end{bmatrix}$$

$$ZSR = L^{-1}\left[\Phi(s)\ B\ U(s)\right]$$

$$\Phi(s) = [SI - A]^{-1} = \begin{bmatrix} \frac{s+3}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{bmatrix}$$

$$y(t) = \Phi(s)\ B\ U(s) = \begin{bmatrix} \frac{s+3}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{bmatrix} \begin{bmatrix} 0 \\ 5 \end{bmatrix} \frac{1}{s} \longrightarrow \text{unit impulse response}$$

$$ZSR = L^{-1}\begin{bmatrix} \frac{5}{s(s+1)(s+2)} \\ \frac{1}{(s+1)(s+2)} \end{bmatrix} = L^{-1}\begin{bmatrix} \frac{5}{2s} - \frac{5}{s+1} - \frac{5}{2(s+2)} \\ \frac{5}{s+1} - \frac{5}{s+2} \end{bmatrix}$$

$$ZSR = \begin{bmatrix} \frac{5}{2} - 5e^{-t} + \frac{5}{2}e^{-2t} \\ 5e^{-t} - 5e^{-2t} \end{bmatrix}$$

$$y(t) = ZIR + ZSR$$

$$= \begin{bmatrix} 2e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} \end{bmatrix} + \begin{bmatrix} 5/2 - 5e^{-t} + 5/2\, e^{-2t} \\ 5e^{-t} - 5e^{-2t} \end{bmatrix}$$

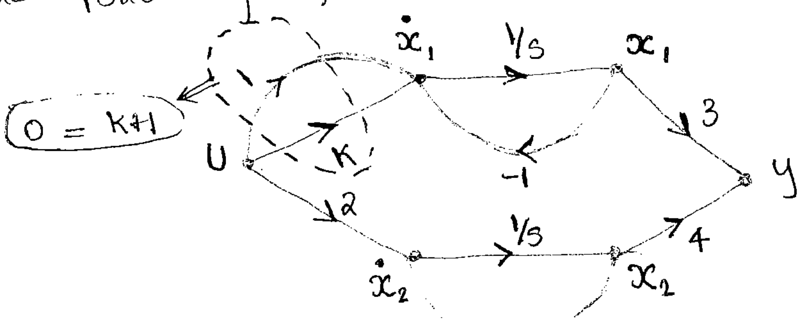
$$y(t) = \begin{bmatrix} 5/2 - 3e^{-t} + 3/2\, e^{-2t} \\ 3e^{-t} - 3e^{-2t} \end{bmatrix}$$

CONTROLLABILITY & OBSERVABILITY

CONTROLLABILITY: A system is said to be controllable if it is possible to transfer the initial states to the desired state in a finite time interval by the controlled input.

If the signal flow graph is given, to check the controllability, observe the path from input to state variable. If the path exists, then the system is controllable.

Q. Find the K value to becomes the system uncontrollable in the following system.



Because No path Exist from U to x_1

to becomes uncontrollable.

$K+1=0$

$K=-1$

KALAMAN'S TEST FOR CONTROLLABILITY (Q_c)

$$Q_c = \left[B \quad AB \quad A^2B \ldots\ldots\ldots A^{n-1}B \right]$$

$$\left.\begin{array}{l} \text{RANK OF } Q_c = \text{RANK OF } A \\ |Q_c| \neq 0 \end{array}\right\} \text{CONTROLLABLE.}$$

Q, $\dfrac{Y(S)}{U(S)} = \dfrac{1}{S^3+2S^2+3S+4}$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -3 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Q_c = \begin{array}{ccc} B & AB & A^2B \end{array}$$
$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

$A^2B = A \times (AB)$

ie, $A \times$ previous column.

$|Q_c| = 1(-1) = -1 \neq 0$ Hence Controllable.

OBSERVABILITY: A system is said to be observable if it is possible to determine the initial ~~sys~~ states of the system by observing the output in a finite Interval.

Q check the controllability and Observability.

$$\dot{X} = \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix} X + \begin{bmatrix} 1 \\ -1 \end{bmatrix} U \quad , \quad Y = \begin{bmatrix} 1 & 1 \end{bmatrix} X$$

$$Q_c = \begin{array}{c} \begin{array}{cc} B & AB \end{array} \\ \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix} \end{array}$$

$|Q_c| = \underline{\underline{0}}$ ∴ Not controllable.

KALAMAN'S TEST FOR OBSERVABILITY (Q_0)

$$\boxed{Q_0 = \left[C^T \;\; A^T C^T \;\; (A^T)^2 C^T \;\cdots\cdots\; (A^T)^{n-1} C^T \right]}$$

RANK OF Q_0 = RANK OF A

$|Q_c| \neq 0$

→ OBSERVABLE.

In above qu,

$$Q_o = \begin{array}{c} C \\ CA \end{array}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow |Q_o| = 0 \quad \text{Not observable.}$$

Q,

$$\dot{x}_1 = -2x_1 + x_2 + U$$

$$\dot{x}_2 = -x_2 + U$$

$$y = x_1 + x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} U, \quad Y = \begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$Q_c = \begin{array}{c} \begin{array}{cc} B & AB \end{array} \\ \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \end{array} \Rightarrow |Q_c| = 0$$

Not controllable.

$$Q_o = \begin{array}{c} C \\ CA \end{array}\begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix} \Rightarrow |Q_o| = 2$$

Observable.

COMPENSATORS AND CONTROLLERS

PURPUSE :

* If the system is unstable, then required a compensator or controller to make it stable and get the required performance.

* If the system is stable, then also required a compensator or controller to get the ~~det~~ desired performance.

* In Type '0' and Type '1' system, the stable operation is possible by adjusting the system Gain.

* In Type '2' and higher order systems are generally Unstable. In this case we must used the Lead Compensator or PD ~~compens~~ Controller (Proportional derivative) to make it stable and achieve the required performance.

consider type 1 system

$$G(s) = \frac{K}{s(s+2)(s+4)} \qquad H(s) = 1$$

$$CE \Rightarrow s^3 + 6s^2 + 8s + K = 0$$

just by varying K directly, we can change stability.

Consider type 2 system,

$$G(s) = \frac{K}{s^2(s+2)(s+4)}, \quad H(s) = 1$$

$$CE \Rightarrow s^4 + 6s^3 + 8s^2 + K$$

whatever variations we make in K, system is unstable.

$\Rightarrow$ So use PD controller.

$$G(s) = \frac{K(K_p + K_D s)}{s^2(s+2)(s+4)} \quad , H(s) = 1$$

$$CE \Rightarrow s^4 + 6s^3 + 8s^2 + KK_D s + \underbrace{KK_p}_{} = 0$$

s^1 term present.

So by varying K, K_D, K_p we can make system Stable.

COMPENSATOR

A compensator is an electrical network which adds a finite pole and finite zeros to the system.

There are three types of compensators.

(i) Lead Compensator.

(ii) Lag Compensator.

(iii) Lag Lead Compensator.

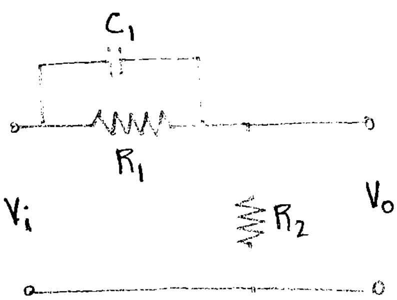
Lead Lag Compensator not used common

	FILTER		POLE-ZERO ORDER
LEAD $\longrightarrow$	HIGH	$\rightarrow$ +ve angle	X O
LAG $\longrightarrow$	LOW	$\rightarrow$ −ve angle	O X
LAG-LEAD $\longrightarrow$	BAND STOP (B.S)	$\rightarrow (\tau_{lag} > \tau_{lead})$	Lead: X O, Lag: O X
LEAD-LAG $\longrightarrow$	BAND PASS (B.P)	$\rightarrow (\tau_{lead} > \tau_{lag})$	Lag: O X, Lead: X O

LEAD COMPENSATOR

* when a sinusoidal input is applied to a network, it produce the sinusoidal steady state output, having a phase lead w.r.t input. Then the network is called Lead compensator.

* The lead compensator improves the transient performance. And also improves the system stability.



steps

① TF

② Time constant form

③ Poles & zeros in s plane.

④ Bode plot.

⑤ Identity filter.

⑥ $\omega_m, \phi_m, M|_{\omega_m}$.

① $$\frac{V_o(s)}{V_i(s)} = \frac{R_2}{\left(\frac{R_1}{sC_1R_1+1}\right) + R_2}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2(sC_1R_1+1)}{R_1 + R_2(1+sC_1R_1)}$$

$$= \frac{R_2(sC_1R_1+1)}{R_1+R_2+sC_1R_1R_2}$$

$$= \frac{R_2(1+sC_1R_1)}{(R_1+R_2)\left(1+\frac{R_2}{R_1+R_2}sC_1R_1\right)}$$

Let $\alpha \Rightarrow$ LEAD CONSTANT $\quad \boxed{\alpha = \frac{R_2}{R_1+R_2} < 1}$

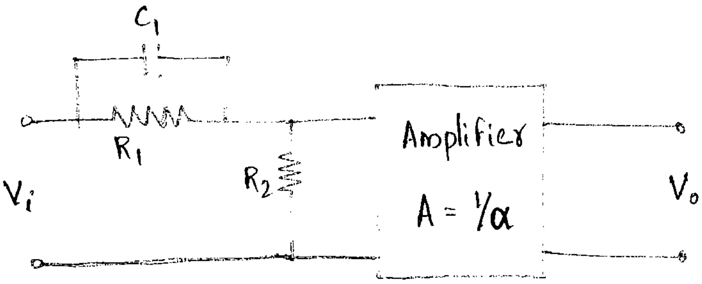
$\tau \Rightarrow$ TIME CONSTANT $\boxed{\tau = R_1C_1}$ $\quad \alpha_{optimum} = 0.1$

$$\frac{V_o(s)}{V_i(s)} = \frac{\alpha(1+\tau s)}{(1+\alpha\tau s)}$$

~~Here Lead compensator makes the system by stable by but~~

Lead compensator creates attenuation in the system ($\alpha < 1$)

To eliminate the attenuation, we need to use an amplifier of Gain $1/\alpha$.

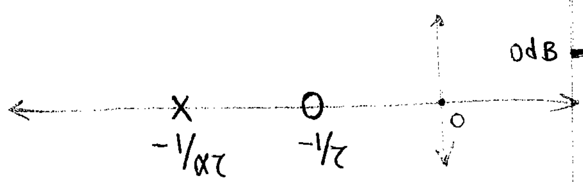


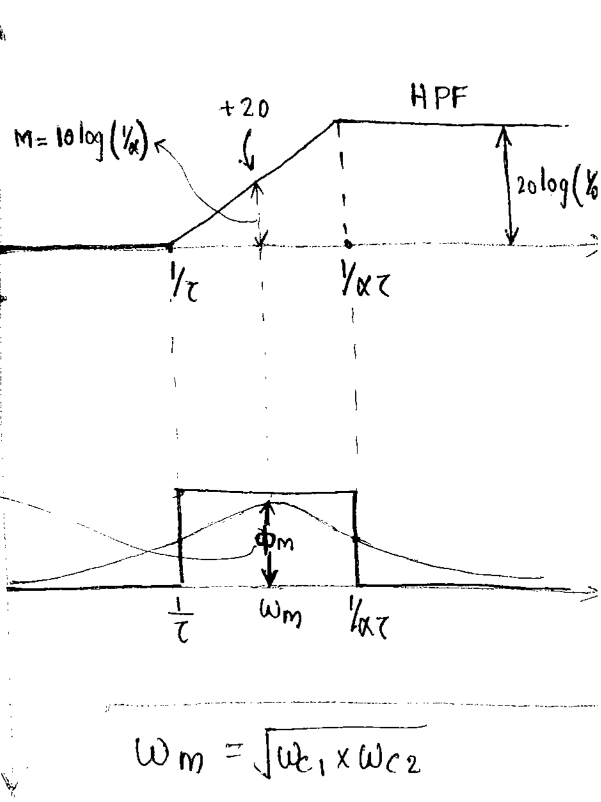
* The α value should not be less than 0.07. ~~oth~~

Finally reduced Transfer function is

$$\boxed{\frac{V_o(s)}{V_i(s)} = \left(\frac{1+\tau s}{1+\alpha\tau s}\right)}$$

BODE PLOT





$$\phi_m = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)$$

$$\omega_m = \sqrt{\omega_{c1} \times \omega_{c2}}$$

$$= \sqrt{\frac{1}{\tau} \times \frac{1}{\alpha\tau}} = \frac{1}{\tau\sqrt{\alpha}} \text{ rad/}$$

* The lead compensator is a Highpass filter, hence the BW of the system increases. As BW increases, the rise time increases ⇒ Transient performance is improved.

* With lead compensator, the damping of the system improved; that means percentage overshoot decreases and settling time also decreases.

* Improves the phase margin and Gain margin, Hence relative stability improved.

* The Lead compensator similar to the P.D controller.

DISADVANTAGE

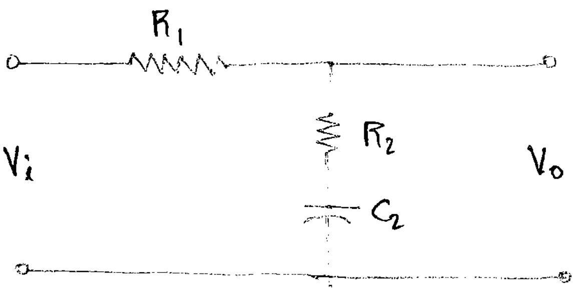
* The lead compensator creates attenuation in the system. Hence ~~re q~~ we require to add an amplifier with the gain of $1/\alpha$ which adds the cost and space

* The lead compensator is High pass filter. So the noise is entered to system. So the signal to noise strength is poorer.

* The maximum phase lead given by lead compensator is 60°. If require more than that, we required to add a multisection compensator.

LAG COMPENSATOR

* When a sinusoidal input applied to a network. It produce the steady state output with phase lag with respect to input. Then the network is called Lag compensator.



S1: Find the transfer function.

$$\frac{V_o(S)}{V_i(S)} = \frac{R_2 + 1/sC_2}{R_1+R_2+1/sC_2}$$

Multiply & ÷ by R_2

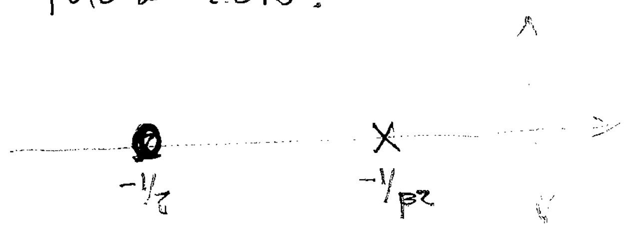
$$\frac{V_o(S)}{V_i(S)} = \frac{sC_2R_2+1}{1+sC_2R_2\frac{(R_1+R_2)}{R_2}}$$

Let $\beta \Rightarrow$ LAG CONSTANT $\boxed{\beta = \frac{R_1+R_2}{R_2} > 1}$ $\beta_{optimum} = 10$

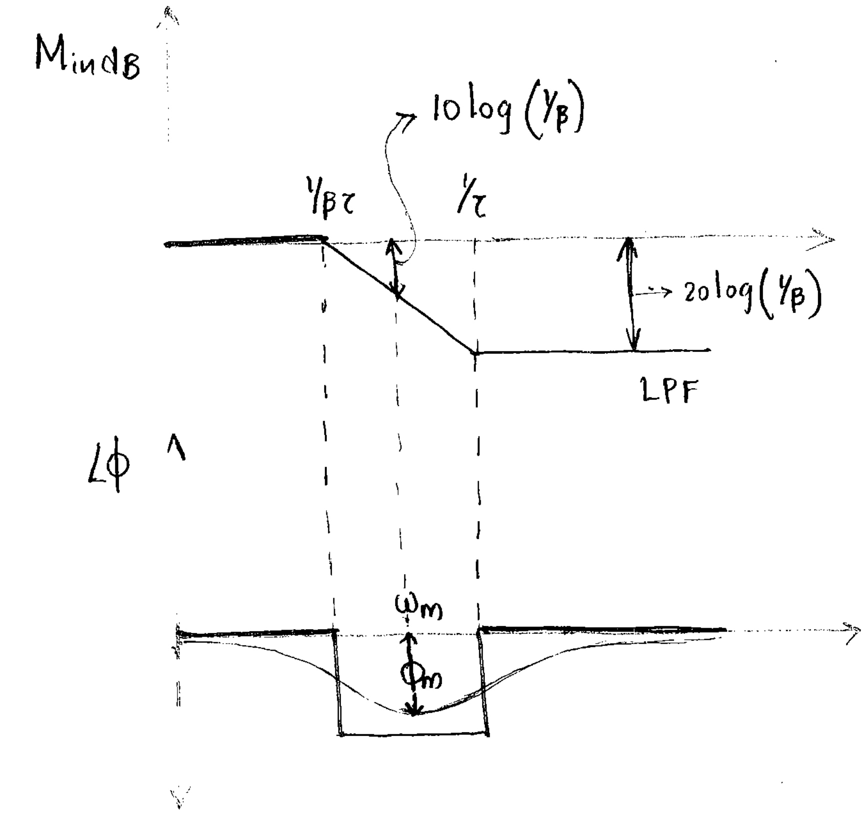
$\tau \Rightarrow$ LAG TIME CONSTANT $\boxed{\tau = R_2C_2}$

Equ becomes $\boxed{\frac{V_o(S)}{V_i(S)} = \frac{1+\tau s}{1+\beta\tau s}}$

S2: Locating pole & zero.



53 : Bode plot.



$\omega_m \rightarrow$ Frequency at which minimum phase shift.

$\phi_m \rightarrow$ Minimum phase shift.

$$\omega_m = \sqrt{\omega_{c_1} \times \omega_{c_2}}$$

$$\omega_m = \sqrt{\frac{1}{\beta\tau} \times \frac{1}{\tau}}$$

$$\boxed{\omega_m = \frac{1}{\tau\sqrt{\beta}} \text{ rad/sec.}}$$

$$\boxed{\phi_m = \sin^{-1}\left(\frac{\beta-1}{\beta+1}\right)}$$

Effects of Lag Compensator.

* The Lag compensator is a lowpass filter, which eliminates the noise in the system. So the signal to noise ratio at the output is improved.

* The Lag compensator improves the steady state performance

* with Lag compensator, the Bandwidth of the system is decreased.

* As bandwidth decreases, rise time increases. The system ~~Pro~~ response becomes slow.

* In Lag compensator, ~~at~~ the attenuation characteristics are used for the compensation whereas there is no use of phase lag characteristics of the lag compensator.

* With Lag compensator, the system become very sensitive with parameter variation

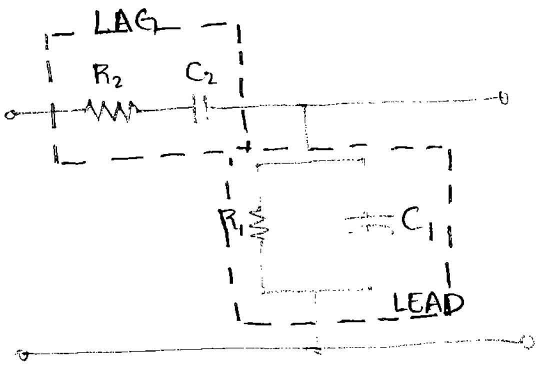
* The Lag compensator similar to PI controller

LAG - LEAD COMPENSATOR

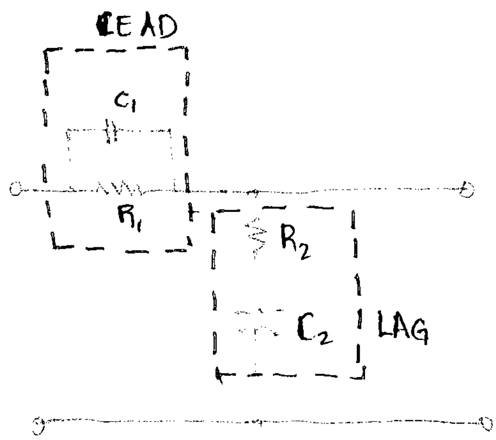
* The lag lead compensator gives the very quick response, good static accuracy. (steady state error decreases, rise time also decreases).

LAG - LEAD COMPENSATOR

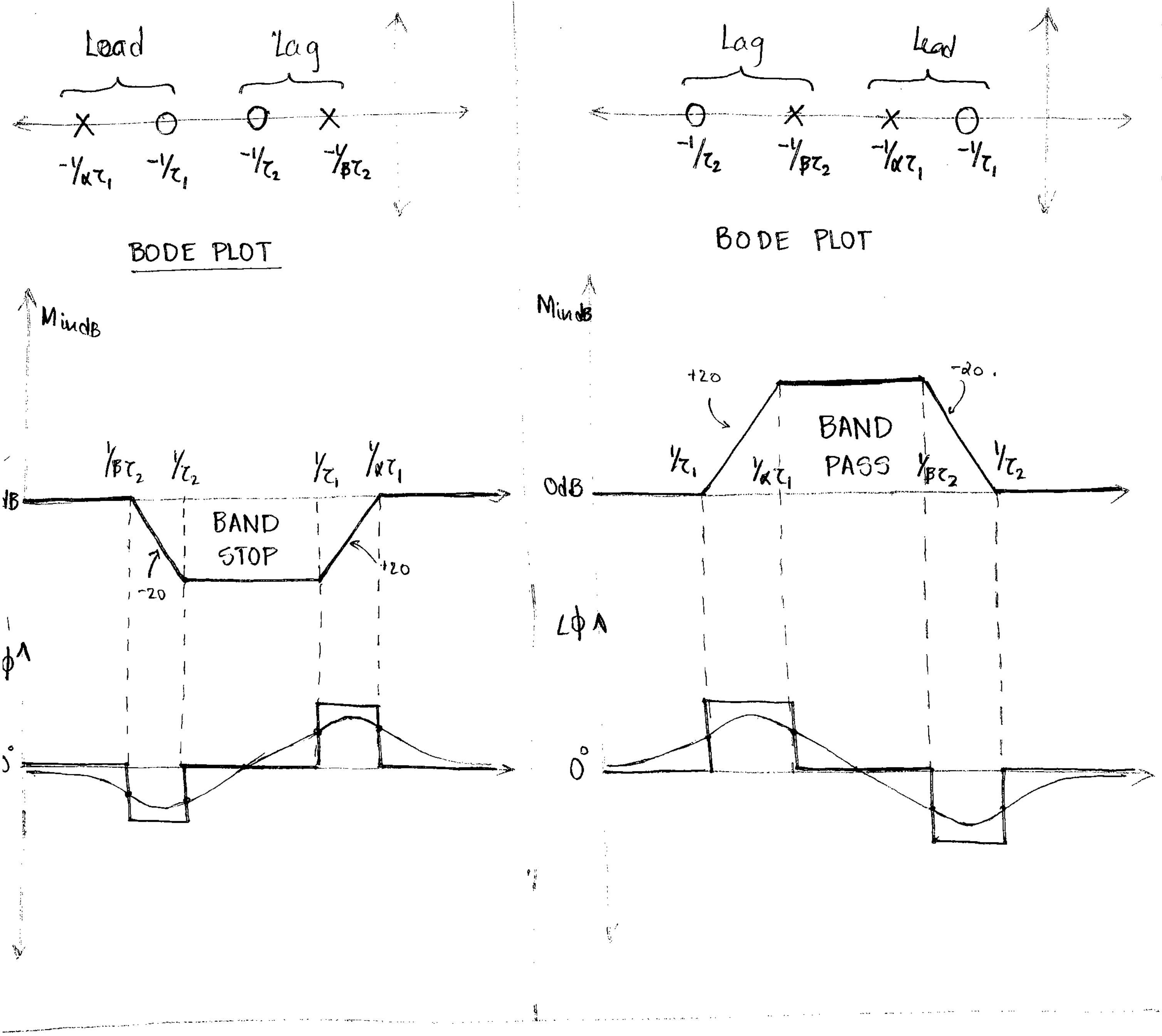
$$(\tau_{lag} > \tau_{lead})$$



LEAD - LAG COMPENSATOR



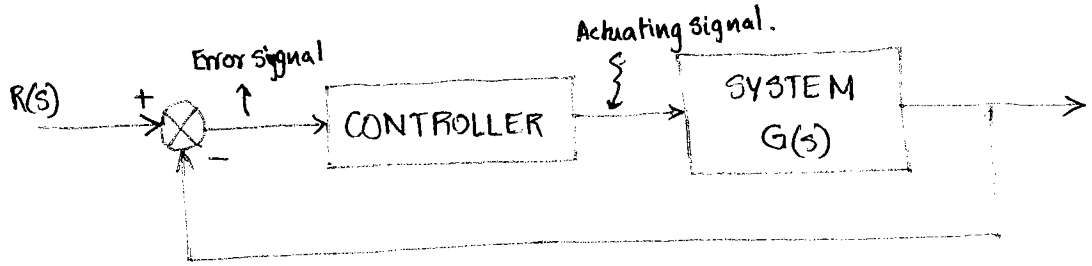
$$\frac{V_o(s)}{V_i(s)} = \underbrace{\left(\frac{1+\tau_1 s}{1+\alpha\tau_1 s}\right)}_{\text{Lead}}\underbrace{\left(\frac{1+\tau_2 s}{1+\beta\tau_2 s}\right)}_{\text{Lag.}}$$



PID CONTROLLER

* A controller is a device which is used to control the transient and steady state performance as per the requirement.
* The Best system demands smallest Rise time, Smallest settling time, smallest peak overshoot, smallest steady state error.
* To get the above requirements, we required to ~~had~~ add a controller to the system.

* A block diagram with the controller is shown in figure.



PROPORTIONAL CONTROLLER

PURPOSE : To change the transient response as per the requirements

The transfer function of proportional controller $= \boxed{K_P}$

For Eg:

Let $G(S)\Big|_{WOC} = \dfrac{1}{S(S+10)}$ CLTF $= \dfrac{1}{S^2+10S+1}$ $\omega_n = 1$ rad/s, $\xi = 5$ (over damped)

$G(S)\Big|_{\text{with controller}} = \dfrac{K_P}{S(S+10)}$ CLTF $= \dfrac{K_P}{S^2+10S+K_P}$ Let $K_P = 100$, $\omega_n = 10$, $\xi = 0.5$ (under damped)

* ~~So by changing~~ So by ~~changing~~ selecting proper value of K_P, we can attain the required response.

Disadvantages:

* The proportional controller cannot eliminate complete error in the system.
* As K_P value increases, to get the better transient response, the damping ratio ξ decreases. Hence the percentage of peak overshoot increase. So the system becomes the less stable and it gives the more oscillation.

GATE

Q Find the maximum phase lead angle occurs and at what frequency it occurs

$$\frac{V_o(s)}{V_i(s)} = \frac{s+1}{s+10}$$

$$= \frac{1+s}{10(1+0.1s)} = \frac{1+\tau s}{1+\alpha\tau s}$$

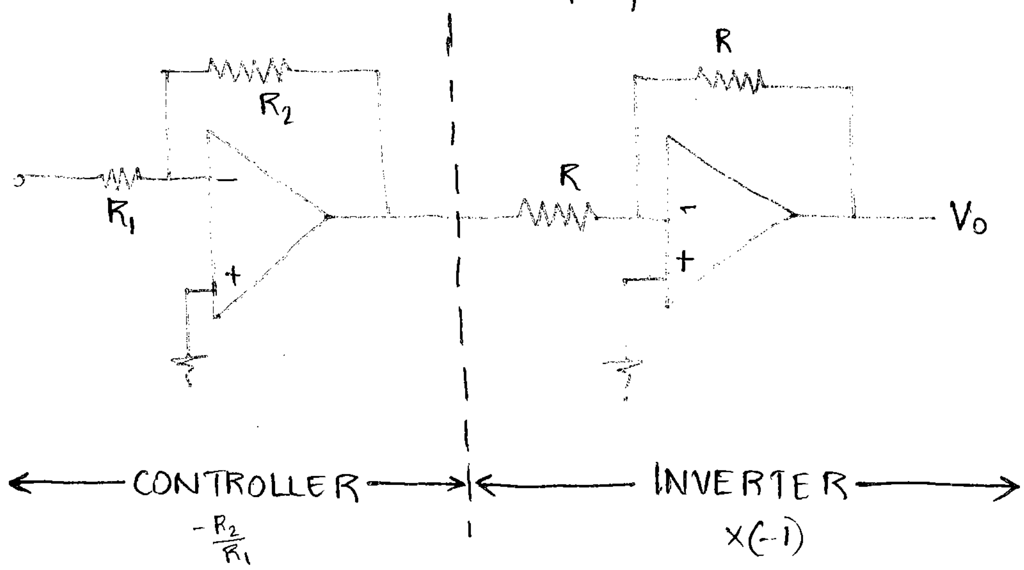
neglect

$$\tau = 1, \ \alpha = 0.1$$

$$\omega_m = \frac{1}{\tau\sqrt{\alpha}} = \frac{1}{1\sqrt{0.1}} = \sqrt{10} \text{ rad/s}.$$

$$\phi_m = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right) = \sin^{-1}\left(\frac{1-0.1}{1.1}\right) = \underline{\underline{54.9^\circ}}$$

The practical circuit of proportional controller is given by



$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_1} = K_p}$$

INTEGRAL CONTROLLER [RESET CONTROLLER]

PURPOSE : To decrease the steady state error.

The transfer function of Integral controller is $\boxed{\frac{K_I}{s}}$

* The Integral controller added one pole at origin which increases the type of the system. As type increases, the steady state error decreases, But the system stability is effected.

For eg:

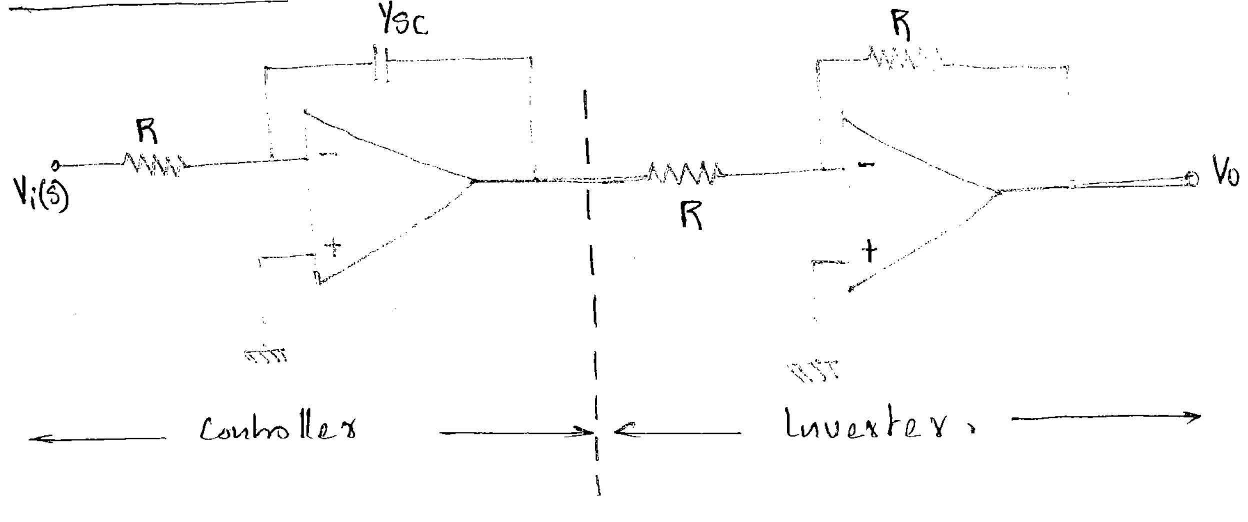
$$G(s)\Big|_{\substack{\text{without} \\ \text{controller}}} = \frac{1}{s(s+10)} \quad \text{Type 1} \quad \xrightarrow{CE} \quad s^2+10s+1=0 \quad \text{Stable.}$$

$$G(s)\Big|_{\substack{\text{with} \\ \text{controller}}} = \frac{K_I}{s^2(s+10)} \quad \text{Type 2}\uparrow \quad \xrightarrow{CE} \quad \underbrace{s^3+10s^2+K_I=0}_{s^1 \text{ term missing}} \quad \text{Unstable}$$

$e_{ss}\downarrow \times$

* The Integral controller effects the system stability. Hence before using the Integral controller, we require to check the system stability. If the system stability is effected, as in above case, the integral controller are not used.

PRACTICAL CKT



DERIVATIVE CONTROLLER [RATE CONTROLLER]

PURPOSE : To improve the system stability.

* The Transfer function of derivative controller is $= \boxed{K_D S}$

* The derivative controller added one zero at origin. Hence the type of system decrease. As type decreases, the system stability is improved. But the steady state error increases. (The system becomes less accurate).

For eg:

$G(S)\Big|_{\text{without controller}} = \dfrac{1}{S^2(S+10)}$ Type 2 $\xrightarrow{CE} S^3+10S^2+1=0$ Unstable

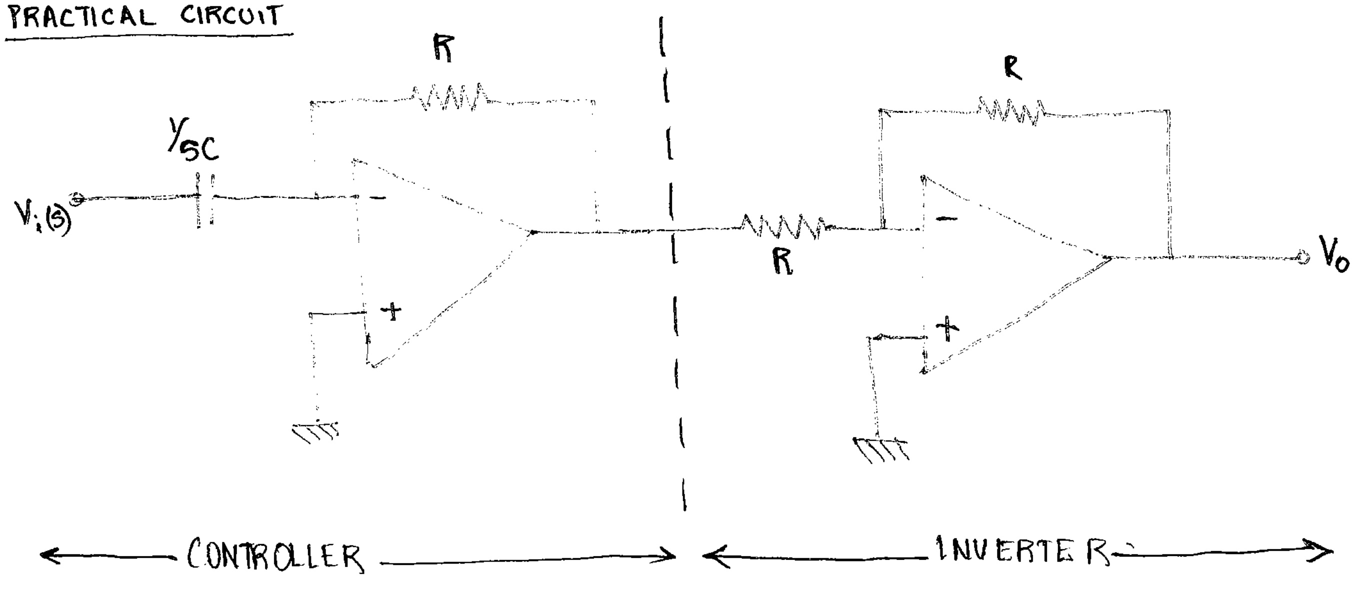
$G(S)\Big|_{\text{with controller}} = \dfrac{K_D}{S(S+10)}$ Type 1 $\xrightarrow{CE} S^2+10S+K_D=0$ Stable

But system becomes less accurate.

* The best example for derivative controller is Tacho meter.

T.F = KS

PRACTICAL CIRCUIT



$$\frac{V_o(s)}{V_i(s)} = \frac{R}{1/sC} = sCR = K_D s = \tau_d s$$

where $K_D = \tau_D = RC$

P-I CONTROLLER

PURPUSE : To decrease the steady state error without effecting the system stablity.

The Transfer function of PI controller is

$$\boxed{T.F = K_p + \frac{K_I}{s} = \frac{sK_p + K_I}{s}}$$

* The PI controller added one pole at origin. Hence type of the system increases. As the type of the system increases, the steady state error decreases.

* The PI controller added one finite zero in the left hand sign which avoid the effect on system stability.

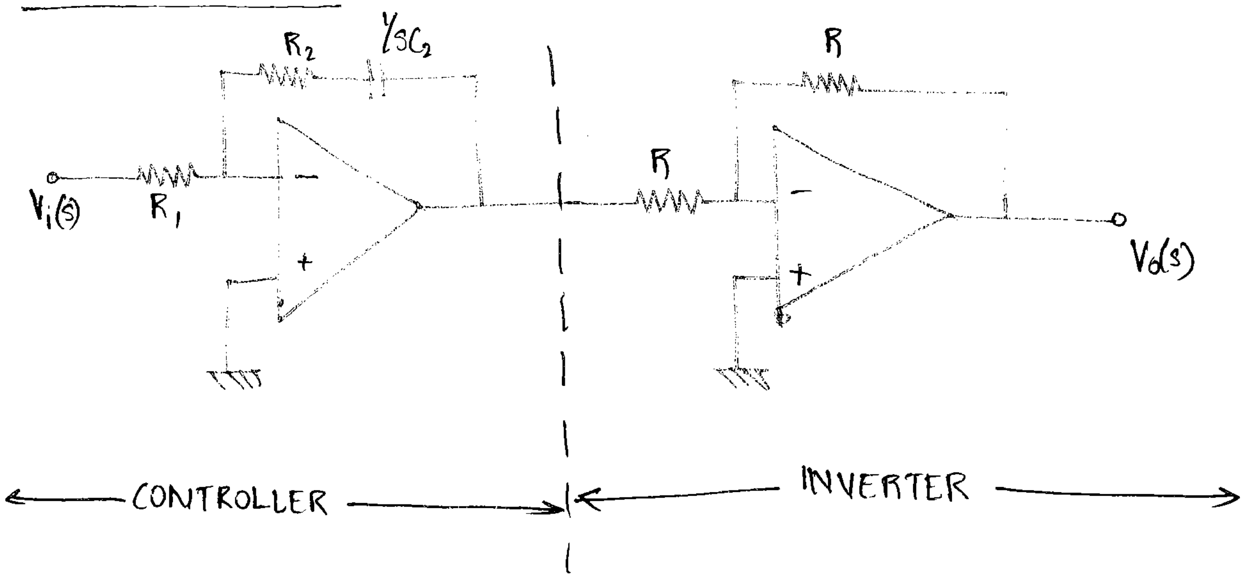
For eg:

$$G(s)\Big|_{\text{without controller}} = \frac{1}{s(s+10)} \quad \text{Type 1} \quad \xrightarrow{CE} s^2+10s+1 \quad \text{stable}$$

Stability effect

$$G(s)\Big|_{\text{with controller}} = \frac{sK_p + K_I}{s^2(s+10)} \quad \text{Type 2}\uparrow \ \ e_{ss}\downarrow \quad \xrightarrow{CE} s^3+10s^2+sK_p+K_I=0 \quad \text{stable}$$

PRACTICAL CIRCUIT



$$\frac{V_o(s)}{V_i(s)} = \frac{R_2 + 1/sC_2}{R_1} = \frac{R_2}{R_1} + \frac{1}{sR_2C_2} = \left(K_P + \frac{K_I}{s}\right)$$

$$\text{where } K_P = \frac{R_2}{R_1}, \quad K_I = \frac{1}{R_1R_2}$$

Effects of PI Controller

* PI controller as Low pass Filter. The steady state performance is increased. (steady state error decreases).

* The B.W of the system decreases. As Bandwidth decrease, rise time increases.

* The damping ratio increases. Hence percentage of peak overshoot decreases.

* Gain Margin and phase Margin increases. Hence the relative stability is improved.

P-D CONTROLLER

PURPOSE:

The PD controller Improves the system stability without effecting steady state error.

* Transfer function of PD controller is

$$\boxed{T.F = K_P + K_D S}$$

* The PD controller added one finite zero in the left hand side. Hence the system stability is improved.
* The PD controller not changes the type. Hence the steady state error is not effected.

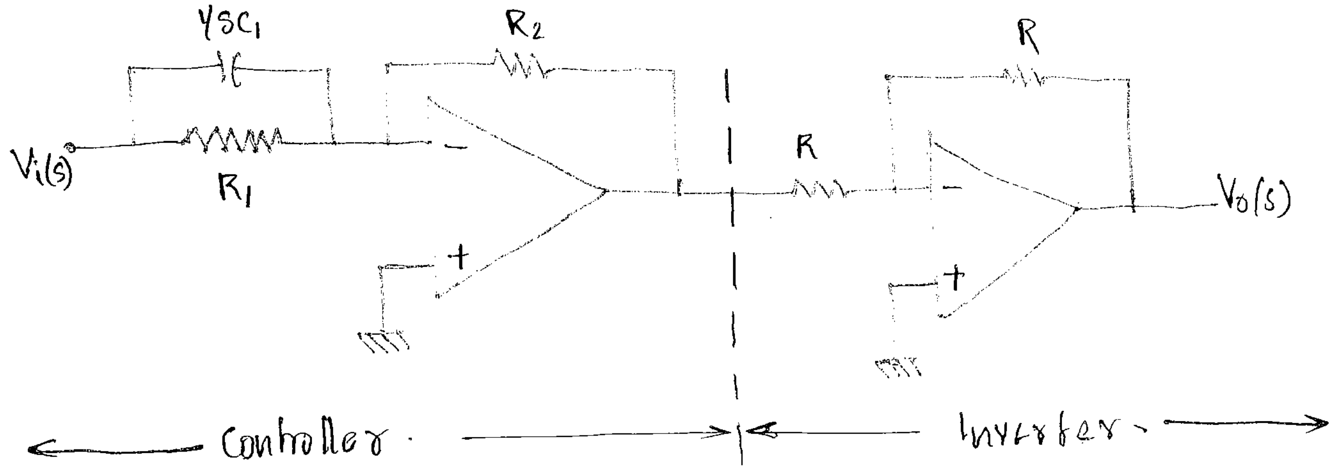
For eg:

$$G(s)\Big|_{\text{without controller}} = \frac{1}{s^2(s+10)} \quad \text{Type 2} \quad \xrightarrow{CE} s^3 + 10s^2 + 1 = 0 \ \text{(Unstable)}$$

$$G(s)\Big|_{\text{with controller}} = \frac{(sK_D + K_P)}{s^2(s+10)} \quad \text{Type 2} \quad \xrightarrow{CE} s^3 + 10s^2 + sK_D + K_P = 0 \ \text{(Stable)}$$

No ~~change~~ effect in ess because no change in type.

PRACTICAL CIRCUIT



$$\frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_1/(sC_1R_1+1)} = \frac{R_2(1+sC_1R_1)}{R_1} = \left(\frac{R_2}{R_1} + sR_2C_1\right)$$

$$= K_p + K_D s$$

Effects of PD controller.

* PD controller is a highpass filter. Hence noise is entered into the system.
* The Bandwidth of the system increases, As BW increases, rise time decreases. Hence the transient performance improved.
* The damping ratio with PD controller is $\boxed{\left(\xi + \frac{\omega_n K_D}{2}\right)}$

$$\boxed{\xi_{PD} = \xi + \frac{\omega_n K_D}{2}}$$

* with PD controller, ξ decreases, percentages of peak overshoot increases.
* Relative stability of system Improved since gain margin and phase margin improved.

PURPOSE

To improve the system stability and decrease the steady state error.

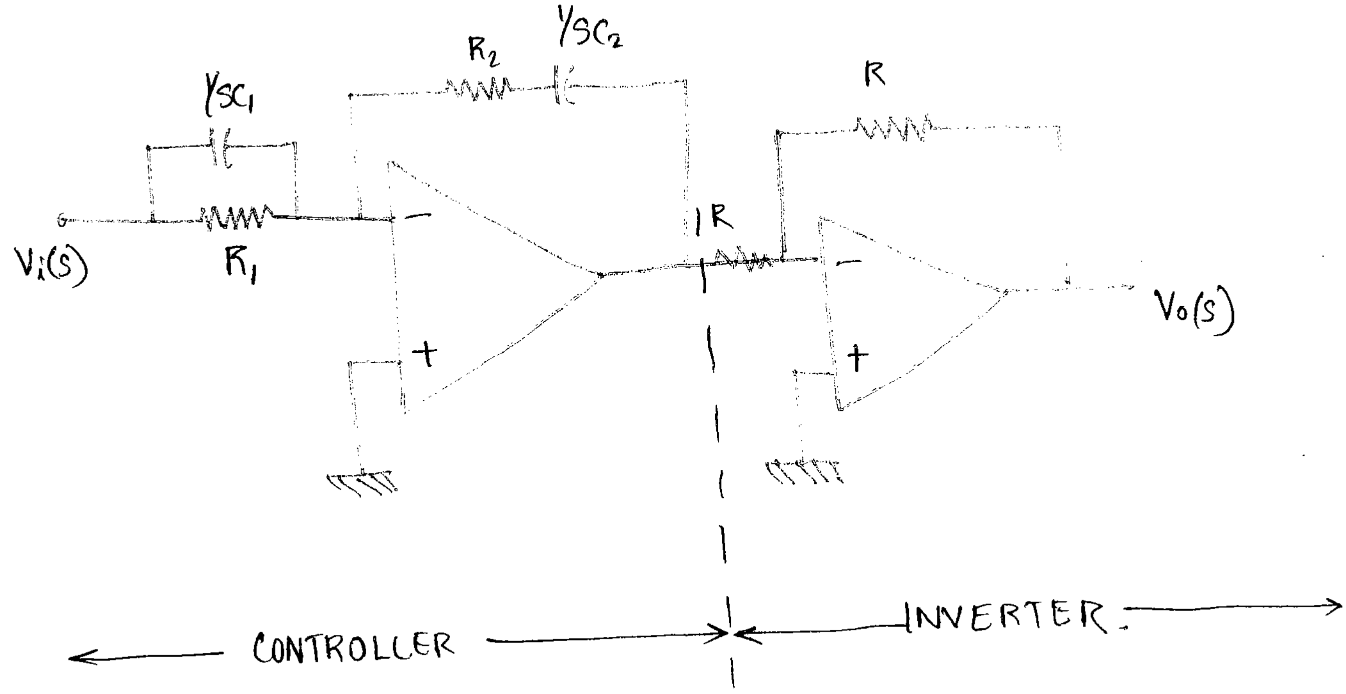
The transfer function of PID controller is

$$\boxed{T.F = \left(K_P + \frac{K_I}{S} + K_D S\right) = \frac{K_D S^2 + K_P S + K_I}{S}}$$

* The PID controller added one pole at origin. Hence type of the system increases. As the type increases, steady state error decrease.

* The PID controller added two finite zeros in the left hand side. One finite zero avoid the effect on system stability effect of pole. Another finite zero ~~incre~~ improves the system stability.

PRACTICAL CIRCUIT



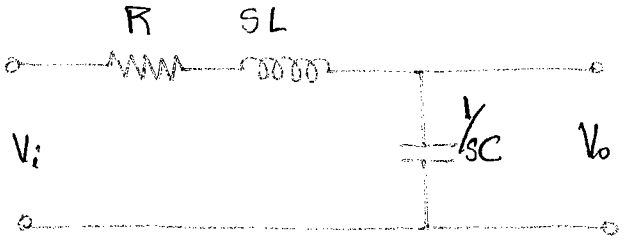
$$\frac{V_o(s)}{V_i(s)} = \frac{R_2 + \frac{1}{sC_2}}{\frac{R_1}{sC_1R_1+1}} = \frac{(sC_2R_2+1)(sC_1R_1+1)}{sC_2R_2} = \underbrace{\left(\frac{R_2}{R_1} + \frac{C_1}{C_1}\right)}_{K_P} + \underbrace{\left(\frac{1}{sC_2R_2}\right)}_{\frac{K_I}{s}} + \underbrace{\left(sC_1R_2\right)}_{K_D s}$$

~~The Gene~~

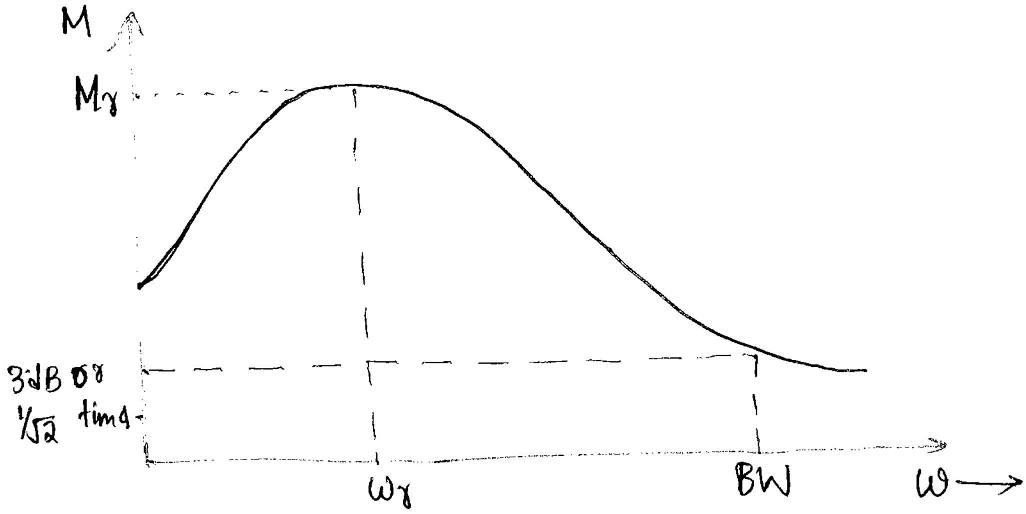
FREQUENCY DOMAIN SPECIFICATIONS

The General frequency response of RLC circuit is shown in figure. fas

RLC → as we need Low pass Ckt.



$$\boxed{\xi = \frac{R}{2}\sqrt{\frac{C}{L}}} \qquad \boxed{Q = \frac{1}{2\xi} = \frac{1}{R}\sqrt{\frac{L}{C}}}$$



RESONANT FREQUENCY: The frequency at which maximum magnitude occurs

$$\boxed{\omega_r = \omega_n\sqrt{1-2\xi^2}}\ \text{rad/sec}$$

The frequency domain specifications are valid when $\xi < \frac{1}{\sqrt{2}}$

RESONANT PEAK: It is the maximum ~~peak~~ magnitude occurs at resonant frequency.

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

BANDWIDTH: It is the range of frequencies at which the magnitude droped by 3 dB or $1/\sqrt{2}$ time from the maximum value.

$$\boxed{BW = \omega_n\sqrt{1-2\xi^2+\sqrt{2-4\xi^2+4\xi^4}}}\ \text{rad/sec}$$

SENSITIVITY

Sensitivity is used to describe the relative variations in parameters like G(S), H(s)

Sensitivity of T.F w.r.t to G(S) = $S_G^T = \frac{\%\text{ of change in T.F}}{\%\text{ of change in } G(s)}$

$$S_G^T = \left(\frac{\partial T/T}{\partial G/G}\right) = \frac{G}{T}\left(\frac{\partial T}{\partial G}\right)$$

$$\boxed{S_G^T = \frac{G}{T}\left(\frac{\partial T}{\partial G}\right)}$$

Similarly $$\boxed{S_H^T = \frac{H}{T}\left(\frac{\partial T}{\partial H}\right)}$$

Q. Find the sensitivity of openloop and closed loop system stability w.r.t. the variations in

(i) $G(s)$

(ii) $H(s)$

CL system $\rightarrow$ $$T = \frac{G}{1+GH}$$

$$S_G^T = \left(\frac{G}{T}\right)\left(\frac{\partial T}{\partial G}\right) = \frac{\cancel{G}}{\left(\cancel{\frac{G}{1+GH}}\right)}\left(\frac{1(1+GH) - GH}{(1+GH)^2}\right) = \frac{1}{1+GH}$$

$$\boxed{S_G^T = \frac{1}{1+GH}}$$

$$S_H^T = \left(\frac{H}{T}\right)\left(\frac{\partial T}{\partial H}\right) = \frac{H}{\frac{G}{1+GH}}\left(\frac{0 - G \times G}{(1+GH)^2}\right) = \frac{-GH}{1+GH}$$

$$\boxed{S_H^T = \frac{-GH}{1+GH}}$$

$$\boxed{S_H^T > S_G^T}$$

feedback N/w more sensitive than forward path.

open loop system.

$$T = G$$

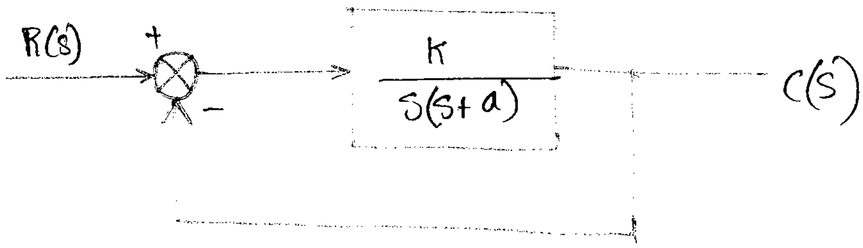
$$S_G^T = \frac{G}{T}\left(\frac{\partial T}{\partial G}\right) = \frac{G}{G} = 1$$

open loop system is more sensitive than Closed loop system

Q. Find the sensitivity of the given ~~OL~~ system, ~~w.r.t~~ w.r.t

(i) K

(ii) a



$$\text{CLTF},\ T = \frac{K}{s^2+as+K}$$

$$S_K^T = \frac{(\partial T/T)}{(\partial K/K)} = \left(\frac{K}{T}\right)\left(\frac{\partial T}{\partial K}\right)$$

$$= \frac{K}{K/(s^2+as+K)}\left(\frac{1(s^2+as+K) - K\cdot 1}{(s^2+as+K)^2}\right)$$

$$\boxed{S_K^T = \frac{s^2+as}{s^2+as+K}}$$

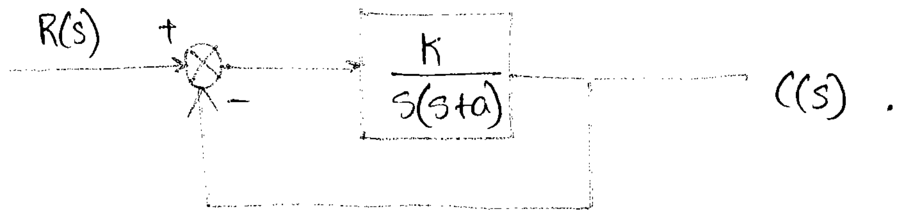
$$S_a^T = \frac{(\partial T/T)}{(\partial a/a)} = \left(\frac{a}{T}\right)\left(\frac{\partial T}{\partial a}\right)$$

$$= \frac{a}{\left(\frac{K}{s^2+as+K}\right)} \left(\frac{0-Ks}{(s^2+as+K)^2}\right)$$

$$\boxed{S_a^T = \frac{-as}{s^2+as+K}}$$

Q Find the Sensitivity of the steady state error w.r.t variations in (i) K

(ii) a for unit ramp input.



Soln

For steady state error we need G(s)

$$G(s) = \frac{K}{s(s+a)}$$

Type → 1 $\quad e_{ss} = \frac{A}{K} = \frac{1}{K/a} = \underline{\underline{\frac{a}{K}}}$

$$S_K^{e_{ss}} = \left(\frac{\partial e_{ss}/e_{ss}}{\partial K/K}\right) = \left(\frac{K}{e_{ss}}\right)\left(\frac{\partial e_{ss}}{\partial K}\right) = \left(\frac{K}{a/K}\right) \times \left(-a/K^2\right) = \underline{\underline{1}}$$

$$S_a^{e_{ss}} = \left(\frac{\partial e_{ss}/e_{ss}}{\partial a/a}\right) = \left(\frac{a}{e_{ss}}\right)\left(\frac{\partial e_{ss}}{\partial a}\right) = \left(\frac{a}{a/K}\right)(1/K) = 1$$